多層重設型選擇權之訂價及避險策略

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I. Abstract

This research provides the closed-form pricing formulas of reset options with $m$ strike resets and $n$ pre-decided reset dates. The exact closed-form pricing formulas of reset options with $m$ strike resets and continuous reset period are also derived. Also, we find that the reset options not only have the phenomena of Delta jump and Gamma jump across reset dates, but also have the properties of Delta waviness and Gamma waviness, especially near the time before reset dates. Furthermore, Delta and Gamma can be negative when the stock price is near the strike resets at the times close to the reset dates.

Keywords: Reset Options, Multiple Strike Resets, Delta waviness, Gamma waviness

中文摘要

本研究推導出含有 $m$ 個重設履約價及 $n$ 個事先決定重設日之重設選擇權封閉解，並且也推導出其在連續重設時間下的公式解。我們發現重設選擇權不僅有 Delta 和 Gamma 跳躍的現象，而且其在重設日期前存在有 Delta 和 Gamma 波浪現象。另外，當股價接近重設履約價且時間接近重設日期時，Delta 和 Gamma 有可能為負。

關鍵字：重設選擇權、多層重設履約價、Delta 波浪、Gamma 波浪

II. Purposes of the Study

Path-dependent options, whose payoffs are influenced by the path of the prices of underlying assets, have become increasingly popular in recent years. One of the path-dependent options is the look-back option, whose payoff depends, in particular, on the minimum or maximum price of the underlying asset during the option’s lifetime. There is another kind of path-dependent option known as a reset option. Unlike the look-back option, the strike price of a reset option will be reset to a new strike price only on the pre-specified reset dates if the price of the underlying asset is lower than one of the strike resets.

Reset options have been issued in practice for many years. The Chicago Board Options Exchange (CBOE) and the New York Stock Exchange (NYSE) both introduced S&P 500 index put warrants with a three-month reset period in late 1996. Morgan Stanley issued a reset warrant with an initial strike price of $44.73 in July 1997. The strike price would be adjusted to $39.76 on August 5, 1997 if the price of its underlying asset fell below $39.76. A more recent example comes from Taiwan, where Grand Cathay Securities had six reset options listed on the Taiwan Stock Exchange (TSE) (Codes in TSE are 0517, 0522, 0523, 0527, 0528, and 0538) from 1998 to 1999. Most reset options, including all of the reset options listed in TSE, are options with multiple strike resets and reset dates. For example, the reset condition of 0522 of TSE is that the strike price would be adjusted if the six-day average closing price of 2323 on the TSE fell below 98%, 96%, 94%, 92%, 90% of the initial strike price of $81 during the first three months after the warrant was issued.

Since the reset warrants are new derivative products in financial markets, few
studies have been done on their pricing problems. Gray and Whaley (1997) examined the pricing of the put warrant with periodic reset and the warrant’s risk characteristics. They further provided a closed-from solution for reset options with a single reset date in a latter paper (Gray and Whaley (1999)). Cheng and Zhang (2000) studied reset options whose strike price will be reset to the prevailing stock price if the option is out of money. A closed-form pricing formula in terms of a multivariate normal distribution is derived under the risk-neutral framework. However, the reset conditions of reset options investigated by Cheng and Zhang (2000) are not the general cases of reset products in practice. Let the underlying asset price at time \( t \) be denoted \( ) (t S \). The terminal payoff of a reset option with \( n \) reset dates and initial strike price \( K_0 \), which was studied by Cheng and Zhang (2000), is as follows:

\[
C(T) = \text{Max} \left[ S(T) - \text{Min} \left[ K_0, S(t_1), \ldots, S(t_n) \right], 0 \right].
\]  

In practice, however, the terminal payoff of the reset option is more often set as

\[
C(T) = \text{Max} \left[ S(T) - K^*, 0 \right] = \left[ S(T) - K^* \right]^+,
\]  

where

\[
K^* = \begin{cases} 
K_0 & \text{if } \text{Min} \left[ S(t_1), \ldots, S(t_n) \right] > D_i, \\
K_i & \text{if } D_i \geq \text{Min} \left[ S(t_1), \ldots, S(t_n) \right] > D_{i-1}, \\
K_m & \text{if } D_m \geq \text{Min} \left[ S(t_1), \ldots, S(t_n) \right],
\end{cases}
\]

and \( K_i, i = 1, \ldots, m \), are the reset strike prices; \( D_i, i = 1, \ldots, m \), are the strike resets.

