Announcement

- **Project proposal** (due on Nov. 7) should include the following sections:

  1. Introduction /Your topic and motivation
  2. Search tricks /Your score formulation
  3. System design /Class diagrams [proposal sample]
  4. Schedule /How and when to accomplish stages
  5. Challenges /Techniques that you need but may have a hard time to learn on your own
Announcement

HWs Review
- BMI
- Generic Progression
- Keyword Counting
- The Ordered List
- HTML Tag Matching
Abstract

Non-linear Data Structures

Trees and their variations
Abstract Data Type (ADT)

- An abstract data type (ADT) is an abstraction of a data structure

- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

- We have discussed Array ADT, List ADT, Stack ADT, and Queue ADT

- All of them are linear ADT
A Hierarchical Structure
Linux/Unix file systems
Tree: A Hierarchical ADT

- A tree (upside down) is an abstract model of a hierarchical structure
- A tree consists of nodes with a parent-child relation
- Each element (except the top element) has a parent and zero or more children elements
Tree Terminology

- **Root**: a node without any parent (A)
- **Internal node**: a node with at least one child (A, B, C, F)
- **External node** (a.k.a. leaf): a node without children (E, I, J, K, G, H, D)
- **Subtree**: tree consisting of a node and its descendants
Tree Terminology

- **Ancestors** of a node: parent, grandparent, grand-grandparent, etc.
- **Depth** of a node: number of ancestors
- **Height** of a tree: maximum depth of any node (3)
- **Descendant** of a node: child, grandchild, grand-grandchild, etc.
Tree ADT

- We use positions to define the tree ADT
- The positions in a tree are its nodes and neighboring positions satisfy the parent-child relationships

<table>
<thead>
<tr>
<th>method</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>root()</td>
<td>Return the tree’s root; error if tree is empty</td>
</tr>
<tr>
<td>parent(v)</td>
<td>Return v’s parent; error if v is a root</td>
</tr>
<tr>
<td>children(v)</td>
<td>Return v’s children (an iterable collection of nodes)</td>
</tr>
<tr>
<td>isRoot(v)</td>
<td>Test whether v is a root</td>
</tr>
<tr>
<td>isExternal(v)</td>
<td>Test whether v is an external node</td>
</tr>
<tr>
<td>isInternal(v)</td>
<td>Test whether v is an internal node</td>
</tr>
</tbody>
</table>
Tree ADT

- **Generic methods (not necessarily related to a tree structure):**

<table>
<thead>
<tr>
<th>method</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>isEmpty()</td>
<td>Test whether the tree has any node or not</td>
</tr>
<tr>
<td>size()</td>
<td>Return the number of nodes in the tree</td>
</tr>
<tr>
<td>iterator()</td>
<td>Return an iterator of all the elements stored in the tree</td>
</tr>
<tr>
<td>positions()</td>
<td>Return an iterable collection of all the nodes of the tree</td>
</tr>
<tr>
<td>replace(v,e)</td>
<td>Replace with e and return the element stored at node v</td>
</tr>
</tbody>
</table>
A Linked Structure for Tree

- A node is represented by an object storing
  - Element
  - A parent node
  - A sequence of children nodes
A Linked Structure for Tree

- Node objects implement the Position ADT
Tree Traversal

- Visit all nodes in a tree
- Do some operations during the visit
Preorder Traversal

- A node is visited (so is the operation) before its descendants

- Application:
  - Print a structured document

Algorithm `preOrder(v)`

```
visit(v)
for each child w of v
    preOrder(w)
```
For your project, you can print a structured web site with its sub links using preorder traversal.
Postorder Traversal

- A node is visited after its descendants

- Application:
  - Compute space used by files in a directory and its subdirectories

Algorithm `postOrder(v)`

```
for each child w of v
    postOrder(w)
visit(v)
```
For your project, you can compute the score of a web site and its sub links Using postorder traversal
A binary tree is a tree with the following properties:

- Each internal node has at most two children
- The children of a node are an ordered pair (left and right)
- We call the children of an internal node left child and right child
Binary Tree

- Alternative recursive definition: a binary tree is either
  - a tree consisting of a single node, or
  - a tree whose root has an ordered pair of children, each of which is a binary tree
Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - external nodes: operands

- Example: arithmetic expression tree for the expression:

\[(2 \times (a - 1) + (3 \times b))\]
Decision Tree

- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - external nodes: decisions
- Example: dining decision

```
Want a fast meal?

On a diet?
- Subway

On expense account?
- Mc Donald’s
- 王品台塑
- 我家牛排
```
Proper Binary Trees

- Each internal node has exactly 2 children
Proper Binary Trees

- $n$: number of total nodes
- $e$: number of external nodes
- $i$: number of internal nodes
- $h$: height (maximum depth of a node)

Properties:

1. $e = i + 1$
2. $n = 2e - 1$
3. $h \leq i$
4. $h \leq (n - 1)/2$
5. $e \leq 2^h$
6. $h \geq \log_2 e$
7. $h \geq \log_2 (n + 1) - 1$
Properties

