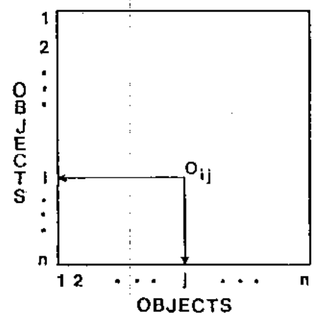
Asymmetric Multidimensional Scaling

Input: Single nxn square asymmetric matrix O of dissimilarities ($o_{ij} \neq o_{ji}$)

Number of ways: 2

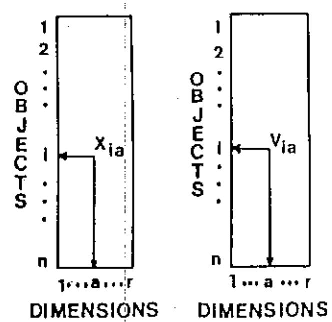
Number of modes: 1



Output: Two rectangular matrices: X is an nxr matrix of coordinates (x_{ia} is coordinate of i'th object on a'th dimension); and V is an nxr matrix of object weights (v_{ia} is weight of object i on dimension a).

Number of ways: 2

Number of components: 2



Model: Asymmetric Euclidian

$$o_{ij} \cong t[d_{ij}]$$

$$d_{ij} = \sqrt{\sum_a^r v_{ia}(x_{ia}-x_{ja})^2}$$

FIG. 6.2. Schematic of asymmetric multidimensional scaling

tangular GEMs can only be applied to rectangular data, thus the five models generate five analyses.

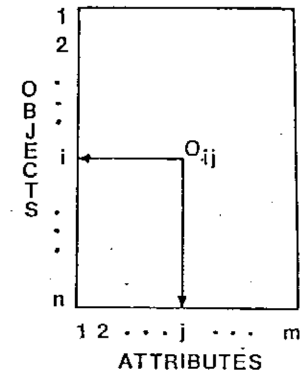
The five rectangular GEMs differ in their specification of $\mathbf{V}_i$. The simplest and most familiar analysis of rectangular data (see Figure 6.3) results from using the rectangular GEM that invokes the assumption that $\mathbf{V}_i = \mathbf{I}$ for all i. This results in

Multidimensional Unfolding

Input: Single nxm rectangular matrix O of ratings of n objects on m attributes (o_{ij} is rating of object i on attribute j).

Number of ways: 2

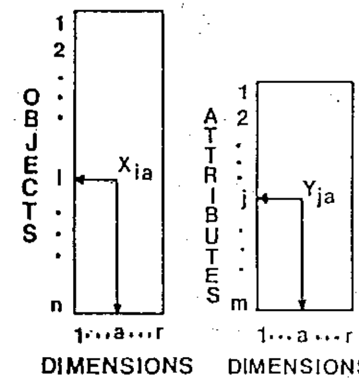
Number of modes: 2



Output: Two rectangular matrices: X, an nxr matrix of the coordinates of n objects on r dimensions; and Y, an mxr matrix of the coordinates of m attributes on r dimensions (x_{ia} is coordinate of i'th object on a'th dimension, y_{ja} is coordinate of j'th attribute on a'th dimension).

Number of ways: 2

Number of components: 2



Model: Joint Euclidian

$$o_{ij} \cong t_i [d_{ij}]$$

$$d_{ij} = \sqrt{\sum_a^r (x_{ia}-y_{ja})^2}$$

FIG. 6.3. Schematic of multidimensional unfolding

This combination of data and model corresponds to Coombs' (1964) proposal for the unfolding of preference data. Schiffman, Reynolds, and Young