Our first contribution in this article is to derive the exact closed-form solution for reset options with \( m \) strike resets and \( n \) pre-decided reset dates, as specified in (2) and (3), under the risk-neutral framework. Furthermore, we also provide the closed-form solution for reset options with \( m \) strike resets and continuous reset dates, which is the limiting case of the former.

Some previous studies, such as Cheng and Zhang (2000), have pointed out the phenomenon of Delta jump across reset dates. The second contribution of this paper is the finding that, in addition to Delta jump, a reset option with \( m \) strike resets also has the phenomena of Gamma jump, Delta waviness, and Gamma waviness as well. The waviness of delta and gamma means that the delta and gamma of reset options will oscillate when the stock price passes across the strike resets. When the time is approaching the reset dates and the stock price is near the strike resets, delta and gamma may change their values from positive to negative. The phenomena of Delta jump and Gamma jump near reset time as well as the properties of Delta waviness and Gamma waviness will make the risk management more difficult.

III. Results and Discussion

III.1. Pricing reset options with \( m \) strike resets and \( n \) reset dates

We assume the dynamics of underlying asset price \( S(t) \) are described by the following stochastic differential equation:

\[
dS(t) = uS(t)dt + \sigma S(t)dW_t,
\]  

where \( u \) and \( \sigma > 0 \) are constants, and \( W_t \) is a one-dimensional standard Brownian motion defined in a filtered probability space \( (\Omega, F, P) \). The money market account, \( B(t) \), corresponds to the wealth accumulated from an initial $1 investment at spot interest rate \( r \) in each subsequent period. Therefore,

\[
\frac{dB(t)}{B(t)} = rB(t)dt.
\]

In view of (2) and (3), the payoff at expiry of the reset option with \( m \) strike resets and \( n \) pre-decided reset dates can be written as

\[
C(T) = \left[ S(T) - K_0 \right]^+ \left[ \text{Min} S(t_j) > D_1 \right]
+ \left[ S(T) - K_1 \right]^+
+ \left[ S(T) - K_{m-1} \right]^+
+ \ldots
+ \left[ S(T) - K_m \right]^+
+ \left[ S(T) - K_m \right]^+ \left[ 1 - \text{Min} S(t_j) > D_m \right],
\]

where \( I(\cdot) \) is an indicator function. Under the risk neutral probability measure \( Q \), the arbitrage-free price of reset option \( C(t) \) at
the explicit solution to (8) is as follows:

\[
C(t) = e^{-r(T-t)} E^Q \left[ (C(T) - K_{i+1})^+ I \left( \text{Min} S(t_j) > D_i \right) \right] F_{i+1}^{i+1} \]

\[
= e^{-r(T-t)} \sum_{i=1}^{m} E^Q \left[ (S(T) - K_i)^+ I \left( \text{Min} S(t_j) > D_i \right) \right] F_{i+1}^{i+1} \\
- e^{-r(T-t)} \sum_{i=1}^{m} E^Q \left[ (S(T) - K_i)^+ I \left( \text{Min} S(t_j) > D_i \right) \right] F_{i+1}^{i+1} \\
+ e^{-r(T-t)} E^Q \left[ (S(T) - K_n)^+ I \right] F_{i+1}^{i+1} \\
\tag{7}
\]

From (7), we know that the key to the solution is to compute the following expression:

\[
e^{-r(T-t)} E^Q \left[ (S(T) - K_h)^+ I \left( \text{Min} S(t_j) > D_i \right) \right] F_{i+1}^{i+1} \\
\tag{8}
\]

We present the result in the following Theorem.

