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訊息風險與持股限制對國際投資策略及證券評價之影響

The Impact of International Risk and Holding Constraints on International Portfolio Strategy and Security Pricing

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摘要

本文探討國外證券訊息差異化對國際投資策略及均衡的證券評價的影響。訊息差異化影響下的國際投資組合及其對應的資產評價與無差異化訊息時相當不同。在外人持股限制下，本國投資者對外國證券依其訊息高低要求付出不同的貼水。而外國投資者對其本國證券依其訊息高低要求不同的折扣依訊息高低而有兩種些貼水與折扣的關係。當證券收益的共變異矩陣為未知時，國際投資組合的證券評價與已知之情況相當不同。

關鍵字：國際投資組合、證券評價、訊息差異、持股限制
Abstract

The effect of differential information on international portfolio demands and equilibrium asset pricing relationships are examined. International portfolio selections and the corresponding asset pricing relationships under differential information are divergent from those without regard for differential information. When a foreign ownership constraint is imposed, the domestic investors pay different premiums for the high and low information foreign securities, whereas the foreign investors demand different discounts for their own domestic high and low information securities. There are two relationships between the premiums and discounts - one for the high information foreign securities and another for the low information foreign securities. When the covariance matrices of security payoffs are unknown, international portfolio demands and the corresponding asset pricing relationships are substantially divergent from those when the covariance matrices were known.

Keyword: international portfolio, asset pricing, differential information, foreign ownership constraint.
I. Introduction

Documented researches on international portfolio selections and equilibrium asset pricing relationships have frequently appeared in the finance literature. Some important changes and contributions to the theory of international finance are numerous as a result. Important contributors in the study of the exchange risk effect on international portfolio behavior and equilibrium asset pricing relationships are, to name a few, Solnik (1974), Grauer, Litzenberger and Stehle (1976), Stulz (1981), and Adler and Dumas (1983). Other contributors who examine barriers to international investments are Black (1974), Stulz (1981), Stapleton and Subrahmanyam (1977), Errunza and Losq (1985, 1989), and Eun and Janakiramanan (1986). All of their research findings have helped us understand better as to how exchange risk affects equilibrium asset pricing through its impact on portfolio behavior and how international asset pricing will be changed under barriers to international investment.

Equilibrium asset pricing relationships in the domestic or international setting are established on the basis of individual portfolio behavior under uncertainty. Individual portfolio decisions involve choices among alternative probability distributions of asset returns (or prices) with known mean returns and return variances. As a result, the empirical implications of equilibrium asset pricing models are traditionally based on the assumption of known mean returns and variances. Thus, individual portfolio choice, the corresponding equilibrium asset pricing relationships and the empirical implications have not recognized the fact that the parameters associated with the probability distributions of asset returns are rarely known with certainty. In the real world, investors have to make their portfolio choices
based on the information available for investment. The amounts of information for investment include historical observations of security returns, reports of security analysts, press releases, firms' financial reports, etc. The amounts of information for investment may not be sufficient and available for all securities. Some securities have high (or sufficient) information available for investors' portfolio choices while others are low (or insufficient) in information for investment. Securities with low information for investment induce more risk due to more parameter uncertainty than those with high information. It is, therefore, important to examine the effect of differential information on investors' portfolio demands for high and low information securities and hence equilibrium asset pricing relationships.

Barry and Brown (1985) examined the effect of differential information on portfolio risk and some possible implications for capital market equilibrium. A portfolio's variance is shown to be greater under differential information than under equal information. In addition, a portfolio consisting of more high information securities have smaller systematic risk than it would under equal estimation risk.1 As a result, a portfolio with high information securities may appear to earn a low (or even negative) abnormal return due to the overestimate of its beta (without regard for differential information), whereas a portfolio with relatively low information securities tends to earn a positive abnormal return. However, portfolio demand functions and equilibrium asset pricing relationships under

1Cole and Loewenstein (1988) examined equilibrium pricing and portfolio selection under symmetric or equal estimation risk, with a more reasonable assumption that the end-of-period security payoffs are given by nature and technology (exogenous) and by investment decisions as made previously. Under their assumption, symmetric estimation risk affects equilibrium asset pricing, expected returns, market weights and betas.
differential information have not explicitly derived and examined, especially in the international setting. In this paper, the effect of differential information on international portfolio selections and equilibrium pricing relationships will be examined in detail. In the international setting, not only the information related to domestic investment is usually insufficient for some domestic securities, but also many foreign securities may have relatively low information for the domestic investors. Differential information is thus an important issue that cannot be ignored in the analysis of international portfolio selections and the corresponding international asset pricing models. Some of the important results of this study are briefly given below.

1. International portfolio demand functions and the equilibrium prices for the high and low information domestic and foreign securities differ from those without regard for differential information. Moreover, international portfolio selections and the equilibrium prices become an extreme case when the covariance matrices of securities payoffs are unknown.

2. When the foreign ownership is restricted (i.e., under the so-called $\delta$ constraint), the premiums and discounts for the high and low information foreign securities are quite different in value from those without regard for differential information such as documented in Eun and Janakiramanan (1986) (EJ, hereafter). There are two relationships between the premiums and discounts, one for the high information foreign securities and another for the low information foreign securities, whereas only one relationship is documented by EJ. However, when the covariance matrices are unknown, the domestic investors are willing to
pay a premium only for the high information foreign securities while the foreign investors receive a discount for the same securities.

3. The equilibrium pricing relationships under the $\delta$ constraint for the high and low information foreign securities are priced distinctly from those without regard for differential information. When the covariance matrices are unknown, the equilibrium pricing relationships are applicable only to the high information securities.

In the second section of this paper, the effect of differential information on international portfolio selections is examined. In Section III, international asset pricing is developed and examined under differential information. The results are extended to investigate the effect of differential information on international asset pricing under an ownership constraint, which is given in Section IV. A conclusion is drawn in Section V.

II. Differential Information and International Portfolio Selections

In the analysis that follows, a world is assumed to have two countries - the domestic country (D) and the foreign country (F). Since the major concern of this paper is to examine the effect of differential information on international portfolio selections and the corresponding equilibrium asset pricing relationships, exchange rate risk is irrelevant to the analysis. Thus, a fixed exchange rate regime is assumed for both countries. The risk-free rate of interest expressed in either domestic or foreign currency is assumed to be identical. In addition, the following assumption are used throughout the analysis.

1. The domestic and foreign capital markets are perfectly competitive.

2. No transaction costs and taxes exist in each country.
3. The distribution of end-of-period payoffs (or prices) on the domestic and foreign securities is a multivariate normal distribution.

4. The domestic and foreign investors have homogeneous expectations regarding to the distributions of security payoffs.

5. Both domestic and foreign investors can borrow or lend at the risk-free rate of interest \( r \).

Additional assumptions will be specified when needed in the subsequent analysis.

Assume that investor \( k \) is endowed with the initial supplies of the domestic and foreign risky securities and other wealth, \( W_{0}^{k} \). Then investor \( k \)'s budget constraint for assets can be expressed as

\[
W_{0}^{k} + Z_{D}^{k} P_{D} + Z_{F}^{k} P_{F} = z_{D}^{k} P_{D} + z_{F}^{k} P_{F} + z_{f} P_{0f}
\]

where \( Z_{D}^{k} (Z_{F}^{k}) \) = vector of the initial supplies of the domestic (foreign) risky securities with typical elements \( z_{iD}^{k} (z_{iF}^{k}) \).