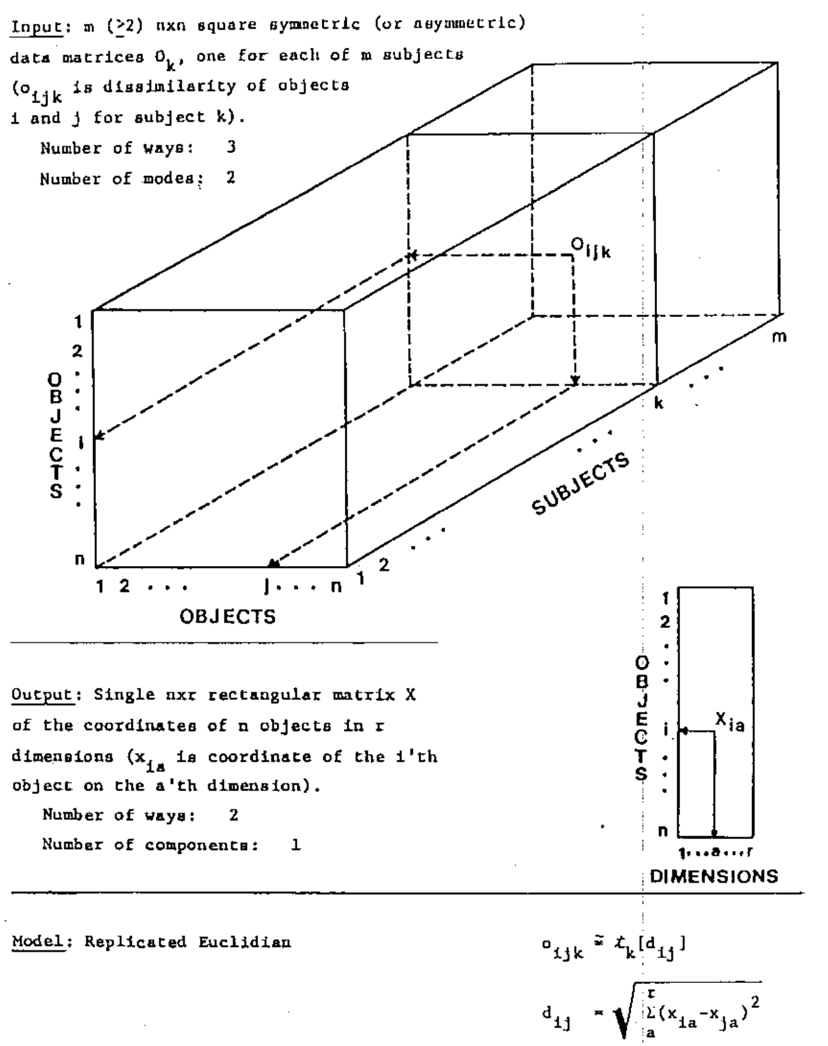


FIG. 6.4. Schematic of replicated multidimensional scaling

Thus, these models are expressed by the GEM

$$d_{ijk}^2 = (\mathbf{x}_i - \mathbf{x}_j)\mathbf{W}_k(\mathbf{x}_i - \mathbf{x}_j)'. \tag{6.6}$$