THEOREM: The explicit solution to (8) is as follows:

\[
e^{-r(T-t)} E^Q \left[ (S(T) - K_h)^+ I \left( \text{Min} S(t_j) > D_i \right) \right] F_{i+1}^{i+1} \\
= \sum_{g=1}^{n} \left[ S(t) N_{n+1}(D^g_{i,h}; \Sigma_g) - K e^{-r(T-t)} N_{n+1}(\hat{D}^g_{i,h}; \Sigma_g) \right] \\
\tag{9}
\]

where \( N_{n+1}(\cdot; \Sigma) \) is the cumulative probability of an \((n+1)\)-dimensional multivariate normal distribution with mean vector 0 and covariance matrix \( \Sigma \). For \( i, h = 1, \ldots, m \), the parameters in (9) are defined as follows:

\[
D^{i,h} = \begin{bmatrix} d_{i,1} & e_{1,2} & \ldots & e_{1,n} & y_h \\ e_{2,1} & d_{i,2} & \ldots & e_{2,n} & y_h \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ e_{n,1} & e_{n,2} & \ldots & d_{i,n} & y_h \end{bmatrix}
\]

and \( D^{j,h} \) stands for the \( j^{th} \) row of \( D^{i,h} \);

\[
\ln \left( \frac{S(t)}{D_j} \right) + (r + \frac{1}{2} \sigma^2)(T-t) \\
d_{i,j} = \frac{(r + \frac{1}{2} \sigma^2)(t_j - t)}{\sqrt{t_j - t}} \\
e_{i,j} = \frac{(r + \frac{1}{2} \sigma^2)(t_j - t)}{\sqrt{t_j - t}} \\
y_h = \frac{\ln(S(t))}{K_h} + (r + \frac{1}{2} \sigma^2)(T-t) \\
\frac{1}{\sqrt{T-t}}
\]

\( \hat{D}^{i,h} \) is similarly defined as \( D^{i,h} \) with the parameters \( d_{i,j}, e_{i,j} \), and \( y_h \) replaced by \( \hat{d}_{i,j}, \hat{e}_{i,j} \), and \( \hat{y}_h \), respectively:

\[
\hat{F}_{i+1}^{i+1} = d_{i,j} - \sigma \sqrt{t_j - t}, \\
\hat{\hat{d}}_{i,j} = \frac{(r - \frac{1}{2} \sigma^2)(t_j - t)}{\sqrt{t_j - t}}, \\
\hat{\hat{y}}_h = y_h - \sigma \sqrt{T-t}, \text{ and the correlation matrix} \\
\sum_g = \left\{ \rho_{ij}^g \right\}_{i=j=1, \ldots, n+1}^{i=1, j=1, \ldots, n+1} \\
\text{where} \rho_{ij}^g \text{ is given by}^1
\]

\[
\begin{cases}
1, & i = j \\
\frac{\sqrt{t_t - t_j}}{\sqrt{t_g - t_t}}, & 1 \leq i < j \leq g - 1 \\
-\frac{\sqrt{t_t - t_j}}{\sqrt{t_g - t_t}}, & 1 \leq i \leq g - 1, j = g, \\
-\frac{\sqrt{t_t - t_j}}{t_t - t}, & 1 \leq i \leq g - 1, j = n + 1, \\
\frac{\sqrt{t_t - t_j}}{t_t - t}, & g + 1 \leq i \leq n, j = n + 1, \\
0, & \text{otherwise.}
\end{cases}
\]

Accordingly, the closed-form solution for a reset option with \( m \) strike resets and \( n \) pre-decided reset dates \( C(t) \) is

\[
C(t) = S(t) \left[ N(y_m) + \sum_{i=1}^{m} \sum_{g=1}^{n} N_{n+1}(D_{i,g}; \Sigma_g) - N_{n+1}(\hat{D}_{i,g}; \Sigma_g) \right] \\
- \sum_{i=1}^{m} K_i \left[ N_{n+1}(D^g_{i,h}; \Sigma_g) - N_{n+1}(\hat{D}^g_{i,h}; \Sigma_g) \right] \\
- K_g \sum_{g=1}^{n} N_{n+1}(\hat{D}^g_{i,h}; \Sigma_g)
\]

\[\text{Here we define } T = t_{n+1}.\]
In view of (10), we can replicate the reset option by borrowing $M$ dollars and purchasing $\Delta$ shares of stock at price $S(t)$. The amount $\Delta$ and $M$ are as follows:

$$
\Delta = N(y_m) + \sum_{g=1}^{n} \sum_{j=1}^{m} \left[ N_{n+1}(D_g^{l,j}, \Sigma_g) + N_{n+1}(\hat{D}_g^{m,j}, \Sigma_g) \right]
$$