\( z_{D}^{k} (z_{F}^{k}) \) = vector of the number of shares of the domestic (foreign) securities demanded by investor \( k \) with typical element \( z_{iD}^{k} (z_{iF}^{k}) \),

\( z_{f} \) = the number of units of the riskless asset demanded by investor \( k \),

\( P_{D} (P_{F}) \) = vector of the initial prices of the domestic (foreign) securities which are known, and

\( P_{0f} \) = the initial price of the riskless asset with infinitely elastic supply at price \( P_{0f} = 1 \).\(^{2}\)

\(^{2}\)With this assumption, equilibrium will exist without explicitly considering the price of the riskless asset.
The random end-of-period wealth of investor $k$ is given by

$$
\tilde{W}^k(z_D^k, z_F^k, P_D^k, P_F^k) = (z_D^k \tilde{p}_D^k + z_F^k \tilde{p}_F^k) + z_F^k P_{lf}^k
$$

$$
- (z_D^k \tilde{p}_D^k + z_F^k \tilde{p}_F^k) + \left[ u^k_0 + (z_D^k - z_D^k)^\prime P_D + (z_F^k - z_F^k)^\prime P_F \right] (1 + r_f^k) \tag{2}
$$

where $\tilde{p}_D^k (\tilde{p}_F^k)$ = vector of the random end-of-period payoffs on the domestic (foreign) securities

$P_{lf}^k$ = the certain end-of-period price per unit of the riskless asset

$r_f^k = (P_{lf}^k - P_{0f}) / P_{0f}$, the riskless rate of return.

Then, the expected end-of-period wealth and its variance under differential information with the known covariance matrices are, respectively, given by

$$
\tilde{w}^k = (z_D^k \tilde{\eta}_D^k + z_F^k \tilde{\eta}_F^k) + [u^k_0 (z_D^k - z_D^k)^\prime P_D + (z_F^k - z_F^k)^\prime P_F] (1 + r_f^k) \tag{3}
$$

$$
\text{Var}(\tilde{w}^k) = z_D^k \Sigma_D z_D^k + 2z_D^k \Sigma_{DF} z_F^k + z_F^k \Sigma_F z_F^k
$$

$$
= \sum_{ij} z_D^i \eta_{Dij}^k h_{ij}^D \eta_{ij}^D + 2 \sum_{ij} z_D^i \eta_{Dij}^k h_{ij}^F \eta_{ij}^F + \sum_{ij} z_F^i \eta_{ij}^F h_{ij}^D \eta_{ij}^D + \sum_{ij} z_F^i \eta_{ij}^F h_{ij}^F \eta_{ij}^F \tag{4b}
$$

where $\tilde{\eta}_D (\tilde{\eta}_F)$ = vector of the predictive mean payoffs on the domestic (foreign) securities;

$\Sigma_D (\Sigma_F)$ = the predictive covariance matrix of payoffs on the domestic (foreign) securities with typical elements $h_{ij}^D \Omega_{ij}^D$ ($h_{ij}^F \Omega_{ij}^F$);

$\Omega_{ij}^D$ ($\Omega_{ij}^F$) is the covariance between the end-of-period payoffs on domestic (foreign) securities $i$ and $j$; $h_{ij}^D$ and $h_{ij}^F$ represent, respectively, the effects of differential information associated with the domestic and foreign securities. 3

$\Sigma_{DF}$ = the covariance matrix of the prices of the domestic and foreign securities;

3The explicit expressions for $h_{ij}^D$ ($h_{ij}^F$) will be given later. See Kalymon (1971) for the predictive covariance matrices under differential information.
\( h_{ij}^{DF} \) is the predictive covariance of a domestic security (i) and a foreign security (j) which is the typical element of the predictive covariance matrix of \( \tilde{P}_D \) and \( \tilde{P}_F \). \( \Sigma_{DF} \).

\( h_{ij}^{DF} \) captures the impact of differential information on the \((i,j)\)th element of the covariance matrix of \( \tilde{P}_D \) and \( \tilde{P}_F \).

Note that both the expected end-of-period wealth \( \tilde{W}^k \) in (3) and the variance \( \text{Var}(\tilde{W}^k) \) in (4a-b) have already reflected the impact of differential information, and do not depend on the unknown parameters such as the original means, variances and covariances.

By taking explicitly differential information into account, the investor's optimization problem is to choose the demand vectors of the domestic and foreign securities \( z_D^k \) and \( z_F^k \) that maximize the unconditional (or predictive) expected utility of the end-of-period wealth given by 4

\[
E_\theta E_{P|\theta} [U(\tilde{W}^k(z_D^k, z_F^k, P_D, P_F)) - \int \int U(\tilde{W}^k(z_D^k, z_F^k, P_D, P_F)) \cdot g_1(\tilde{P}|\theta) g_2(\theta) d\tilde{P} d\theta - f[\tilde{W}^k, \text{Var}(\tilde{W}^k)]
\]

\[(5a)\]

where \( U(\cdot) \) is a von Neumann-Morgenstern utility function,

\( \theta \) denotes the parameter space of payoffs, \( E(\tilde{P}_D) \) and \( E(\tilde{P}_F) \),

\( g_1(\tilde{P}|\theta) \) is the distribution of the end-of-period payoffs on the domestic and foreign securities \( \tilde{P}_D \) and \( \tilde{P}_F \), conditional on the parameter space \( \theta \)

\( g_2(\theta) \) is the distribution of parameters, and

\[
\int g_1(\tilde{P}|\theta) g_2(\theta) d\theta
\]

is the predictive distribution of the end-of-period payoffs on the domestic and foreign securities.

\[4\] See Klein and Bawa (1976) for a detailed examination of the unconditional solution for utility maximization under parameter uncertainty.
Note that, in defining the unconditional utility function given in (5a-b), the end-of-period payoffs are assumed to be governed by primitive uncertainty such as nature, weather, etc., and by investors' portfolio decisions that reflect differential information. Since investors' portfolio decisions will reflect differential information, the equilibrium prices and expected returns will be adjusted for the impact of differential information, and hence differential information is relevant to market equilibrium. In contrast, previous studies as documented by Barry (1978), Bawa and Brown (1979), and Brown (1979) treated security returns and expected returns as exogenous - depending on initial asset prices. As a result, estimation risk was found, in their studies, to be irrelevant to market equilibrium.⁵ We shall show that differential information is relevant to international portfolio selections and asset pricing relationships in the following section.

Using the unconditional expected utility given in (5b), the first-order conditions for investor k's optimal portfolio choice can be obtained from (5) and shown to be

\[
\hat{\eta}_D - P_D (1 + r_F) = \left[ \frac{-2f_{\sigma}^k}{f_{\sigma}^k} \right] \left( \hat{\eta}_D z_D^k + \hat{\eta}_D z_F^k \right)
\]

(6a)

\[
\hat{\eta}_F - P_F (1 + r_F) = \left[ \frac{-2f_{\sigma}^k}{f_{\sigma}^k} \right] \left( \hat{\eta}_F z_F^k + \hat{\eta}_D z_D^k \right)
\]

(6b)

where \( f_{\sigma}^k \) and \( f_{\sigma}^k \) are the derivatives of the investor's utility function with respect to the variance of wealth and the variance of the asset returns, respectively.

The term, \(-2f_{\sigma}^k / f_{\sigma}^k\) in (6) represents the measure of investor k's risk aversion which depends on the predictive (or unconditional) expected utility of end-of-period wealth. Differential information, thus, changes the investor's risk aversion measure. In addition, differential information also

⁵See Coles and Loewenstein (1988) for a detailed examination of this issue based on symmetric estimation risk.
affects the investor’s demands for the domestic and foreign securities \( z_D^k \) and \( z_F^k \) through its impact on the equilibrium prices \( P_D \) and \( P_F \) and the predictive covariance matrices which consist of information quantity indicators \( h_{ij}^D \), \( h_{ij}^F \), and \( h_{ij}^{DF} \) induced by differential information.  