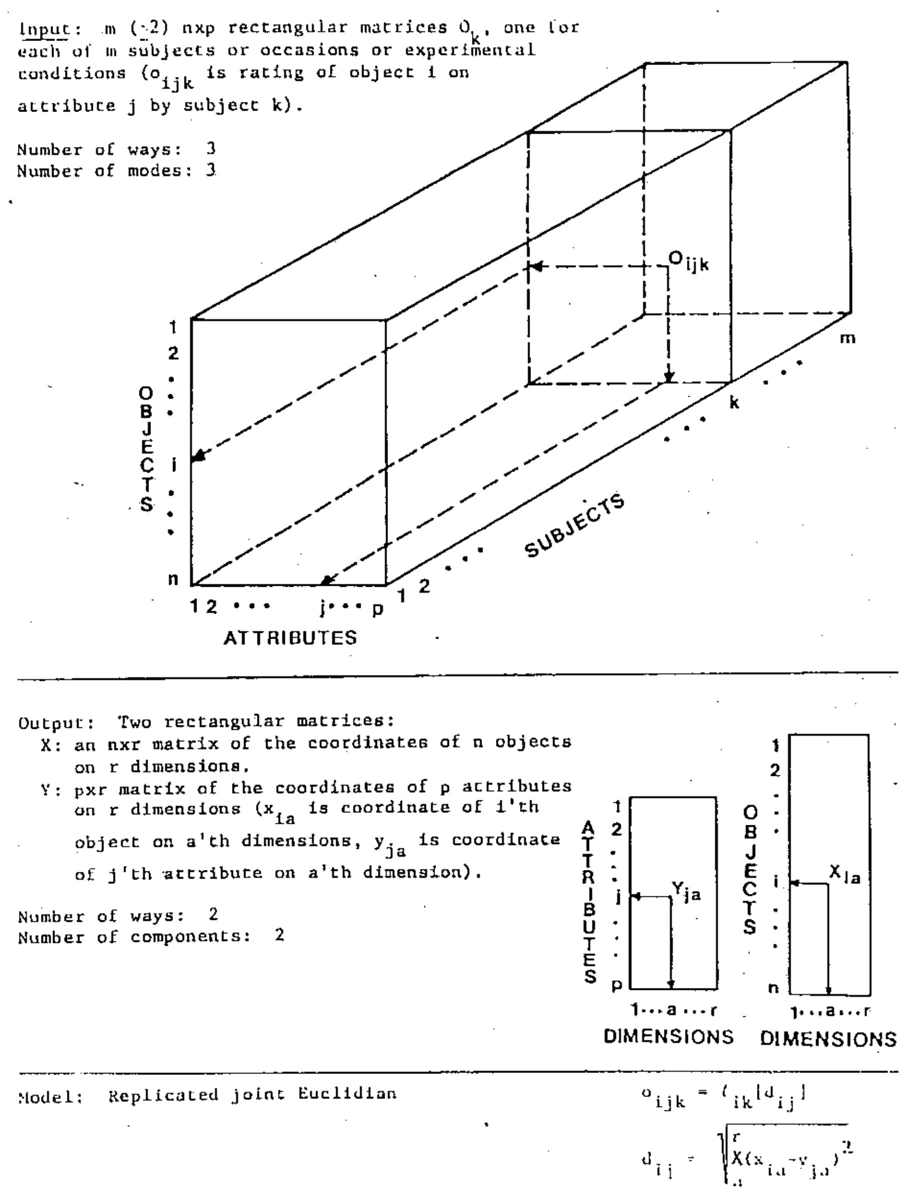


FIG. 6.5. Schematic of replicated multidimensional unfolding

a matrix is asymmetric, then the adoption of a symmetric GEM in our data analysis implies that we think the asymmetry is error.

Certainly, the most well-known of these eight analyses is the one proposed by Carroll and Chang (1970). They called this Individual Differences Scaling (INDSCAL), since they used the weights to model differences between individuals. Their work involves the assumption that the third-way weights $\mathbf{W}_k$ are diagonal, and was proposed for symmetric data. This analysis is schematized in Figure 6.6, where the diagonals of $\mathbf{W}_k$ are shown as the *rows of* $\mathbf{W}$. Chapter 9 of this book, by Hoffman and Perreault, uses this analysis.

Shortly after this very famous development, Carroll and Chang (1972) extended their thinking to include weights $\mathbf{W}_k$ which were full rank. They called this Individual Differences in Orientation Scaling (IDIOSCAL). Bloxom (1978) and Young and Lewyckyj (1979b) have also discussed the case where the weights are reduced in rank and represent principal directions (Figure 6.7). Chapter 10 of this book, by Easterling, uses this analysis. Finally, Young (1982) attended to the rank-one situation, and Young (1984) included the asymmetric data situation as well. Thus, all eight MDS analyses and four models have been previously discussed.

The family of symmetric GEMs share many common aspects: (a) They may be used when the researcher has obtained, from a number of sources (individuals, occasions, experimental conditions) data about the (dis)similarity of pairs of things to each other; (b) They develop a *common space* that represents the structure of the things that is shared in common between the several sources of data, the representation being as points in a multidimensional Euclidean space; and (c) They portray variation across the third way of the data (people, occasions, or conditions) in a geometric fashion. The models differ, however, in their assumptions about the basic nature of the variation across the third way, and, thus, in their geometric representation of this variation. These differences are reflected in the different types of $\mathbf{W}_k$.

The nature of the GEM representation just given can be restated in the original terminology used by the psychometricians who developed these models. In this terminology, (a) the models can be used when the researcher has obtained data from several individuals about the dissimilarity of pairs of stimuli; (b) the model represents the stimulus structure that is shared by the group of individuals in a *group stimulus space*, which has points for each stimulus; and (c) individual differences in perception or cognition are represented geometrically in a weight space, whose nature depends on the specific characteristics of the weight matrix $\mathbf{W}_k$.