1. \( e = i + 1 \)
2. \( n = e + i = 2e - 1 = 2i + 1 \)
Properties

- 3. $h \leq i$
- 4. $h \leq (n-1)/2$
Properties

- 5. $e \leq 2^h$
- 6. $h \geq \log_2 e$
- 7. $h \geq \log_2 \left(\frac{n+1}{2}\right) = \log_2(n+1) - 1$
BinaryTree ADT

- The BinaryTree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT

- Additional methods:
  - position left(p)
  - position right(p)
  - boolean hasLeft(p)
  - boolean hasRight(p)

- Update methods may be defined by data structures implementing the BinaryTree ADT
Inorder Traversal

- A node is visited after its left subtree and before its right subtree

Algorithm inOrder(v)

if hasLeft (v)
    inOrder (left (v))
visit(v)
if hasRight (v)
    inOrder (right (v))
Print Arithmetic Expressions

- Specialization of an inorder traversal
  - print operand or operator when visiting node
  - print “(“ before traversing left subtree
  - print “)“ after traversing right subtree

Algorithm `printExpression(v)`

```python
if hasLeft(v):
    print("(")
    printExpression(left(v))
    print(v.element())
    if hasRight(v):
        printExpression(right(v))
        print ")")
```

$$((2 \times (a - 1)) + (3 \times b))$$
Evaluate Arithmetic Expressions

- Specialization of a postorder traversal
  - recursive method returning the value of a subtree
  - when visiting an internal node, combine the values of the subtrees

**Algorithm** $\text{evalExpr}(v)$

```plaintext
if isExternal $(v)$
  return $v$.element ()
else
  $x \leftarrow \text{evalExpr}(\text{leftChild}(v))$
  $y \leftarrow \text{evalExpr}(\text{rightChild}(v))$
  ◊ \leftarrow \text{operator stored at } v$
  return $x \diamondsuit y$
```
Euler Tour Traversal

- Generic traversal of a binary tree
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)
A template method pattern

- A generic computation mechanism
- Specialized for an application by redefining the visit actions

**Algorithm** eularTour(T, v)

Perform the action for visiting node v on the left

*If* v has a left child u in T *then*

- eularTour(T, u)

Perform the action for visiting node v from below

*If* v has a right child w in T *then*

- eularTour(T, w)

Perform the action for visiting node v on the right
An Application of EularTour

- printExpression
  - On the left action: print ( 
  - From below action: print v 
  - On the right action: print )

Algorithm printExpression(T,v)
if T.isInternal(v) then print “(”
If v has a left child u in T then
  printExpression(T, u)
print(v)
if v has a right child w in T then
  printExpression(T, w)
if T.isInternal(v) then print “)”
A Linked Structure for Binary Trees

- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node
A Linked Structure for Binary Trees
An Array-Based Representation

- Nodes are stored in an array \( A \)
- Node \( v \) is stored at \( A[\text{rank}(v)] \)
  - \( \text{rank}(\text{root}) = 1 \)
  - Left in even: if node is the left child of parent(node),
    \( \text{rank}(\text{node}) = 2 \cdot \text{rank}(\text{parent}(\text{node})) \)
  - Right in odd: if node is the right child of parent(node),
    \( \text{rank}(\text{node}) = 2 \cdot \text{rank}(\text{parent}(\text{node})) + 1 \)
- \( A[0] \) is always empty
- \( A[i] \) is empty if there is no node in the \( i \)th position
- The array size \( N \) is \( 2^{(h+1)} \)
An Array-Based Representation

```
A
B   D
E   F   C   J
G   H
0  1  2  3  4  5  6  7  10  11
```

```
Compute the score of a website!

- Construct a tree and its nodes according to a given website
  - An element (referred by a node) represents one web page and has three fields: (name, url, score)

- Given a keyword and its weight, compute the score of each node
  - Score = number of appearance * weight
  - The score of a node = the score of the content of its url + the scores of its children
  - This can be done by a postorder traversal of a tree

- Output the hierarchy of the website (with names and scores) using parentheses
  - This can be done by an eular tour
An example input

You will be given a website like:

- Projects, http://soslab.nccu.edu.tw/Projects.html
- Stranger, https://vlab.cs.ucsb.edu/stranger/
- Member, http://soslab.nccu.edu.tw/Members.html
An example output

Given a set of keywords, (Yu, 1.2), (Fang, 1.8) you shall output something like:

```
( Soslab, 56.6
  (Publication, 18)
  (Projects, 15.6
    (AppBeach, 2.6)
    (Stranger, 8.8)
  )
  (Member, 9.2)
  (Course, 4.8)
)
```

**Fang Yu, 56.6** indicates that the sum of the score in the content of the given url (http://soslab.nccu.edu.tw) and its sub links.
Coming Up…

- The project proposal is due on Nov. 5
- We will talk about heap (some kind of a tree) on Nov. 5.
  - Read Chapter 8