A. Differential Information and Portfolio Demand by the Domestic Investors

To find the demand equations for investor \( k \) in the domestic country, we solve the system of demand functions in (6a) and (6b) for \( z_D^k \) and \( z_F^k \) to yield the following:

\[
z_D^k = \frac{1}{G^k} \left( V_D \left[ \hat{\eta}_D - P_F (1 + r_F) \right] + V_{DF} \left[ \hat{\eta}_F - P_F (1 + r_F) \right] \right)
\]

\[
z_F^k = \frac{1}{G^k} \left( V_{DF} \left[ \hat{\eta}_D - P_D (1 + r_D) \right] + V_F \left[ \hat{\eta}_F - P_F (1 + r_F) \right] \right)
\]

where \( G^k = (-2\theta^k_{\alpha}/\theta^k_{\alpha}) \), the measure of investor \( k \)'s risk aversion under differential information,

\[
V_D = \begin{bmatrix} \Sigma_D & \Sigma_{DF} \Sigma_F^{-1} \Sigma_{DF} \end{bmatrix}^{-1},
\]

\[
V_{DF} = -\Sigma_D^{-1} \Sigma_{DF} \left( \Sigma_F - \Sigma_{DF} \Sigma_D^{-1} \Sigma_{DF} \right)^{-1},
\]

\[
V_F = \left( \Sigma_F - \Sigma_{DF} \Sigma_D^{-1} \Sigma_{DF} \right)^{-1}.
\]

The aggregate demand functions for the domestic investors are obtained by aggregating the individual demand functions in (7) and (8) over all domestic investors. This leads to the following:

\[
z_D^d = \frac{1}{G^D} \left( V_D \left[ \hat{\eta}_D - P_D (1 + r_D) \right] + V_{DF} \left[ \hat{\eta}_F - P_F (1 + r_F) \right] \right)
\]

\( ^6 \)Securities with high information have their information quantity indicators close to 1 whereas low information securities are associated with information quantity indicators departing far from 1. The information quantity indicators will be defined later [(11), (12), and (13)].
\[
    z_F^d = \frac{1}{G^D} \left( V_{DF} [\eta_D - P_D (1 + r_F^D)] + V_F [\eta_F - P_F (1 + r_F^F)] \right)
\]

where \( z_D^d = \sum_{k \in D} z_D^k \), the aggregate demand function for the domestic securities by the domestic investors,

\( z_F^d = \sum_{k \in D} z_F^k \), the aggregate demand function for the foreign securities by the domestic investors, and

\[
    1 - \frac{1}{G^D} = \sum_{k \in D} \frac{1}{G^k},
\]

the measure of aggregate risk aversion of the domestic investors under differential information.

The demand functions for the domestic investors in (9) and (10) are determined by the predictive covariance matrices of the domestic and foreign security payoffs. To examine the differential information effect on the demand functions, the domestic and foreign securities are classified into two sets of securities according to the amounts of information available to the domestic investors for investment. Suppose that among \( S_F^D \) (\( S_F^F \)) domestic (foreign) securities, there are \( S_D^D \) (\( S_F^F \)) domestic (foreign) securities with a high amount of information for investment. The remaining subset of the domestic (foreign) securities with a low amount of information is denoted by \( S_D^D - S_D^D \) (\( S_F^F - S_F^F \)). In particular, suppose that there are \( n_D^D \) (\( n_F^F \)) observed returns for the first \( S_D^D \) (\( S_F^F \)) domestic (foreign) securities, but only \( n_D^D < N_D \) (\( n_F^F < N_F \)) observed returns for the remaining \( S_D^D - S_D^D \) (\( S_F^F - S_F^F \)) domestic (foreign) securities.\(^7\)

Assume that the covariance matrices characterizing

\(^7\)In reality, the number of the high-information foreign securities from the viewpoint of the domestic investors (\( N_F^F \)) is most likely to be smaller than the number of the low-information foreign securities (\( n_F^F \)). This is due to the fact that information regarding to the foreign securities is costly and usually difficult to obtain by the domestic investors. The foreign investors face a similar situation in which information associated with most domestic securities is insufficient from their viewpoint. However, due to increasing liberalization of world financial markets, domestic and foreign investors can
the payoff distributions of the domestic and foreign securities are known and positive definite. Then, the only insufficient information for investment is the information regarding to the mean payoffs on the domestic and foreign securities. With diffuse prior for the mean payoffs, the predictive distribution of security payoffs is multivariate normal. Kalymon (1971) and Barry and Brown (1985) have provided the predictive covariance matrices under differential information. Using their results, the predictive covariance matrices \( \Sigma_D, \Sigma_{DF}, \) and \( \Sigma_F \) contained in \( V_D, V_F \) and \( V_{DF} \) of equation (7) and (8) can be expressed as follows:

\[
\Sigma_D = \begin{bmatrix}
  h(N_D^D)\Sigma_{HH}^D & h(N_D^D)\Sigma_{HL}^D \\
  h(N_D^D)\Sigma_{LH}^D & h(n_2^D)\Sigma_{LL}^D
\end{bmatrix}
\]

(11)

\[
\Sigma_F = \begin{bmatrix}
  h(N_F^F)\Sigma_{HH}^F & h(N_F^F)\Sigma_{HL}^F \\
  h(N_F^F)\Sigma_{LH}^F & h(n_2^F)\Sigma_{LL}^F
\end{bmatrix}
\]

(12)

\[
\Sigma_{DF} = \begin{bmatrix}
  h(N_1^D)\Sigma_{HH}^{DF} & h(N_2^D)\Sigma_{HL}^{DF} \\
  h(N_3^D)\Sigma_{LH}^{DF} & h(n_4^D)\Sigma_{LL}^{DF}
\end{bmatrix}
\]

(13)

where \( h(n) = 1 + 1/n, \) \( n = N_D, n_2^D, N_F \) and \( n_2^F, \)

\( h(N_1^D) = 1 + 1/\max(N_D, N_F), \)

\( h(N_2^D) = 1 + 1/\max(N_D, n_2^F), \)

\( h(N_3^D) = 1 + 1/\max(n_2^D, N_F), \)

\( h(N_4^D) = 1 + 1/\max(n_2^D, n_2^F). \)

Invest globally via international mutual funds and country funds that are provided by mutual fund companies and large brokerage firms. These mutual fund companies and large brokerage firms at home and abroad should have a high amount of information for domestic and foreign investments. As a result, we may assume that, through investments in international mutual funds and country funds, both domestic and foreign investors have the same amount of information for investment in our analysis. Otherwise, the resulting demand functions and equilibrium prices cannot be solved in an explicit form, and a meaningful economic analysis cannot be carried out.
B. Differential Information and Portfolio Demand by Foreign Investors

In a similar manner as shown above, the aggregate demand functions for the domestic (foreign) securities by the foreign investors, \( z_D^f (z_F^f) \), can be written as follows:

\[
\begin{align*}
z_D^f &= \frac{1}{G^F} \left( V_D \left[ \hat{\eta}_D - P_D (1 + r_f) \right] + V_{DF} \left[ \hat{\eta}_F - P_F (1 + r_f) \right] \right) \\
z_F^f &= \frac{1}{G^F} \left( V_{DF} \left[ \hat{\eta}_D - P_D (1 + r_f) \right] + V_F \left[ \hat{\eta}_F - P_F (1 + r_f) \right] \right)
\end{align*}
\] (14) (15)

where \( \frac{1}{G^F} = \sum_{q \in F} \frac{1}{G^q} \), and \( \frac{1}{G^q} \) is the measure of foreign investor \( q \)'s risk aversion in the presence of differential information. Again, the aggregate demand functions for the foreign investors depend on the predictive variances and covariances of the domestic and foreign securities, which capture the impact of differential information for investment. The aggregate demand functions for the domestic and foreign investors \( z_D^d, z_F^d, z_D^f, \) and \( z_F^f \) with differential information have been shown to be different in equilibrium value from those assuming away differential information such as given in EJ\(^8\). As a result, the following proposition is established.

**Proposition 1.** With differential information, the aggregate demand functions for the domestic and foreign investors differ in equilibrium value from those when differential information is assumed away. That is,

\[
\begin{align*}
&z_D^d = *z_D^d, \ z_F^d = *z_F^d, \\
&z_D^f = *z_D^f, \ z_F^f = *z_F^f.
\end{align*}
\]

In addition, \( \frac{1}{G^D} = \frac{1}{G^D} \) and \( \frac{1}{G^F} = \frac{1}{G^F} \), where the variables associated with asterisks represent the aggregate demand functions in the absence of differential information.