6.3.2. Analyses Using Asymmetric Three-Way GEMs

The asymmetric three-way GEMs include GEMs that differ in the assumptions made about both the first-way and third-way weight matrices $\mathbf{V}_i$ and

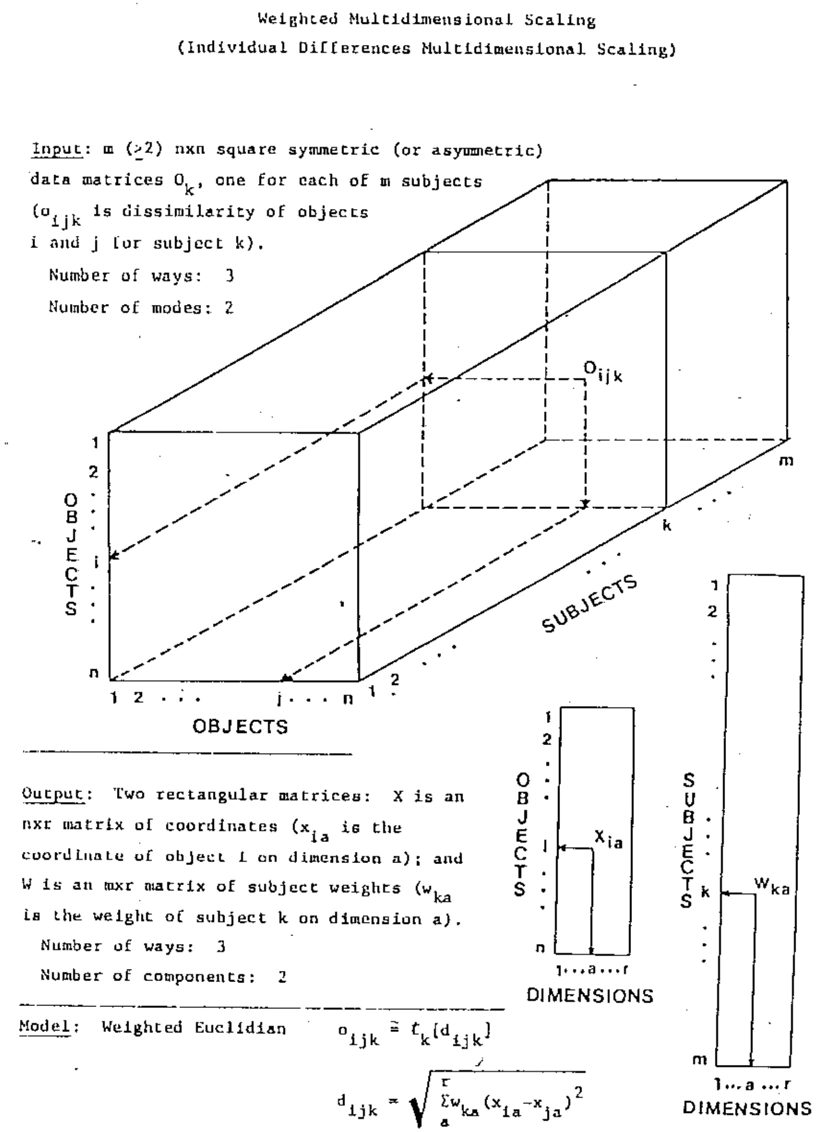


FIG. 6.6. Schematic of weighted multidimensional scaling

$\mathbf{W}_k$. Since the data must have square asymmetric matrices, all these models are required to have

