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研究計畫成果報告

策略性債務融資與可轉換公司債之評價
－最適資本結構模型

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I. Abstract

This research derives the analytic valuation of the callable convertible bonds. The model allows the optimal strategies for call, bankruptcy, and voluntary conversion to be endogenously determined by the bond issuer and the bondholder. The numerical results predict when the optimal voluntary conversion, forced conversion, and pure redemption of a callable convertible bond may occur. The empirical literature findings of late calls associated with dividend payments and tax benefits are confirmed by the model.

Keywords: Convertible Bonds, Call Strategy, Conversion Strategy, Bankruptcy Strategy

II. Purposes of the Study

Convertible bonds, spanning the dimensions from common stocks on the one hand to straight bonds on the other, are one of the most popular hybrid financing instruments. Most convertible bonds have call features, which adds some complexity to the valuation of the convertible bonds as well as to the determination of the optimal strategies for call and conversion. Like straight bonds, investors of convertible bonds are entitled to receive coupons and principal payments; hence, the default risk of the bond issuer must be incorporated into the valuation of convertible bonds. Furthermore, the optimal strategies for call, conversion, and bankruptcy must be simultaneously determined since these strategies are mutually interacted.

Rogers (1999) notes that it is hard to deal realistically with convertible bonds in the reduced-form model: “The existence of convertible bonds really forces one to consider firm value – so maybe we should go for a structural approach anyway?” Consequently, this research provides a simple structural model to price callable convertible bonds where the optimal strategies for call, conversion and bankruptcy are endogenously determined at the same time. First of all, we apply the pricing methodology of double-barrier options to value a non-callable convertible bond and decide the optimal voluntary conversion strategy. Take it as given, we then utilize the same methodology to value a callable forced-convertible bond and determine the optimal policies for call and bankruptcy. Finally, from the above results, the closed-form pricing formula for a callable convertible bond with the bond issuer’s default risk is derived and the optimal strategies for voluntary conversion, call and
bankruptcy are therefore obtained.

III. Results and Discussion

III.1. Valuation Framework

Consider a bond issuer (or an objective firm) where the callable convertible bond is the only senior issue, which continuously pays a constant coupon flow, $C$, with the time to maturity, $T$, and the par value, $P$. The other claim of the firm is the common share. Let $V(t)$ designate the unleveraged asset value of the bond issuer at time $t$. The dynamics of $V(t)$ on the risk-neutral probability space are given by

$$dV(t) = V(t)\left((r - q)dt + \sigma dW^Q(t)\right),$$

where $r$ denotes the constant risk-free interest rate, $q$ is the constant payout rate of the issuer, $\sigma$ is the constant volatility of the rate of return of $V(t)$, and $W^Q$ is a Wiener process on the same space. Moreover, we assume there is no incentive problem between the management of the objective firm and the shareholders of the firm.

Next, we are going to characterize the callable convertible bond of the objective firm. As usual, if the bondholders convert the convertible bonds into the common shares, then they will receive a fraction $\gamma$ of the unleveraged asset value of the issuer. One advantage of the structural model for valuing convertible bonds is that $\gamma$ captures the dilution effect of the conversion, which represents the ratio between the total converted shares and the total outstanding shares after conversion. Here we implicitly assume the conversion in the model is the block conversion. If the issuer of the callable convertible bonds calls back all outstanding callable convertible bonds at the same time, then all the bondholders have to immediately choose either to convert the callable convertibles into the common shares or to receive the pre-specified call price (the redemption value), $(1 + \beta)P$, where $\beta P$ is the call premium of the callable convertible bonds. At the initial time, assumed to be time zero for simplicity, we suppose that the upper constant call barrier, $V^0_{\text{Call}}$, which are both greater than the initial unleveraged asset value of the bond issuer, $V(0)$. As soon as the unleveraged asset value of the bond issuer goes up and touches either $V^0_{\text{Call}}$ or $V^0_{\text{Con}}$, then either the call of the bond issuer or the voluntary conversion of the bondholder is triggered. Therefore, two first passage times can be further defined as $\tau^0_{\text{Call}} = \inf\{t > 0 : V(t) \geq V^0_{\text{Con}}\}$ and $\tau^0_{\text{Con}} = \inf\{t > 0 : V(t) \geq V^0_{\text{Con}}\}$, where $\tau^0_{\text{Call}}$ and $\tau^0_{\text{Con}}$ are the times that the bond issuer decides to call back the bonds and the bondholder determines to voluntarily convert the bonds into the common shares, respectively. Subsequently, another lower constant bankruptcy barrier is defined as $V^0_B$, which is less than $V^0$. As soon as the unleveraged asset value of the bond issuer goes down and touches $V^0_B$, the bankruptcy of the bond issuer is triggered. Once the bond issuer declares bankruptcy, the bondholder receives the recovery value, $(1 - \alpha)V^0_B$, at the time of default, where $\alpha$, between 0 and 1, is the ratio of bankruptcy costs or restructuring costs. Again, another first passage time can be denoted as $\tau^0_B = \inf\{t > 0 : V(t) \leq V^0_B\}$, where $\tau^0_B$ is the time that the bond issuer announces bankruptcy. In the next section, we will endogenously determine the above three barriers, $V^0_{\text{Call}}$, $V^0_{\text{Con}}$ and $V^0_B$, which are actually time-dependent, by taking the desired objectives of the bond issuer and the bondholder into account.