\(^8\)See Footnote 12
differential information.\footnote{The demand-functions without regard for differential information are similar to those given in (9), (10), (14), and (15). For example, with the relevant variables affixed with asterisks, the aggregate demand function for the domestic risky assets by the domestic investors when differential information is not admitted is given by}

The aggregate demand functions for the domestic and foreign securities by the domestic and foreign investors are influenced by differential information in three ways. First, the aggregate domestic and foreign demand functions with differential information depend on the initial prices of the domestic and foreign securities \( (P_D \text{ and } P_F) \) which are endogenously determined equilibrium prices, whereas those without regard for differential information are exogenously determined. Second, differential information enters into the determination of the aggregate domestic and foreign demand functions through its impact on the predictive variance-covariance submatrices \( (\Sigma^D_{HH}, \Sigma^D_{HL}, \Sigma^D_{LL}, \Sigma^F_{HH}, \Sigma^F_{HL}, \text{ etc.}) \), which consist of the information quantity indicators. In contrast, the variance-covariance matrices \( (\Sigma^D, \Sigma^F \text{ and } \Sigma^{DF}) \) when differential information is assumed away involve no adjustment factors for differential information. Third, the aggregate demand functions are determined by domestic and foreign investors' risk aversion which capture the differential information impact. As a result, international portfolio selections of the domestic and foreign investors under differential information will deviate from those without regard for differential information. The corresponding international asset pricing relationships will also be affected when differential information is recognized in the

\[ z^*_d = \frac{1}{C^*_d} \left( V^*_D \left[ \eta_D - P_D^* (1 + r_e) \right] + V^*_D \left[ \eta_F - P_F^* (1 + r_e) \right] \right). \]

But, \( V^*_D = V_D, \ P^*_D = P_D, \ V^*_D = V_D, \text{ and } \ P^*_F = P_F. \)
analysis. This issue will be explored later.

C. Differential Information and Portfolio Selection When Covariance Matrices Are Unknown

In the domestic setting, Klein and Bawa (1976 and 1977) examined the diversification effect of symmetric estimation risk for the cases where covariance matrices are both known and unknown. They showed that the adjustment factors for (symmetric) estimation risk ($h(n)$) when the covariance matrices are unknown are larger than those with the known covariance matrices. Using this result, Barry and Brown (1985) provided a simple example showing that the effect of estimation risk can be substantially more pronounced with the unknown covariances than with the known covariances.

Moreover, Klein and Bawa (1977) extended their study of the diversification effect to the case of the unknown covariance matrices when the amounts of information available for investment are asymmetric (or differential). They showed that if the prior distribution of means and variance-covariance matrices is noninformative, securities with (relatively) low information will introduce an arbitrarily large amount of estimation risk into a portfolio's risk. This leads to an infinite predictive portfolio variance. As a result, a risk-averse investor will limit his portfolio demand only for high information securities. Hence, differential information with the unknown covariance matrices has a much more profound impact on portfolio selections than the case with symmetric estimation risk.

In the international setting, the domestic investors look for foreign investment opportunities to diversify their portfolio risk. Without estimation risk, international diversification can be achieved on the basis of the fundamental risk-return characteristics of the foreign securities in relation to the domestic securities. However, with differential information,
some foreign securities that are thought to be desirable for international diversification will become undesirable if their investment information for the domestic investors is insufficient. As a result, international portfolio selections by the domestic investors under differential information will be quite different from those without regard for differential information. Since the information for investment associated with the foreign securities is generally more difficult to obtain by the domestic investors than by the foreign investors, the effect of differential information actually reduces the number of foreign securities that are desirable for international diversification. The implication is that international diversification may be costlier or less effective under differential information than otherwise.

Under differential information with the unknown covariance matrices, the domestic (or foreign) investors will limit their portfolios to investment in the high-information domestic and foreign securities. In particular, the domestic (or foreign) risk-averse investors will entirely avoid holding (relatively) low information domestic and foreign securities since those securities contain arbitrarily large estimation risk. In an international market equilibrium, the null demands for the low information domestic and foreign securities imply that the prices of those securities will be driven to zero. This result has an interesting implication for the pricing of the foreign securities with a constraint on the fraction of the foreign securities that are allowed to be held by foreigners. According to EJ, the domestic investors will be willing to pay premiums for the foreign securities under the ownership constraint and the foreign investors will demand for a discount for the purchase of their own domestic securities. However, under differential information with the unknown covariance matrices, those foreign securities with insufficient information will have, in equilibrium, a (or
almost) zero demand by the domestic (and foreign) risk-averse investors. A null demand for the low information foreign securities will evaporate the premiums and discounts for the purchase and sale of this type of securities. In addition, the risk-averse investors will be likely to pay higher premiums for the high information foreign securities than for the otherwise identical securities, since differential information with the unknown covariance matrices reduces the international investment opportunity set in terms of the number of desirable foreign securities available for international diversification. Similarly, the high information foreign securities will be selling at a lower discount for the foreign investors than will the otherwise identical securities. Nevertheless, the low information foreign and domestic securities will be selling at extremely low prices or even at zero value.

In the environment of international investment nowadays, information for foreign investment is usually either insufficient or even not accessible to the domestic investors for some foreign securities. The effect of differential information is thus important to be recognized in the pricing of international securities.

### III. Differential Information and International Asset Pricing

To consider international asset pricing with differential information, the world aggregate demand functions for the domestic and foreign securities are obtained by aggregating (9), (10), (14) and (15) across the two countries with the market-clearing conditions given by

\[ z_i^d + z_i^f = Z_i, \text{ i} \in D \quad \text{(or } z_D^d + z_D^f = Z_D) \]

and \[ z_i^d + z_i^f = Z_i, \text{ i} \in F \quad \text{(or } z_F^d + z_F^f = Z_F) \].

where \( z_i^d \) (\( z_i^f \)) is the number of shares of domestic security \( i \) held by the
domestic (foreign) investors and $z_i^d (z_i^f)$ for $i \in F$ has a similar definition.

A. With the Known Covariance Matrices

It is easy to show that with differential information the world aggregate demand functions for the domestic and foreign securities ($Z_D$ and $Z_F$) are given by

$$
\begin{pmatrix}
Z_D \\
Z_F
\end{pmatrix} = \frac{1}{G^W} \begin{pmatrix}
V_D & V_{DF} \\
V_{DF} & V_F
\end{pmatrix} \begin{pmatrix}
\hat{\eta}_D - \tilde{P}_D (1 + r_f^D) \\
\hat{\eta}_F - \tilde{P}_F (1 + r_f^F)
\end{pmatrix} \tag{16}
$$

where $\frac{1}{G^W} = \frac{1}{G^D} + \frac{1}{G^F}$. Solving (16) for $P_D$ and $P_F$ yields the equilibrium prices for the domestic and foreign securities under differential information:

$$
\begin{pmatrix}
P_D \\
P_F
\end{pmatrix} = \frac{1}{1 + r} \begin{pmatrix}
\dot{\eta}_D - G^W \begin{pmatrix}
\Sigma_D & \Sigma_{DF} \\
\Sigma_{DF} & \Sigma_F
\end{pmatrix} \begin{pmatrix}
Z_D \\
Z_F
\end{pmatrix}
\end{pmatrix} \tag{17}
$$

To analyze the effect of differential information, the vector of equilibrium prices in (17) is decomposed into two classes of equilibrium prices corresponding to the high and low information securities. By applying the definitions of the predictive covariance matrices given in (11), (12), and (13) to (17), the equilibrium prices of the high and low information domestic securities, denoted respectively by $P_{DH}$ and $P_{DL}$, can be written as follows:

$$
P_{DH} = \frac{1}{1 + r} \left\{ \hat{\eta}_{DH} - G^W \begin{pmatrix}
[h(N^D) \Sigma_{HH} Z_{DH} + h(N^D) \Sigma_{HL} Z_{DL}] \\
+ [h(N_1^D) \Sigma_{HH} Z_{FH} + h(N_2^D) \Sigma_{HL}^D Z_{FL}] 
\end{pmatrix} \right\} \tag{18}
$$

and

$$
P_{DL} = \frac{1}{1 + r} \left\{ \hat{\eta}_{DL} - G^W \begin{pmatrix}
[h(N^D) \Sigma_{HH} Z_{DH} + h(N_2^D) \Sigma_{LL} Z_{DL}] \\
+ [h(N_3^D) \Sigma_{HH} Z_{FH} + h(N_4^D) \Sigma_{LL}^D Z_{FL}] 
\end{pmatrix} \right\} \tag{19}
$$
where $\tilde{\eta}_{DH}$ ($\tilde{\eta}_{DL}$) = the mean payoff vector of the domestic high (low) information securities,

$Z_{DH}$ ($Z_{DL}$) = the vector of the world demand functions for the domestic high (low) information securities, and

$Z_{FH}$ ($Z_{FL}$) = the vector of the world demand functions for the foreign high (low) information securities.