(b) $\mathbf{Y} = \mathbf{X}$

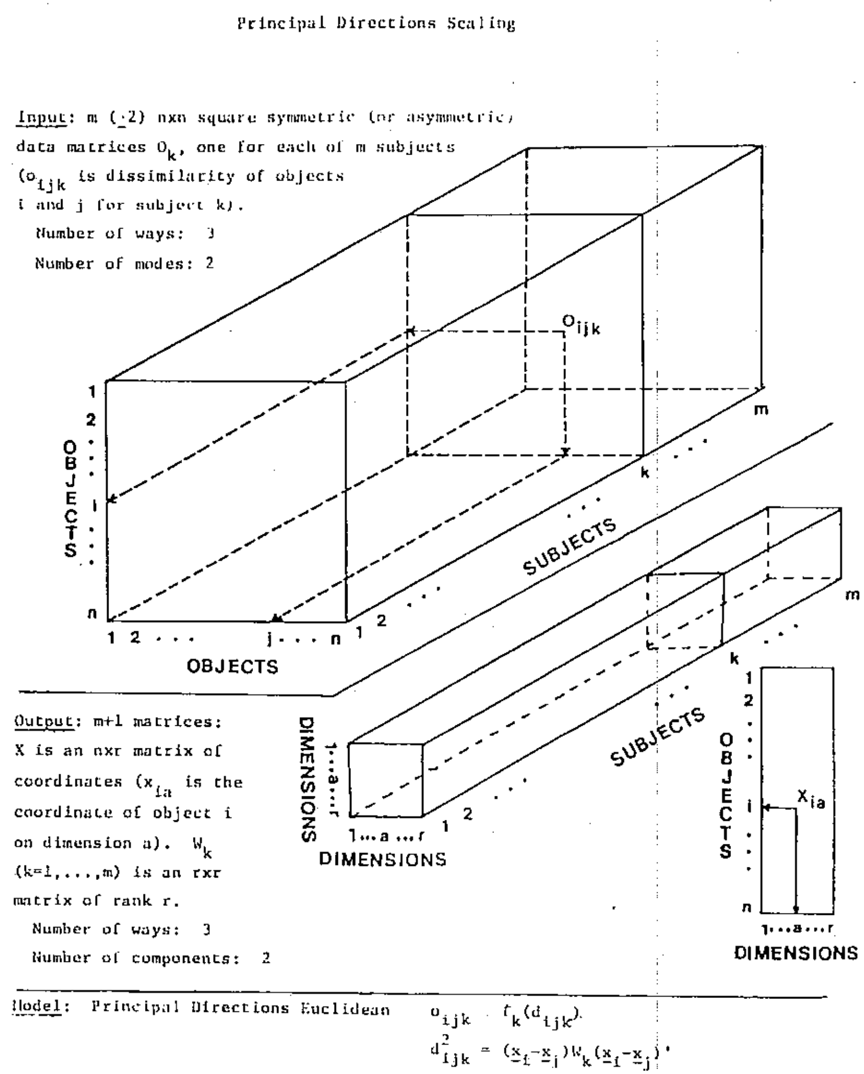


FIG. 6.7. Schematic of principal directions scaling

Thus, these models have the GEM equation

$$d_{ijk}^2 = (\mathbf{x}_i - \mathbf{x}_j)\mathbf{V}_i\mathbf{W}_k(\mathbf{x}_i - \mathbf{x}_j)'. \tag{6.7}$$

There are sixteen models that result from factorially combining four of the five types of $\mathbf{V}_i$ (excluding identity weights, which were discussed in the previous section) with four of the five types of $\mathbf{W}_k$ (again excluding identity weights, discussed in the replicated two-way section). Of these sixteen models, only the model having two diagonal weight matrices has been discussed, to our knowledge. This is the Asymmetric Individual Differences (ASINDSCAL) model of Young and Lewyckyj (1979a). However, the four models that involve a diagonal $\mathbf{V}_i$ and diagonal, rank one, reduced rank or full rank $\mathbf{W}_k$ are implemented in the ALSCAL program (Young, 1982), even though not explicitly discussed. The ASINDSCAL analysis is schematized in Figure 6.8, where the diagonals of $\mathbf{V}_i$ and $\mathbf{W}_k$ form the *rows* of V and W.

6.3.3. Analyses Using Rectangular Three-Way GEMs

The family of twenty GEMs for three-way rectangular (preference) data can be grouped according to those where $\mathbf{V}_i = \mathbf{I}$, and those without this assumption. For the four GEMs with $\mathbf{V}_i = \mathbf{I}$, the full rank $\mathbf{W}_k$ model is at the heart of an analysis method proposed by De Sarbo (1978). Young and Lewyckyj (1979b) and Young (1982, 1984) have discussed the full, reduced, rank-one, and diagonal $\mathbf{W}_k$ models. The GEM equation is

$$d_{ijk}^2 = (\mathbf{y}_i - \mathbf{x}_j)\mathbf{W}_k(\mathbf{y}_i - \mathbf{x}_j)'. \tag{6.8}$$

The diagonal $\mathbf{W}_k$ is used in an analysis called Weighted Multidimensional Unfolding (WMDU) by Young and Lewyckyj (1979a). This is schematized in Figure 6.9, with the diagonals, of $\mathbf{W}_k$ becoming the rows of W.

The second family of GEMs for three-way rectangular data consists of the sixteen models for which neither the $\mathbf{W}_k$ nor the $\mathbf{V}_i$ are required to be identity matrices. This is the most general GEM, corresponding to the full equation (Equation 6.1). To our knowledge, no one has explicitly discussed analyses based on any of these models. This family is a set of three-way models that parallel Carroll's (1972) family of two-way weighted unfolding models, but with additional weights for the several matrices.