$$
M = \sum_{j=1}^{m-1} K_j e^{-r(T-t)}
\left\{ \sum_{g=1}^{n} \left[ N_{n+1}(\hat{D}_g^{l,j}, \Sigma_g) - N_{n+1}(D_g^{l,j}, \Sigma_g) \right] \right\}
+ K_0 e^{-r(T-t)} \sum_{g=1}^{n} N_{n+1}(\hat{D}_g^{1,0}, \Sigma_g)
+ K_m e^{-r(T-t)} \left[ N(\hat{y}_m) - \sum_{g=1}^{n} N_{n+1}(\hat{D}_g^{m,m}, \Sigma_g) \right]
$$

Similar to the closed-form valuations of exotic options, such as options on the maximum or minimum of several assets (Johnson (1987)), discrete partial barrier options (Heynen and Kat (1996)), reset options (Cheng and Zhang (2000)), or economic models with limited dependent variables, including multinomial probit, panel studies, spatial analysis, and time series analysis, the closed-form solutions for reset options involve the multivariate normal distribution functions.

Among the methods of evaluating multivariate normal cumulative probabilities, as pointed out by Gollwitzer and Rackwitz (1987), Deák (1988), and Vijverberg (1997), Monte Carlo simulator methods seem to be the most promising for higher-order probabilities, preferable over analytical approximations or numerical integration methods. Hajivassiliou, McFadden, and Ruud (1996) surveyed eleven Monte Carlo techniques of evaluating multivariate normal probabilities, they found that the Geweke-Hajivassiliou-Keane (GHK) simulator is overall the most reliable method. Consequently, for the closed-form solution for reset options with a large number of reset dates, we suggest using the GHK simulator to compute the multivariate normal cumulative probabilities.

III.2. Characteristics of Reset Options

First, we discuss some properties of reset options. Consider a one-year maturity reset option with an initial strike price at 100. The strike price will be adjusted if the closing price of the underlying stock falls below 90% or 80% of the initial strike price. We will compare the prices of the reset options with two strike resets and one, two and three reset dates to the plain vanilla call option. We can see that some characteristics of reset options are similar to the standard European call option. For example, the values of reset options are increasing functions of stock price, risk-free interest rate, and the volatility of stock returns. In addition, there are four properties that uniquely exist in reset options. First, the values of reset options are increasing with the number of reset dates. Second, under the same strike resets $D_j$, lower reset strike prices $K_j$ will result in higher values of reset options. Third, due to the more protection toward the holders of reset options, the values of reset options are always greater than that of standard European call option. Finally, in the case of higher values of stock price than the strike reset and smaller volatility of stock returns, the difference between the prices of reset options and plain vanilla call options is insignificant. Take a stock price of 115 and a volatility of stock returns of 10% as an example. In this case, the prices of reset option and plain vanilla call option are almost the same.

When $n$ approaches infinity with a remaining time to maturity $T-t$, the set of discrete reset dates become a continuous reset period. The terminal payoff of a reset option with continuous reset period is as follows:

$$
C(T) = C_t^m + \sum_{j=1}^{m} C_t^{l,j} I_{\min S(t) > D_l}
\left( \sum_{l=1}^{m} C_t^{l} I_{\min S(t) > D_l} \right),
$$

where $C_t^{l} = (S(T) - K_j)^+$. In view of (11), we can replicate the reset option with the following strategy:

1. Purchase one European call option with strike price $K_m$. 

2. Purchase one European down and out call option with strike price \( K_{i-1} \) and barrier \( D_i \), for each \( i = 1, \ldots, m \).
3. Short sell one European down and out call option with strike price \( K_i \) and barrier \( D_i \), for each \( i = 1, \ldots, m \).