Similarly, the equilibrium prices of the high and low information foreign securities ($P_{FH}$ and $P_{FL}$) can be represented, respectively, by

$$
P_{FH} = \frac{1}{1 + r} \left\{ \tilde{\eta}_{FH} - c^U \left[ h(N_{1}) \sum_{HH}^{DF} Z_{DH} + h(N_{2}) \sum_{HL}^{DF} Z_{DL} \right] 
+ [h(N_{1}) \sum_{HH}^{F} Z_{FH} + h(N_{2}) \sum_{HL}^{F} Z_{FL}] \right\} 
$$

(20)

and

$$
P_{FL} = \frac{1}{1 + r} \left\{ \tilde{\eta}_{FL} - c^U \left[ h(N_{3}) \sum_{HH}^{DF} Z_{DH} + h(N_{4}) \sum_{LL}^{DF} Z_{DL} \right] 
+ [h(N_{3}) \sum_{HH}^{F} Z_{FH} + h(N_{4}) \sum_{LL}^{F} Z_{FL}] \right\} 
$$

(21)

where $\tilde{\eta}_{FH}$ ($\tilde{\eta}_{FL}$) is the mean payoff vector on the high (low) information foreign securities.

Consider the effect of differential information on the equilibrium prices for the case where the variance and covariance matrices of security payoffs are known. The domestic and foreign securities with low information are riskier than those with high information. In addition, the low information securities introduce more risk to a portfolio's overall risk.\(^{10}\) As a result, the equilibrium prices of the low information domestic and foreign securities ($P_{DL}$ and $P_{FL}$) have to lower to a level which is commensurate with the added risk induced by insufficient information for foreign securities.

\(^{10}\) See Klein and Bawa (1976, 1977) and Barry and Brown (1985) for the discussion.
investment. Otherwise, the demands for those securities ($Z_{DL}$ and $Z_{FL}$) will be reduced, perhaps, to zero. This, in turn, means that the expected payoffs on the low information domestic and foreign securities ($\eta_{DL}$ and $\eta_{FL}$) will be lower than those without regard for differential information, whereas their equilibrium expected returns will be higher. Equations (19) and (21) determine the values of the equilibrium prices and expected payoffs on the low information domestic and foreign securities under differential information. Their values depend on the variance and covariance matrices of the high and low information securities ($\Sigma^D_H, \Sigma^D_H', \Sigma^F_H, \Sigma^F_H', \Sigma^{DF}_H, \Sigma^{DF}_H'$, etc.) and the information-quantity indicators ($h(n^D_2), h(n^F_2), h(n^D), h(n^F)$ and $h(N^i_1), i = 1, 2, 3, 4$).

The high information domestic and foreign securities are subject to a lower degree of estimation risk than those securities with low information. The impact of differential information should be less profound on the equilibrium prices and expected payoffs of the high information securities than those of the low information securities. The direction of its impact on the equilibrium prices and expected payoffs of the high information securities is not as clearly understood as those of the low information securities. Nevertheless, the equilibrium prices and expected payoffs of the high information domestic and foreign securities will certainly differ in value than those when differential information is not properly taken into account.\footnote{The equilibrium prices of the foreign and domestic securities without regard for differential information have an "expression" similar to (18), (19), (20) and (21) [or (17)] with all information-quantity indicators setting to one. Even setting all indicators in equations (18) to (21) equal to one will not lead to the identical equilibrium prices under both treatments. As indicated in Section II, the initial prices without regard for differential information are exogenously determined equilibrium prices, while the initial prices with differential information are endogenously determined equilibrium prices which are impacted by differential information.} Equations (18) and (20) define the values of the equilibrium
prices of the high information domestic and foreign securities under differential information.

An interesting implication of the above analysis is that the world market value of the domestic and foreign securities with differential information differ from the world market value of the otherwise identical securities. As a result, the world capital market line (WCML) under differential information will be tangent at an expected world market return which is different from the one associated with the otherwise identical WCML. Thus, the equilibrium expected return on a domestic or foreign security under differential information is divergent from that without regard for differential information.

Note that the measures of risk aversion for the domestic and foreign investors (\(G^D\) and \(G^F\)) are influenced by differential information since the unconditional expected utility of the end-of-period wealth depends on the unconditional distribution of the investors' end-of-period wealth. As a result, the equilibrium prices of the high and low information securities are also determined by the domestic and foreign investors' risk aversion measures which are impacted by differential information.

B. Equilibrium Prices with Unknown Covariance Matrices

When the covariance matrices of security payoffs are unknown, the low information domestic and foreign securities introduce arbitrarily large estimation risk to a portfolio's risk. The domestic and foreign investors will shun away from holding the low information securities. As a result, to diversify a portfolio's risk, the domestic and foreign investors will restrict their portfolio selections only on high information domestic and foreign securities. This extreme portfolio behavior leads to a (nearly) zero
demand for the low information domestic and foreign securities \( Z_{DL} = 0 \), \( Z_{FL} \) and an increase in the demand for the high information securities. In equilibrium, the prices of the high information domestic and foreign securities \( P_{DH}^U \) and \( P_{FH}^U \) will be higher than those when the covariance matrices are known \( P_{DH}^U \) and \( P_{FH}^U \). This consequence can also be observed from equations (18) to (21). Under differential information with the unknown covariance matrices, we have, in equilibrium,

\[
Z_{DL}^D = 0 = Z_{FL}^D, \quad \Sigma_{HL}^D = 0 = \Sigma_{HL}^F,
\]

\[
\Sigma_{DF}^D = \Sigma_{LL}^D = \Sigma_{LL}^F = 0.
\]

The above analysis leads to Proposition 2.

**Proposition 2.** Under differential information with the unknown covariance matrices of security payoffs, the following statements hold in equilibrium:

(i) for the high information domestic and foreign securities,

\[
P_{DH}^U = \frac{1}{1 + r} \left\{ \eta_{DH}^U - C_U^W \left[ h(N_D^D) \Sigma_{DH}^D Z_{DH}^U + h(N_F^D) \Sigma_{DF}^D Z_{DF}^U \right] \right\}
\]

\[> P_{HD} \quad \text{ (since } Z_{UDH}^U > Z_{UDH}^D), \quad \text{(22)}\]

\[
P_{FH}^U = \frac{1}{1 + r} \left\{ \eta_{FH}^U - C_U^W \left[ h(N_F^D) \Sigma_{DF}^D Z_{DF}^U + h(N_F^D) \Sigma_{DF}^D Z_{DF}^U \right] \right\}
\]

\[> P_{FH} \quad \text{ (since } Z_{UFDH}^U > Z_{UFDH}^D). \quad \text{(23)}\]

(ii) for the low information domestic and foreign securities,

\[
P_{DL}^U \quad P_{FL}^U \quad (Z_{DL}^U \quad Z_{FL}^U \quad \text{in equilibrium}), \quad \text{(24)}
\]

where \( P_{DH}^U \) (\( P_{FH}^U \)) represents the vector of the equilibrium prices of the high information domestic (foreign) securities under the unknown

22
covariance matrices; $Z^U_{DH}$ ($Z^U_{FH}$) is the world demand for the high information domestic (foreign) securities under the unknown covariance matrices; and all other remaining variables associated with superscript $U$ have similar interpretations as their counterparts under the known covariance matrices.