6.3.4. Analyzing Four-Way Data with Three-Way GEMs

Just as with the family of two-way GEMs, we can take three-way GEMs and apply them to higher-way data. Each one of the forty-four different three-way GEMs can be used with higher-way data, generating an additional set of forty-four replicated three-way analyses. The essential nature of these analyses is the same as when the models are applied to three-way data, the main difference being that we have several sets of three-way data that we view as being the same except for non-systematic error. We are unaware of any of these replicated three-way models being explicitly discussed previously in the literature, nor of any computer programs for fitting them to data.

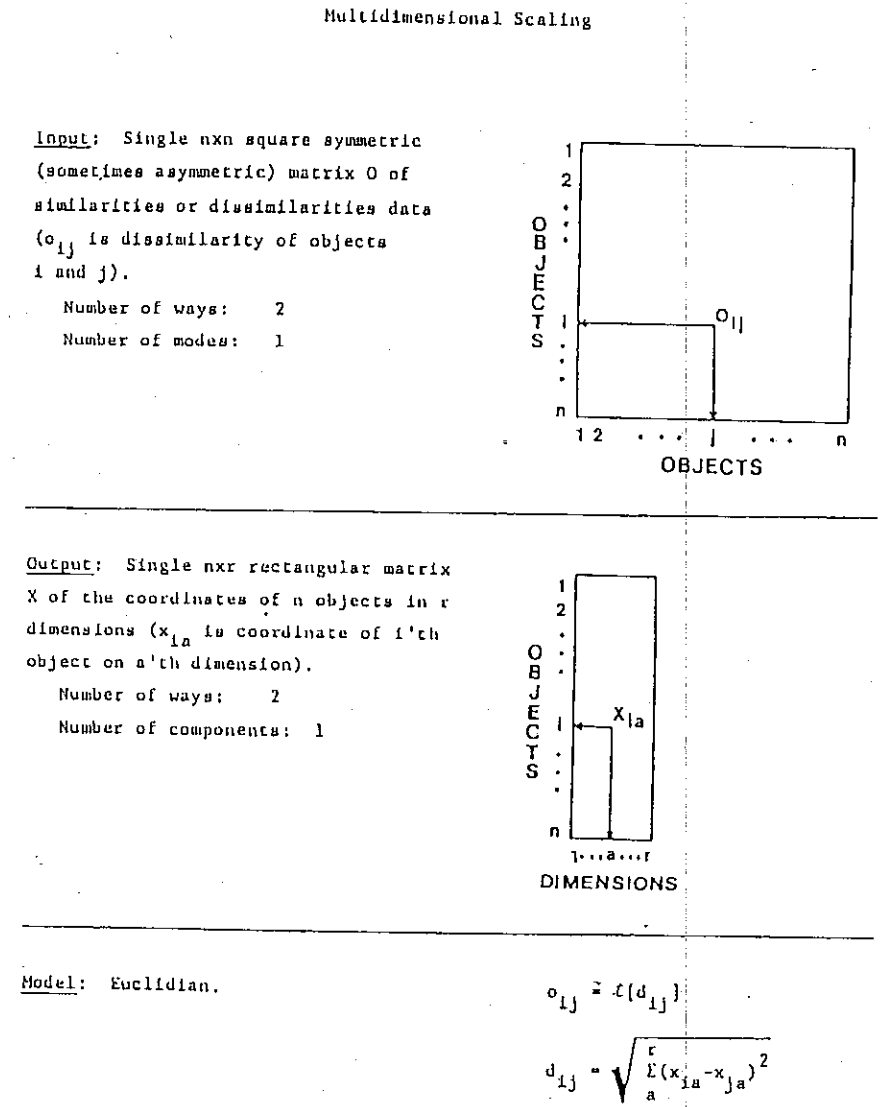


FIG. 6.1. Schematic of classical multidimensional scaling

It is important to note that the symmetric two-way GEM can be also used for asymmetric two-way data. When this is done our analysis implies that the asymmetry is nothing more than error, since the model used in the analysis does not incorporate an asymmetric component. This analysis has no special name in the literature ...

tion, the two kinds of data that it can be applied to generate two unique analyses.

6.2.2. Analyses Using Asymmetric Two-Way GEMs

If we have a single square matrix of asymmetric dissimilarities we can employ the symmetric, two-way GEM given by Equation (6.2), as just mentioned. However, it is important to realize that we are not forced to have the first-way weights $V_i = I$ for all stimuli i: Rather, we choose to assume (or not assume) that this is the case. Note that our freedom to choose does not work in the reverse direction: We are not free to apply an asymmetric model to symmetric data. When the data are symmetric, we must use a symmetric model.