Consequently, we can derive the pricing formulas of reset options with a continuous reset period by discovering the prices of down and out call options. Based on the closed-form solutions of European single-barrier options provided by Rubinstein and Reiner (1991)², we have

\[
e^{-r(T-t)} E_0 \left[ \frac{S(T) - K_j}{\ln(S(T)/K_j)} \right] + \sum_{i=1}^{m} I_{\min \{S(t) > D_{i+1}, t \}} [F_i] \]

\[
= \left\{ \begin{array}{l}
S(t) \left[ N(y_j) - \left( \frac{D_{i+1}}{S(t)} \right)^{2r(0.5 \sigma^2)} N\left(f_{i+1}^{(j)}\right) \right] \\
- K_i e^{-r(T-t)} \left[ N\left(\hat{y}_j\right) - \left( \frac{D_{i+1}}{S(t)} \right)^{2r(0.5 \sigma^2)} N\left(f_{i+1}^{(j)}\right) \right]
\end{array} \right.
\]

(12)

where

\[
f_{i+1}^{(j)} = \frac{\ln(D_{i+1}^2/S(t)K_j) + (r \pm 1/2 \sigma^2)(T-t)}{\sigma \sqrt{T-t}}.
\]

Therefore, the price of a reset option with a continuous reset period is

\[
C(t) = S(t) \left[ N(y_j) + \sum_{i=1}^{m} \left( \frac{D_i}{S(t)} \right)^{2r(0.5 \sigma^2)} N\left(f_i^{(j)}\right) \right] - K_i e^{-r(T-t)} \left[ N\left(\hat{y}_j\right) + \sum_{i=1}^{m} \left( \frac{D_i}{S(t)} \right)^{2r(0.5 \sigma^2)} N\left(f_i^{(j)}\right) \right] + \left[ N\left(f_{i+1}^{(j)}\right) + K_i e^{-r(T-t)} \left( \frac{D_{i+1}}{S(t)} \right)^{2r(0.5 \sigma^2)} N\left(f_{i+1}^{(0)}\right) \right]
\]

III. Delta jump, Gamma jump, delta waviness and gamma waviness

We now consider some important properties of reset options, such as Delta jump and Gamma jump. When reset options are issued, the issuers must hedge the risk exposure induced by the reset options. To describe the phenomena of Delta jump and Gamma jump, without loss of generality, we simplify the reset options with only one reset date. Let us define the following expressions:

\[
X(i, j) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-d_{i,j}^2}{2}\right) N(G_{i,j})
\]

\[
Y(i, j) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-y_{i,j}^2}{2}\right) N(Z_{i,j})
\]

\[
DX(i, j) = \frac{\exp\left(\frac{-2y_{i,j}}{2}\right)}{\sigma S(t) \sqrt{2\pi}} \left[ N\left(G_{i,j}\right) d_{i,j} + \rho \exp\left(\frac{-2y_{i,j}^2}{2}\right) \right]
\]

\[
DY(i, j) = \frac{\exp\left(\frac{-2y_{i,j}}{2}\right)}{\sigma S(t) \sqrt{2\pi}} \left[ N\left(Z_{i,j}\right) y_{i,j} + \rho \exp\left(\frac{-2y_{i,j}^2}{2}\right) \right]
\]

where \( d_{i,j} = \frac{d_{i,j-1} - \rho y_{i,j}}{\sqrt{1 - \rho^2}} \) ; \( \hat{X}, \hat{Y}, \hat{D}\hat{X}, \) and \( \hat{D}\hat{Y} \) are the expressions with \( d_{i,j} \) and \( y_{i,j} \) replaced by \( \hat{d}_{i,j} \) and \( \hat{y}_{i,j} \), respectively. Thus the delta and gamma of reset options with one reset date are as follows:

\[
\text{Delta}(t, S(t)) = N(y_m) + \sum_{i=1}^{m} \left[ N(d_{i,1}, y_{i,1}, \Sigma_1) - N(d_{i,1}, y_i, \Sigma_1) \right] + \frac{1}{\sigma \sqrt{t_i - t}} \left[ X(l-1, l) - X(l, l) \right]
\]

\[
\Delta(t, S(t)) = \left\{ \begin{array}{l}
\frac{1}{\sigma \sqrt{T-t}} \left[ Y(l, l) - Y(l, l) \right] \\
+ \frac{1}{\sigma \sqrt{T-t}} \left[ X(l, l+1) - X(l, l) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ \hat{X}(0,1) + \frac{1}{\sigma \sqrt{T-t}} \hat{Y}(0,1) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ \hat{X}(m, m) + \frac{1}{\sigma \sqrt{T-t}} \hat{Y}(m, m) \right] \\
\end{array} \right.
\]
\]

