Fall 2017

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# Data Structures
# Lecture 6

# Announcement

Project Proposal due on Nov. 9

- Remember to **bring a hardcopy of your project proposal to the class next week**
- Also submit your project proposal file to your TAs
- Discuss potential difficulties of your project
- We can select some topics and explain in the lab

# Announcement

- We will have programming lectures and tests on Nov. 6 in 逸仙樓 5F.
- HWs Review
  - BMI
  - Generic Progression
  - Keyword Counting
  - The Ordered List
  - HTML Tag Matching
  - Web Site Score

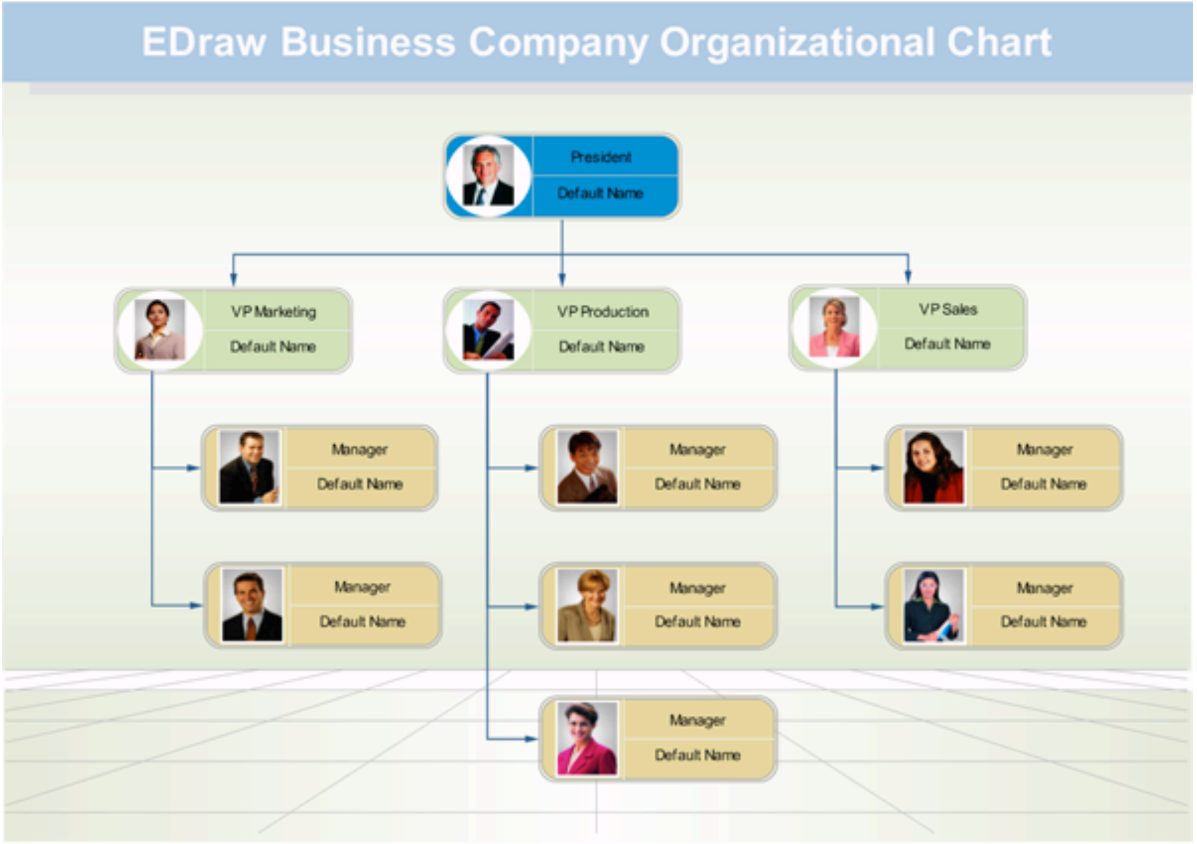
# Abstract
# Non-linear Data Structures

Trees and their variations

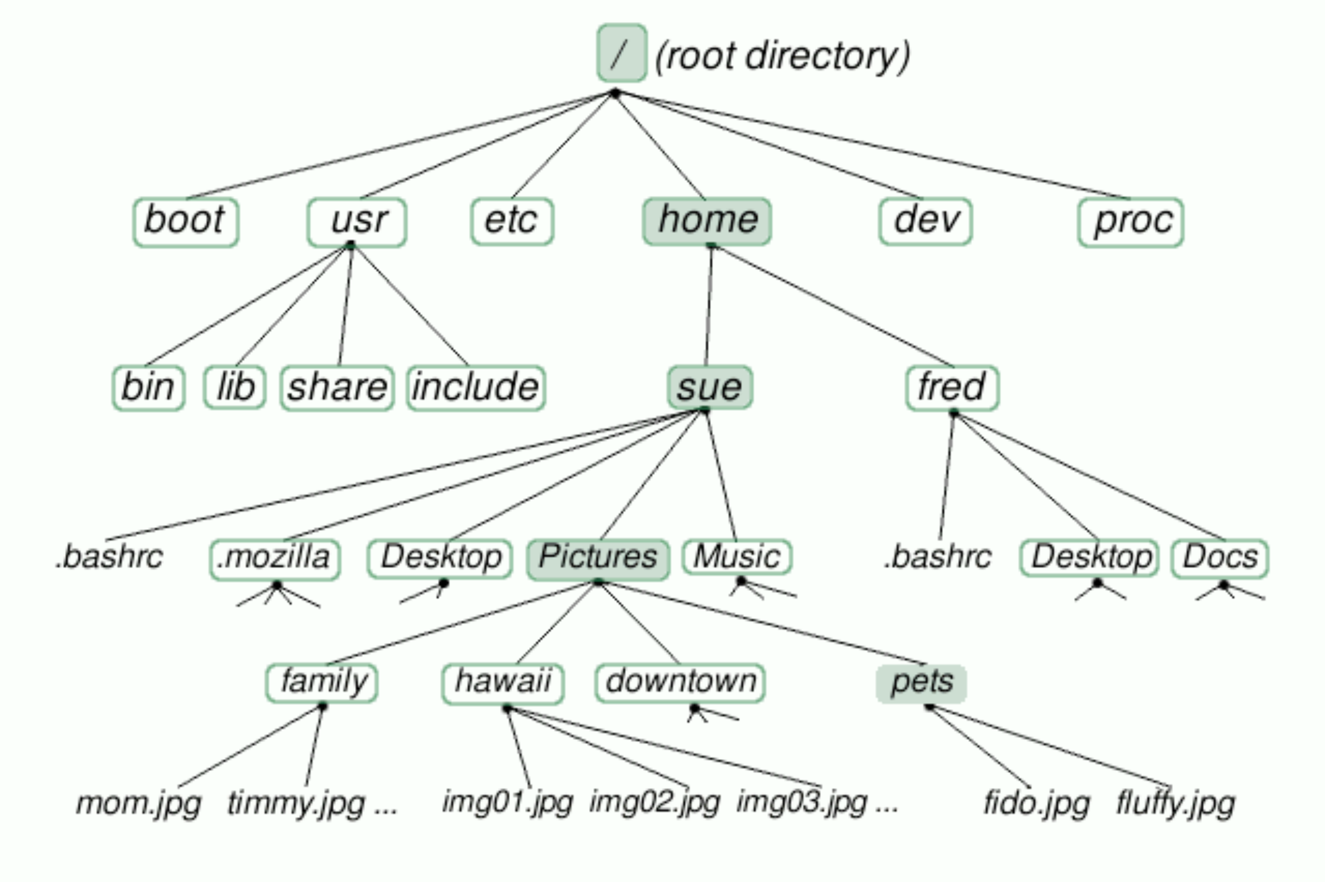
# Abstract Data Type (ADT)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations
- We have discussed Array ADT, List ADT, Stack ADT, and Queue ADT
- All of them are linear ADT