An implication of Proposition 2 is that the domestic and foreign investors pay a higher price for international diversification in a greatly risky world where the covariance matrices are unknown than in a world with the known covariance matrices or in the absence of differential information.

IV. Differential Information and International Asset pricing with an Ownership Constraint

A. Premiums and Discounts Under Differential Information With Known Covariance Matrices

In order to show the differential information effect on international asset pricing, we examine only one case of international asset pricing where the foreign investors are limited to hold an equal fraction of shares ($\delta$) across all firms of a country. This constraint is so-called the $\delta$ constraint by Eun and Janakiramanan (EJ) (1986). The $\delta$ constraint is binding only on all securities of the foreign country. It follows that the foreign securities will be held long, in aggregate, by the domestic investors. However, no restriction is imposed on short sales by the foreign investors either in the domestic country or in the foreign country. Then, the $\delta$ constraint increases the demand for the foreign securities by the domestic investors. With the demand in excess of the supply, the domestic investors will be willing to pay higher premiums than if no constraint were imposed. For the foreign investors, the demand for their domestic securities is less than the supply under the $\delta$ constraint. Thus, the foreign securities will be
selli ng at a discount for the foreign investors. Then the prices of the foreign securities for the domestic and foreign investors, when differential information with the known covariance matrices is recognized, can be expressed as follows:

\begin{align}
\mathcal{P}_{FH,6}^d &= \mathcal{P}_{FH}^d + \alpha_H \\
\mathcal{P}_{FH,6}^f &= \mathcal{P}_{FH}^f - \beta_H \\
\mathcal{P}_{FL,6}^d &= \mathcal{P}_{FL}^d + \alpha_L \\
\mathcal{P}_{FL,6}^f &= \mathcal{P}_{FL}^f - \beta_L
\end{align}

where \( \mathcal{P}_{FH,6}^d \) (\( \mathcal{P}_{FH,6}^f \)) denotes the equilibrium prices of the high information foreign securities for the domestic (foreign) investors under the \( \delta \) restriction;

\( \mathcal{P}_{FL,6}^d \) (\( \mathcal{P}_{FL,6}^f \)) is the equilibrium prices of the low information foreign securities for the domestic (foreign) investors under the \( \delta \) restriction;

\( \alpha_H \) (\( \alpha_L \)) is the premium paid by the domestic investors when they demand the high (low) information foreign securities under differential information; and

\( \beta_H \) (\( \beta_L \)) is the discount for the high (low) foreign securities demanded.

\(^{12}\) To avoid arbitrage opportunities due to two different prices for the foreign securities, it is assumed that the foreign government has imposed some restrictions on the foreign investors that they cannot purchase the foreign securities at a lower price and sell them at a higher price to the domestic investors. See Eun and Janakiramanan (1986) for those governments that impose such constraints.

\(^{13}\) The utility function used in Eun and Janakiramanan (1986) is a negative exponential utility function which is a class of Von Neumann-Morgenstern utility functions. Thus, if the utility function in (5a-b) is specifically replaced by a negative exponential utility function as given in EJ, the analysis and implications of this paper will remain valid.
by the foreign investors.

The explanations for equations (25) through (28) are given below. When differential information is recognized, the risk-averse domestic investors will not pay the same premium for all foreign securities. The amount of premium that they are willing to pay depends on the quantity of information for foreign investment available to them. The high information foreign securities provides more information for investment to the risk-averse domestic investors than do the low information foreign securities. As a result, the low information foreign securities are perceived by the domestic investors as (much) riskier than the high information foreign securities. Klein and Bawa (1977) showed that under differential information investors will diversify more into the high information securities and less into the low information securities. Thus, to achieve effective international diversification, the domestic investors will tend to hold more the high information foreign securities than the low information foreign securities. The demand for the high information foreign securities by the domestic investors will be greater under differential information than will the demand for the otherwise identical foreign securities, whereas the demand for the low information foreign securities by the domestic investors will be lower. Under the $\delta$ constraint, the risk-averse domestic investors will be willing to pay a higher premium for the high information foreign securities than for the otherwise identical securities. And they offer a lower premium for the low information foreign securities than for the otherwise identical foreign securities.

Similarly, under differential information the risk-averse foreign investors will diversify more into the high information foreign securities (their own domestic securities) and less into the low information foreign
securities than if differential information were not recognized. Since the supply of the foreign securities is limited by the $\delta$ constraint, the low information foreign securities with a less demand will be selling at a greater discount to the foreign investors than the otherwise identical foreign securities. And the high information foreign securities with a greater demand will be selling at a lower discount than if differential information were not taken into account.

The above analysis leads to Proposition 3.

**Proposition 3.** Under the assumption of the known covariance matrices of security payoffs, the premiums offered by the domestic investors with and without regard for differential information are related in the following ways:

$$\alpha_H > \pi_H \text{ and } \alpha_L < \pi_L$$

where $\pi_H$ ($\pi_L$) represents the premium offered by the domestic investors for the otherwise identical high (low) information foreign securities (i.e., when differential information were not recognized). In addition, the discounts demanded by the foreign investors with and without regard for differential information are related in the following ways:

$$\beta_H < \lambda_H \text{ and } \beta_L > \lambda_L$$

where $\lambda_H$ ($\lambda_L$) denotes the discount demanded by the foreign investors for the otherwise identical high (low) information foreign securities.

The above result is in a sharp contrast with EJ. They showed that the domestic investors will be willing to pay the same premiums for the foreign securities and the foreign investors will purchase their own domestic securities at the same discounts, regardless of the amounts of information available to the domestic and foreign investors. That is, without regard for
differential information, all premiums are equal: $\alpha_H = \alpha_L = \pi$, and all discounts are identical: $\beta_H = \beta_L = \lambda$ where $\pi$ and $\lambda$ are, respectively, the premiums and discounts when differential information is not recognized. However, it cannot hold under differential information.

The relationship between the premiums and discounts for the high and low information foreign securities are next determined. Proposition 4 provides such relationships.

**Proposition 4.** The relationship between the premium and discount exists only within the same class of the foreign securities - with either high or low information for investment. No crossover relationship exists between the premium and discount for the high and low information foreign securities. For either the high or low information foreign securities, the premium offered by the domestic investors is proportional to the discount demanded by the foreign investors where the proportionality factor is the ratio of the aggregate measures of the domestic and foreign investors' risk aversion which are influenced by differential information.

**Proof:** Using the equations for premiums and discounts [(25) to (28)], the world aggregate demand functions in (16) can be rewritten in an expression corresponding to the low and high information foreign securities:

$$
\begin{bmatrix}
Z_D \\
Z_{FH} \\
Z_{FL}
\end{bmatrix} = \frac{1}{G^W} \begin{bmatrix}
V_D & V_{DF} \\
V_{DF} & V_F \\
\eta_D^* & P_D(1+r) \\
\eta_{FH}^* & P_{FH}(1+r) \\
\eta_{FL}^* & P_{FL}(1+r)
\end{bmatrix}
- \frac{1}{A^D} \begin{bmatrix}
V_D & V_{DF} \\
V_{DF} & V_F \\
\alpha_H^*(1+r) & 0 \\
\alpha_L^*(1+r) & 0
\end{bmatrix}
- \frac{1}{A^F} \begin{bmatrix}
V_D & V_{DF} \\
V_{DF} & V_F \\
\beta_H^*(1+r) & 0 \\
\beta_L^*(1+r) & 0
\end{bmatrix}
$$

(29)
Rewrite (29) to yield

\[
\begin{bmatrix}
P_D \\
P_{FH} \\
P_{FL}
\end{bmatrix}
= \frac{1}{1 + r}
\begin{bmatrix}
\eta_D \\
\eta_{FH} \\
\eta_{FL}
\end{bmatrix}
- G W
\begin{bmatrix}
\Sigma_D & \Sigma_{DF} \\
\Sigma_{DF} & \Sigma_F
\end{bmatrix}
\begin{bmatrix}
Z_D \\
Z_{FH} \\
Z_{FL}
\end{bmatrix}
+ \frac{G^W}{G^D}
\begin{bmatrix}
\alpha_H \\
\alpha_L
\end{bmatrix}
- \frac{G^W}{G^F}
\begin{bmatrix}
\beta_H \\
\beta_L
\end{bmatrix}.
\]  

(30)

If there is no \( \delta \) restriction, the pricing relationship in (30) reduces to that given in (17) or alternative equations (18) to (21), which are expressed explicitly in terms of the low and high information domestic and foreign securities. By comparing (17) and (30), the relationship between the premiums and discounts can be written as follows:

(i) for the high information foreign securities, the relationship is given by

\[
\alpha_H = \frac{G^D}{G^F} \beta_H
\]  

(31)

Thus, the premium associated the high-information foreign securities which is offered by the Domestic investors (\( \alpha_H \)) is propositional to the discount associated with the same type of securities which is demanded by the foreign investors (\( \beta_H \)).