When we have asymmetric data, and we assume that $V_i = I$ the implication is that the asymmetry is error. If we do not think that the asymmetry is error, but is meaningful, systematic information, we can choose to model the asymmetry via the weights V_i. This is the case since, except in certain special cases, the weights V_i generate asymmetric distances when at least one $V_i \neq I$. There are four members of the family of asymmetric GEMs, the simplest one being obtained when we assume that

(c) V_i is a diagonal for all i.

This assumption generates the specific asymmetric GEM

$$d_{ij}^2 = (x_i - x_j)V_i(x_i - x_j)'. \tag{6.3}$$

which was first proposed by Young (1975a) as the basis for his ASYMSCAL analysis (see also Young & Lewyckyj, 1979a). This analysis is used by Collins in Chapter 8 of this book, and is schematized in Figure 6.2 (where the diagonals of V_i are schematized as the *rows* of V). The three other asymmetric GEMs (V_i is full or reduced rank or rank one) have not been investigated, to our knowledge. Note that the four asymmetric models can only be applied to asymmetric data; thus, there are only four kinds of analyses.

6.2.3. Analyses Using Rectangular Two-Way GEMs

The family of five rectangular GEMs differ in the type of assumptions made about V_i. All members of the family involve assuming that

(a) $W_1 = I$, and
(b) $Y \neq X$.

Note that we must assume that the coordinates Y for the rows of our data ...