III.2. Derivation of the Formula

For a non-callable convertible bond, the bond issuer can decide when to go bankrupt and the bondholder can determine when to voluntarily convert the convertible bonds into the common shares.

Under our framework, the initial value of a non-callable convertible bond, $NCCB(0)$, can be written by

$$NCCB(0) = \mathbb{E}^Q\left[e^{-r\tau^0_B}\mathbf{1}_{\tau^0_{\text{Call}} < \tau^0_{\text{Con}} < \tau^0_{\text{Call}}} \right] (1 - \alpha)V^0_B$$
equal to the initial total firm value minus the initial value of the non-callable convertible bond.

To endogenously decide the optimal voluntary conversion policy, \( V_{C0}^{*,0} \), and the optimal bankruptcy strategy, \( V_{B1}^{*,0} \), we apply the following smooth-pasting conditions:

\[
\frac{\partial E_{NCNB}(0)}{\partial V(0)} |_{V(0)=V_{B1}^{*,0}} = \frac{\partial E_{NCNB}(0)}{\partial V_{B1}^{*,0}} |_{V(0)=V_{B1}^{*,0}} = 0 ,
\]

\[
\frac{\partial NCCB(0)}{\partial V(0)} |_{V(0)=V_{C0}^{*,0}} = \frac{\partial NCCB(0)}{\partial V_{C0}^{*,0}} |_{V(0)=V_{C0}^{*,0}} = \gamma .
\]  

Consider a callable forced-convertible bond, where the bond issuer can decide when to go bankrupt and when to call the bond, and the bondholder cannot convert voluntarily. Once the bond issuer announces to call the bond, the bondholder can, at the same time, choose either to accept and then receive the redemption price, or to involuntarily convert the bond into the common shares. Therefore, the initial value of a callable forced-convertible bond, \( CFCB(0) \), can be written by

\[
CFCB(0) = E^Q \left[ e^{-r t^{\tau}_{B1}} \alpha V(t^{\tau}_{B1})_I \right] ,
\]

\[
+ E^Q \left[ e^{-r t^{\tau}_{B1}} 1_{[t^{\tau}_{B1} < t^{\tau}_{C0} \land t^{\tau}_{B1} < t]} P \right] ,
\]

\[
+ E^{Q} \left[ \tau \mathbb{E} \left[ \int_{0}^{t^{\tau}_{C0}} \mathcal{C} e^{-r \tau} d\tau \right] \right] ,
\]

(7)

The other analyses are similar to the non-callable convertible bond. In what follows, we deal with the callable convertible bond by the following two assumptions.

**Assumption 1:** Whenever it is optimal to voluntarily convert a non-callable convertible bond, it will also be optimal to convert a callable convertible bond which is otherwise identical.

**Assumption 2:** The possibility of a voluntary conversion does not affect the optimal call policy.

Owing to the above assumptions, we derive the analytical valuation of a callable convertible bond at the initial time, \( CCB(0) \), subject to the default risk of the bond issuer,
which can be expressed as follows:

\[
CCB(0; V_{B2}^{*,0}, V_{Con}^{*,0}, V_{Call}^{*,0}) =
\begin{cases}
CFCB(0; V_{B2}^{*,0}, V_{Con}^{*,0}), & \text{if } V_{Con}^{*,0} > V_{Call}^{*,0} \\
NCCB(0; V_{B2}^{*,0}, V_{Con}^{*,0}), & \text{if } V_{Con}^{*,0} \leq V_{Call}^{*,0}
\end{cases}
\]

Finally, it is should be emphasized that the optimal strategies for call, voluntary conversion and bankruptcy are all time-dependent and are endogenously determined in our model.

### III.3. Numerical Results

In summary of numerical results, our model predicts that (i) late calls are in most of the cases, and higher coupon, lower risk-free interest rate, greater volatility, and medium time to maturity will lead to the extremely late calls where the optimal call trigger is extraordinarily high, which is mainly consistent with Ederington et al. (1997); (ii) the voluntary conversion may occur in the cases of the callable convertible bond with very low coupon payments, or with shorter time to maturity, smaller volatility, and higher risk-free interest rate; (iii) the pure redemption may take place in the case of the callable convertible bond with greater call premium, higher coupon payments, shorter time to maturity, smaller volatility and the risk-free interest rate in a middle range.

### IV. Evaluation of the Study

In this research, we extend and combine the structural models of Ingersoll (1977a) and Leland and Toft (1996) to derive the closed-form valuation of a callable convertible bond. The model allows for bankruptcy costs, tax benefits, and the time to maturity of the bond. Not only are the optimal call and bankruptcy strategies endogenously determined when the equity value is maximized, but also the optimal voluntary conversion strategy is obtained when the value of the callable convertible bond is maximized.

For future study, we may use the market data to evaluate the model. Also, we may extend our model with asymmetric and incomplete information. After these works have been done, the models can be used efficiently for the financial industry.

### V. Reference