(ii) for the low information foreign securities, it is

\[
\alpha_L = \frac{G^D}{G^F} \beta_L
\]  

(32)

Q.E.D.

Thus, the premium associated with the low-information foreign securities which is offered by the domestic investors (\( \alpha_L \)) is proportional to the discount associated with the same low-information foreign securities which is demanded by the foreign investors.
There are two sets of relationship for the premiums and discounts under differential information; one for the high information foreign securities and another for the low information foreign securities. No crossover relationship between the premium and discount exists for both the high and low information foreign securities. In contrast, as given by EJ only one relationship is applied to all foreign securities when differential information is not taken into account. In addition, the proportionality factor \( (G^D/G^F) \) is a function of the impact of differential information, whereas the proportionality is free from the impact when differential information is not recognized. Hence, the premium that the domestic investors are willing to offer increases with an increase in their aggregate risk aversion measure \((G^D)\). However, the increase in the premium with \(G^D\) that the domestic investors are willing to pay for the low information foreign securities has an upper limit \(\pi_L\) which is the premium offered by the domestic investors when there is no differential information for investment (see Proposition 3, \(\alpha_L < \pi_L\)). That is, the premiums \((\alpha_L)\) that the domestic investors are willing to offer for the low information foreign securities are restricted to the premiums \((\pi_L)\) they are willing to pay for the same foreign securities under no differential information. The minimum premium the domestic investors are willing to pay for the low information foreign securities is \((G^D/G^F)\lambda_L\). Thus, the range of the premium the domestic investors are willing to pay for the low information foreign securities is restricted by \((G^D/G^F)\lambda_L < \alpha_L < \pi_L\). Similarly, the increase in the premium for the high information foreign securities due to a rise in \(G^D\) is restricted by an upper bound \((G^D/G^F)\lambda_H\). The range of the premium the domestic investors are willing to pay for the high information foreign securities is given by \(\pi_H < \alpha_H < (G^D/G^F)\lambda_H\). In contrast, the premium that the domestic investors are
willing to pay for the foreign securities does not have such a constraint when differential information is not taken into account.

B. The International Asset Pricing Relationships Under Differential Information With Known Covariance Matrices

In the following section, we focus our attention on the equilibrium pricing relationships under the δ constraint only for the high and low information foreign securities since the equilibrium pricing relationships for the high and low information domestic securities have been examined in the third section. A similar procedure as adopted by EJ is used to derive the equilibrium pricing relationships with differential information under the δ constraint. By using the market clearing conditions for the high and low information foreign securities under the δ constraint [i.e., \( z_{FH}^d - δz_{FH}^d, z_{FL}^d - δz_{FL}^d, z_{FH}^f - (1 - δ)z_{FH}^f \) and \( z_{FL}^f - (1 - δ)z_{FL}^f \)] and substituting equations (25) to (28) into (10) and (15), it can be shown that the premiums and discounts for the high and low information foreign securities are given by¹⁴

\[
\alpha_H = \frac{1}{1 + r} \left( G^W - G^D \right) (Q_{HH}^F z_{FH} + Q_{HL}^F z_{FL}) 
\]

\[
\alpha_L = \frac{1}{1 + r} \left( G^W - G^D \right) (Q_{HH}^F z_{FH} + Q_{HL}^F z_{FL}) 
\]

\[
\beta_H = \frac{1}{1 + r} \left[ G^F(1 - δ) - G^W \right] (Q_{HH}^F z_{FH} + Q_{HL}^F z_{FL}) 
\]

\[
\beta_L = \frac{1}{1 + r} \left[ G^F(1 - δ) - G^W \right] (Q_{HH}^F z_{FH} + Q_{HL}^F z_{FL}) 
\]

where \( Q_{HH}^F, Q_{HL}^F \) and \( Q_{LL}^F \) are the submatrices of the matrix \( V_F^{-1} \) which is made up of the predictive covariance matrices as given in (11), (12) and (13). That

¹⁴A proof of equations (33) to (36) is given in Appendix.
is, $V_F^{-1}$ is partitioned into four submatrices corresponding to the high and low information foreign securities:

$$V_F^{-1} - \Sigma_F = \Sigma_F \Sigma_F^{-1} \Sigma_F = \begin{pmatrix} Q^F_{HH} & Q^F_{HL} \\ Q^F_{HL} & Q^F_{LL} \end{pmatrix}.$$  \hspace{1cm} (37)

As observed from equations (33) to (36), under differential information the premiums offered by the domestic investors ($\alpha_H$ and $\alpha_L$) and the discounts demanded by the foreign investors ($\beta_H$ and $\beta_L$) are determined by the predictive covariance matrices of security payoffs and the demands for the high and low information foreign securities ($Z_{FH}$ and $Z_{FL}$). Differential information also enters into the premiums and discounts through its impact on the aggregate domestic and foreign risk aversion measures.

Once the premiums and discounts under differential information are determined, the equilibrium asset pricing relationships under the $\delta$ constraint for the high and low information foreign securities can be expressed as follows: \hspace{1cm} 15

(i) for the high information foreign securities demanded by the domestic investors,

$$p^d_{FH, \delta} = \frac{1}{1 + r} \left\{ \frac{\eta_{FH}}{h(N_F)^{\Sigma^F_{HH} Z_{DH}} + h(N_F)^{\Sigma^F_{HL} Z_{DL}}} + [h(N_F)^{\Sigma^F_{HH} Z_{FH}} + h(N_F)^{\Sigma^F_{HL} Z_{FL}}] + (G^{W} - G^{D})(Q_{HH}^{F} Z_{FH} + Q_{HL}^{F} Z_{FL}) \right\}.$$  \hspace{1cm} (38)

(ii) for the high information foreign securities demanded by the foreign investors,

\hspace{1cm} 15 The pricing relationships for the high and low information domestic securities have been given in (18) and (19) when the covariance matrices are known, and equations (22) and (23) are for the case where the covariance matrices are unknown.
\[ P_{FH, \delta}^f = \frac{1}{1 + \tau} \left\{ \hat{r}_{FH} - G^W \left[ h(N_1') \sum_{HH}^{DF'} Z_{DH} + h(N_2') \sum_{HL}^{DF'} Z_{DL} \right] \\
+ [h(N_1') \sum_{HH}^{DF'} Z_{FH} + h(N_2') \sum_{HL}^{DF'} Z_{FL}] \\
- [G^F(1 - \delta) - G^W] \left( Q_{HH}^{F} Z_{FH} + Q_{HL}^{F} Z_{FL} \right) \right\} \]  

(39)

(iii) for the low information foreign securities demanded by the domestic investors,

\[ P_{FL, \delta}^d = \frac{1}{1 + \tau} \left\{ \hat{r}_{FH} - G^W \left[ h(N_3') \sum_{LL}^{DF'} Z_{DH} + h(N_4') \sum_{LL}^{DF'} Z_{DL} \right] \\
+ [h(n_3') \sum_{HH}^{DF'} Z_{FH} + h(n_2') \sum_{HL}^{DF'} Z_{FL}] \\
+ (G^W - G^D \delta) \left( Q_{HL}^{F} Z_{FH} + Q_{LL}^{F} Z_{FL} \right) \right\} \]  