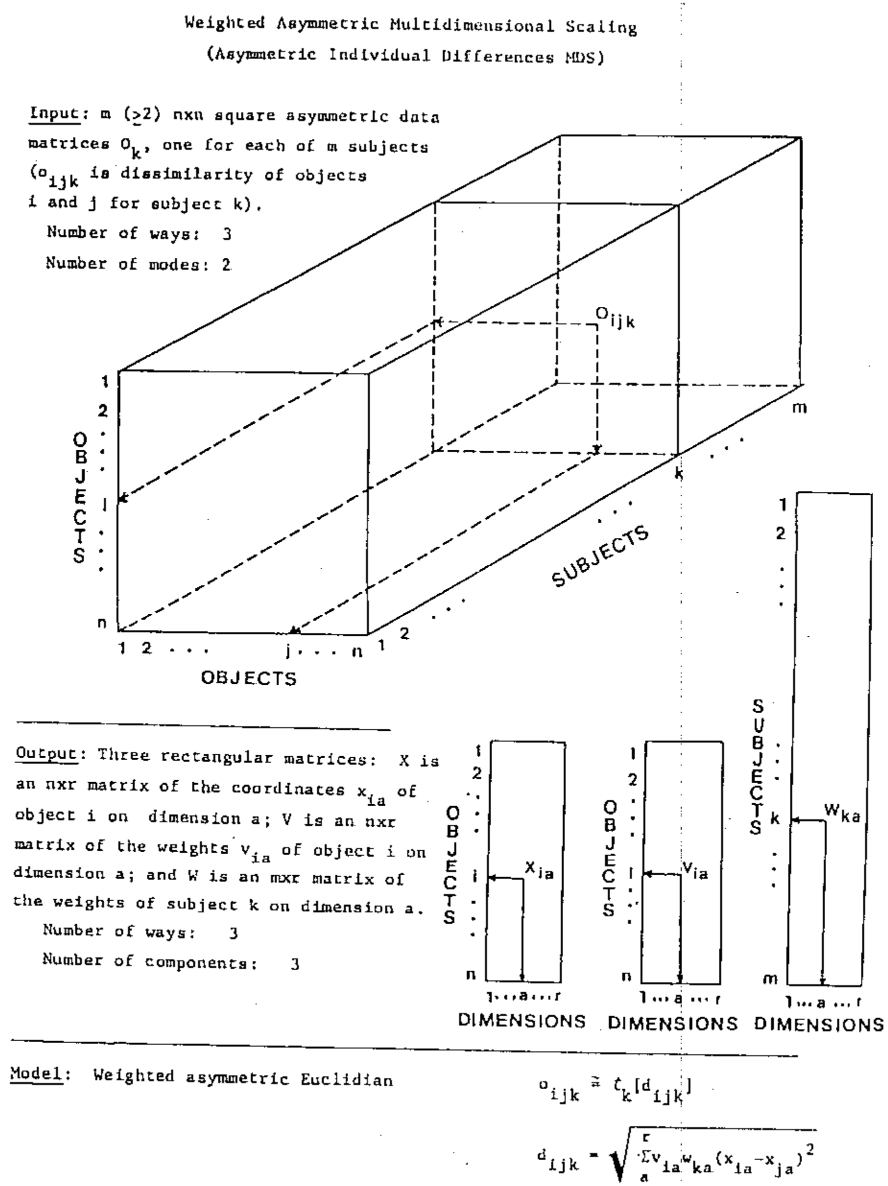


FIG. 6.8. Schematic of weighted asymmetric multidimensional scaling

6.4. DIAGONAL WEIGHTS

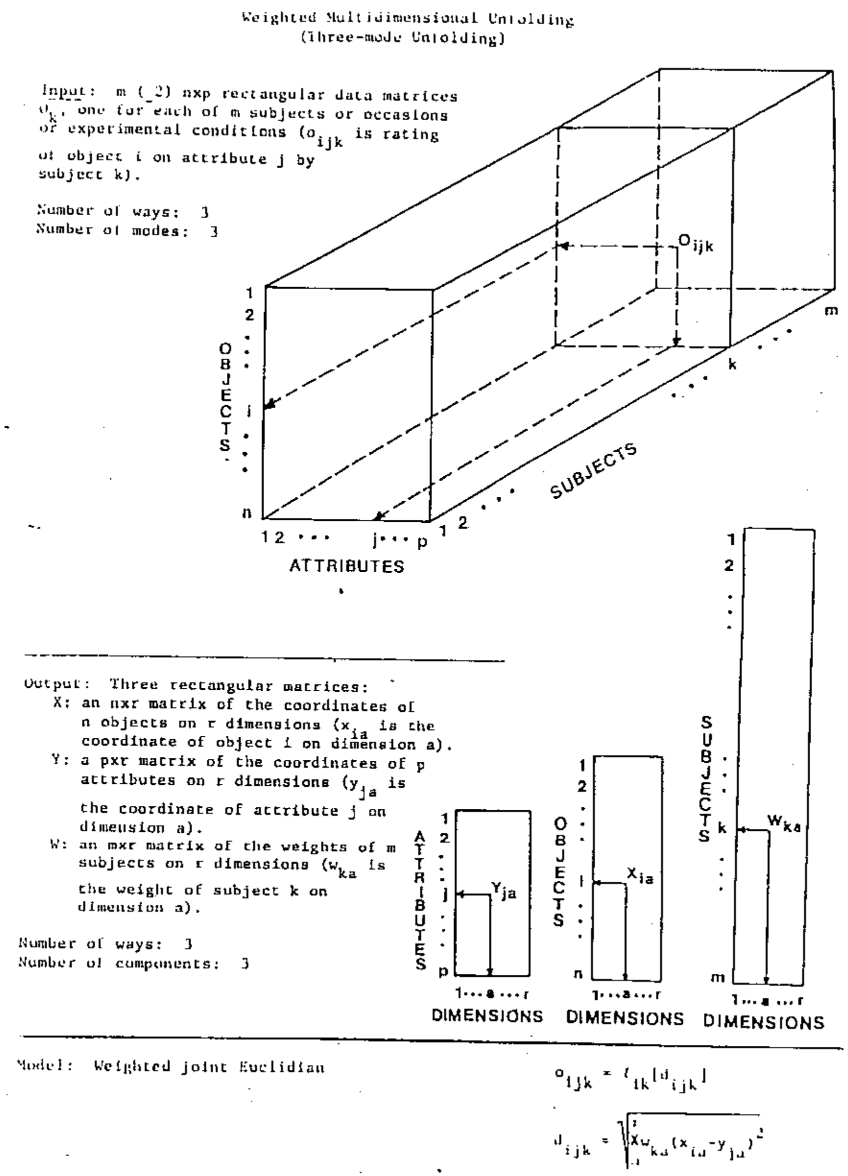


FIG. 6.9. Schematic of weighted multidimensional unfolding

modify the Euclidean space containing the stimulus points X and, possibly,

垃圾分類問題

作答說明：請依據每配對垃圾間的相似性，用五點評定量表（1點為最不相似，5點為最相似）評出其等第。

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)可口可樂玻璃瓶										
(2)保麗龍便當餐盒										
(3)黑松楊桃汁鋁箔包										
(4)揉掉的A4影印紙										
(5)損毀的木頭椅子										
(6)打破的玻璃杯										
(7)報廢的機車輪胎										
(8)維大力汽水鋁罐										
(9)用完的原子筆										
(10)拋棄的抹布										