\text{Gamma}(t, S(t)) = \frac{\exp(-\frac{y^2}{2})}{S(t)\sigma \sqrt{2\pi(T-t)}} + \sum_{j=1}^{m} \frac{1}{\sqrt{t_j - t}} \left[ X(l, l) - X(l, l) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ Y(l, l) - Y(l, l) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ DX(l, l) - DX(l, l) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ DY(l, l) - DY(l, l) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ \hat{X}(0,1) + \frac{1}{\sigma \sqrt{T-t}} \hat{Y}(0,1) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ \hat{X}(m, m) + \frac{1}{\sigma \sqrt{T-t}} \hat{Y}(m, m) \right] \\
\]

\]

Consequently, the Delta and Gamma at time \( t_i \) are as follows:
\[\Delta(t_i, S(t_i)) = N(y_0)_{i=t_i}\]
\[\text{Gamma}(t_i, S(t_i)) = \exp(-\frac{y_i^2}{2}) S(t_i)\sigma \sqrt{2\pi(T-t_i)} \mid_{t=t_i}\]

However, the delta and gamma at \( t > t_1 \)
are given by the following expressions:
\[\Delta(t, S(t)) = N(y_0)_{i=t} + \sum_{j=1}^{m} N(y_j)_{i=t} \left[ D_j \geq S(t_i) > D_{j+1} \right] \]
\[\text{Gamma}(t, S(t)) = \frac{\exp(-\frac{y_0^2}{2})}{S(t)\sigma \sqrt{2\pi(T-t)}} + \sum_{j=1}^{m} \frac{1}{\sqrt{t_j - t}} \left[ X(l, l) - X(l, l) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ Y(l, l) - Y(l, l) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ DX(l, l) - DX(l, l) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ DY(l, l) - DY(l, l) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ \hat{X}(0,1) + \frac{1}{\sigma \sqrt{T-t}} \hat{Y}(0,1) \right] \\
+ \frac{1}{\sqrt{T-t}} \left[ \hat{X}(m, m) + \frac{1}{\sigma \sqrt{T-t}} \hat{Y}(m, m) \right] \\
\]

We can see that the Delta and Gamma at \( t_i \) are continuous only when the condition \( S(t_i) \geq D_1 \)
holds. Therefore, Delta jump and Gamma jump exist when the stock price at \( t_i \) is below \( D_1 \). In other words, we
should carefully implement the Delta and Gamma hedges on the reset dates in the case that the stock price is below the highest strike
reset.

In addition to the properties of Delta jump and Gamma jump on the reset dates,
there exist the phenomena of Delta waviness and Gamma waviness before the reset dates,
especially near the reset dates. Consider the following example. The stock price is currently $100, and the strike price of the
reset option will be adjusted if the stock price falls below 80%, 70%, 60%, 50%, and 40% of initial strike price $100 three months later.
Assume the risk-free interest rate is 5% and
the volatility of stock returns is 30%. We illustrate the properties of Delta waviness and Gamma waviness in Figure 1 and Figure 2, respectively. As shown in the Figures, unlike the Delta and Gamma of the plain vanilla call options, which are definitely non-negative, the Delta and Gamma of reset options will fluctuate dramatically and can be negative as the time approaches the reset dates. When the stock prices are away from the neighborhoods of strike resets, the behaviors of Delta and Gamma are the same as that of plain vanilla call options. However, if the stock prices are near strike resets, the Delta and Gamma will oscillate. The phenomena are more significant when the time approaches the reset dates. From Figure 1, if the time approaches the reset dates, the Delta is a local minimum when the stock price touches strike reset, but the Delta is a local maximum when the stock price is at about the middle of two adjacent strike resets. The dramatic change of Delta between two adjacent strike resets also increases the difficulty of risk management. The wavinesses of Delta and Gamma are as important as Delta jump and Gamma jump in hedging reset options.

IV. Evaluation of the Study

We have provided the closed-form pricing formula for reset options with \( m \) strike resets and \( n \) pre-decided reset dates. In addition to the Delta jump and Gamma jump across the reset dates, we have also discovered the phenomena of Delta waviness and Gamma waviness near the reset dates. For future research, it would be interesting to investigate the hedging strategies of reset options due to the phenomena of Delta jump and Delta waviness across the reset dates.
across the strike resets. The phenomenon of Gamma waviness is more significant when time approaches the reset dates.

V. References