(40)

(iv) for the low information foreign securities demanded by the foreign investors,

\[ P_{FL, \delta}^f = \frac{1}{1 + \tau} \left\{ \hat{r}_{FH} - G^W \left[ h(N_3') \sum_{LL}^{DF'} Z_{DH} + h(N_4') \sum_{LL}^{DF'} Z_{DL} \right] \\
+ [h(N_3') \sum_{HH}^{DF'} Z_{FH} + h(n_2') \sum_{HL}^{DF'} Z_{FL}] \\
+ [G^F(1 - \delta) - G^W] \left( Q_{HH}^{F} Z_{FH} + Q_{LL}^{F} Z_{FL} \right) \right\} \]  

(41)

Equations (38) to (41) describe the equilibrium asset pricing relationships for the high and low information foreign securities under the \( \delta \) constraint. The high (low) information foreign securities under the \( \delta \) constraint are priced in the domestic country in a manner different from the way they are priced in the foreign countries. In addition, the domestic and foreign markets price the high and low information foreign securities in a manner different from those when different information is not taken into
account. Specifically, the high and low information foreign securities are priced in the domestic country according to equations (38) and (40), and priced in the foreign country according to (39) and (41). These equilibrium prices are different from the equilibrium prices when differential information is not taken into account. For example, all of the foreign securities under zero differential information are priced in the same manner in either the domestic or the foreign country [see EJ’s (37) and (38)].

V. Conclusions

International portfolio demands and equilibrium asset pricing relationships have been examined in a two-country world where the amounts of information for investment are insufficient. International portfolio demand functions and the equilibrium prices for the high and low information domestic and foreign securities differ from those without regard for differential information. In addition, the total value of the domestic and foreign markets under differential information is divergent from that of the otherwise identical markets. The world capital market line under differential information will be tangent at an expected world market return which differs from the one without regard for differential information. Thus, the equilibrium expected return on a domestic or foreign security is divergent from the one when differential information is not recognized. In addition, when differential information is considered with the unknown covariance matrices of security payoffs, international portfolio selections and the equilibrium prices for the high and low information securities are substantially divergent from those under the assumption of the known covariance matrices.
Under the $\delta$ constraint, the domestic investors pay different premiums for the high and low information foreign securities, whereas the foreign investors demand for different discounts for their own domestic high and low information securities. The premiums and discounts under differential information differ in value from those otherwise. Two relationships between the premiums and discounts exist for the foreign securities, one for the high information foreign securities and another for the low information foreign securities. In contrast, there is only one relationship between the discount and premium when differential information is not recognized. However, when the covariance matrices are unknown, the domestic investors are willing to pay a premium only for the high information foreign securities, while the foreign investors receive a discount for the same securities.

The equilibrium pricing relationships with the $\delta$ constraint are different under differential information from those otherwise. The high and low information foreign securities are priced in the domestic country in a separate manner, and so are in the foreign country. In contrast, the foreign securities under the $\delta$ constraint are priced in the same manner in the domestic and foreign countries when differential information is not taken into account. Therefore, the overall results have shown that differential information alters investors' international portfolio behavior and asset pricing relationships.
References


Appendix

A proof of equations (33) to (36):

By using the market clearing conditions for the high and low information foreign securities under the δ constraint \( z_{FH}^d - \delta z_{FH}, z_{FL}^d - \delta z_{FL}, z_{FH}^f - (1 - \delta) z_{FH}^f \) and \( z_{FL}^f - (1-\delta) z_{FL}^f \) and substituting (23), (24), (25) and (26) into (10) and (15), equations (10) and (15) can be rewritten in terms of the high and low information foreign securities:

\[
G^D \delta \begin{bmatrix} z_{FH}^d \\ z_{FL}^d \end{bmatrix} - V_D \begin{bmatrix} \eta_{DH} \\ \eta_{DL} \end{bmatrix} - \begin{bmatrix} p_{DH} \\ p_{DL} \end{bmatrix} \begin{bmatrix} 1+r \end{bmatrix} + V_F \begin{bmatrix} \eta_{FH} \\ \eta_{FL} \end{bmatrix} - \begin{bmatrix} p_{FH} \\ p_{FL} \end{bmatrix} \begin{bmatrix} 1+r \end{bmatrix} - V_F \begin{bmatrix} \alpha_H \\ \alpha_L \end{bmatrix} \begin{bmatrix} 1+r \end{bmatrix} \tag{A1}
\]

\[
G^F \begin{bmatrix} 1 - \delta \end{bmatrix} \begin{bmatrix} z_{FH}^f \\ z_{FL}^f \end{bmatrix} - V_D \begin{bmatrix} \eta_{DH} \\ \eta_{DL} \end{bmatrix} - \begin{bmatrix} p_{DH} \\ p_{DL} \end{bmatrix} \begin{bmatrix} 1+r \end{bmatrix} + V_F \begin{bmatrix} \eta_{FH} \\ \eta_{FL} \end{bmatrix} - \begin{bmatrix} p_{FH} \\ p_{FL} \end{bmatrix} \begin{bmatrix} 1+r \end{bmatrix} + V_F \begin{bmatrix} \beta_H \\ \beta_L \end{bmatrix} \begin{bmatrix} 1+r \end{bmatrix} \tag{A2}
\]

Subtracting (A2) from (A1) yields

\[
\begin{bmatrix} G^F - (G^D + G^F) \delta \end{bmatrix} \begin{bmatrix} z_{FH}^d \\ z_{FL}^d \end{bmatrix} - V_F \begin{bmatrix} \alpha_H + \beta_H \\ \alpha_L + \beta_L \end{bmatrix} \begin{bmatrix} 1+r \end{bmatrix} - V_F \begin{bmatrix} 1 + \frac{G^D}{G^F} \end{bmatrix} \begin{bmatrix} \beta_H \\ \beta_L \end{bmatrix} \begin{bmatrix} 1+r \end{bmatrix} \tag{A3}
\]

Solving (A3) for \( \beta_H \) and \( \beta_L \) yields

\[
\begin{bmatrix} \beta_H \\ \beta_L \end{bmatrix} = \frac{1}{1 + r} \begin{bmatrix} G^F (1 - \delta) - G^W \end{bmatrix} V_F^{-1} \begin{bmatrix} z_{FH}^d \\ z_{FL}^d \end{bmatrix} \tag{A4}
\]

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where $V_{F}^{-1} = \Sigma_{F} - \Sigma'_{DF} \Sigma_{DF}^{-1} \Sigma_{DF}$ as defined in (7) and (8). To solve for $\beta_{H}$ and $\beta_{L}$ explicitly, $V_{F}^{-1}$ has to be partitioned into four submatrices corresponding to the high and low information foreign securities. Thus, we let

$$
\Sigma_{F} - \Sigma'_{DF} \Sigma_{DF}^{-1} \Sigma_{DF} = \begin{pmatrix}
Q_{HH}^{F} & Q_{HL}^{F} \\
Q_{HL}^{F} & Q_{LL}^{F}
\end{pmatrix}
$$

(A5)

Substituting (A5) into (A4) and solving for $\beta_{H}$ and $\beta_{L}$, we obtain

$$
\beta_{H} = \frac{1}{1 + r} \left[ G^{F}(1 - \delta) - G^{W} \right] (Q_{HH}^{F}Z_{FH}^{F} + Q_{HL}^{F}Z_{FL}^{F}), \quad \text{which is (35)}
$$

$$
\beta_{L} = \frac{1}{1 + r} \left[ G^{F}(1 - \delta) - G^{W} \right] (Q_{HL}^{F}Z_{FH}^{F} + Q_{LL}^{F}Z_{FL}^{F}), \quad \text{which is (36)}
$$

Finally, substituting (35) and (36) into (31) and (32), we obtain (33) and (34).