# A Hierarchical Structure

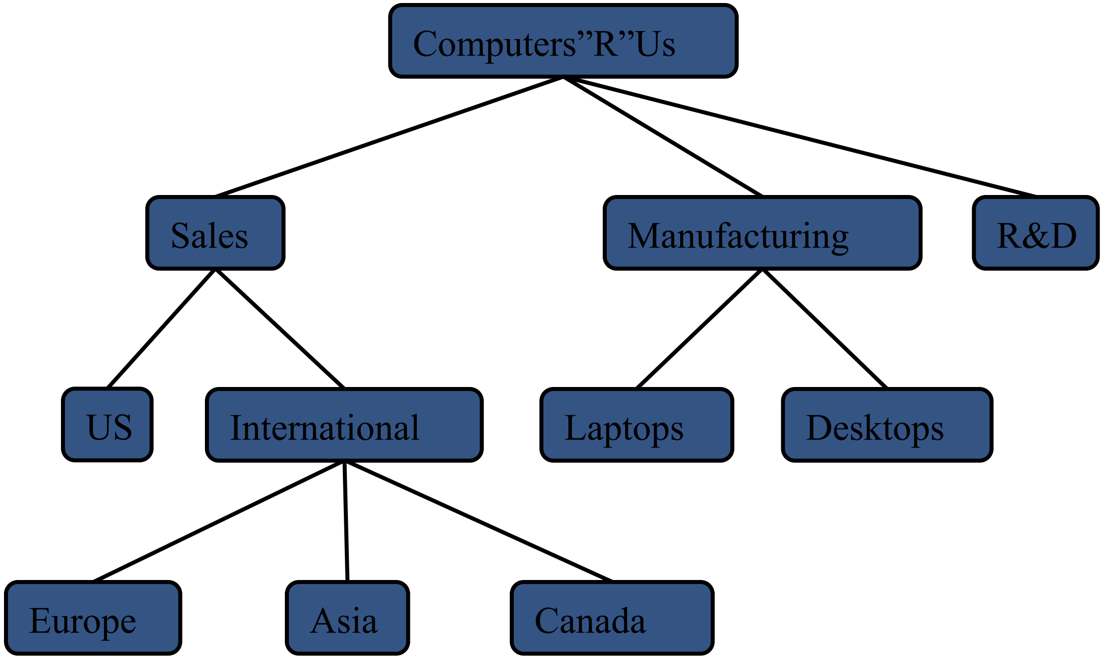


# Linux/Unix file systems



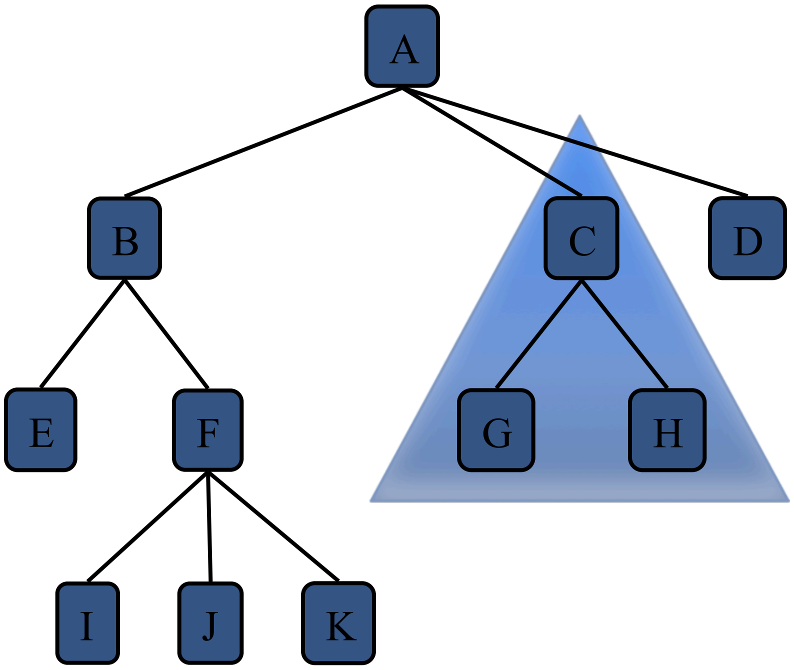
# Tree: A Hierarchical ADT

- A tree (upside down) is an abstract model of a hierarchical structure
- A tree consists of nodes with a parent-child relation
- Each element (except the top element) has a parent and zero or more children elements



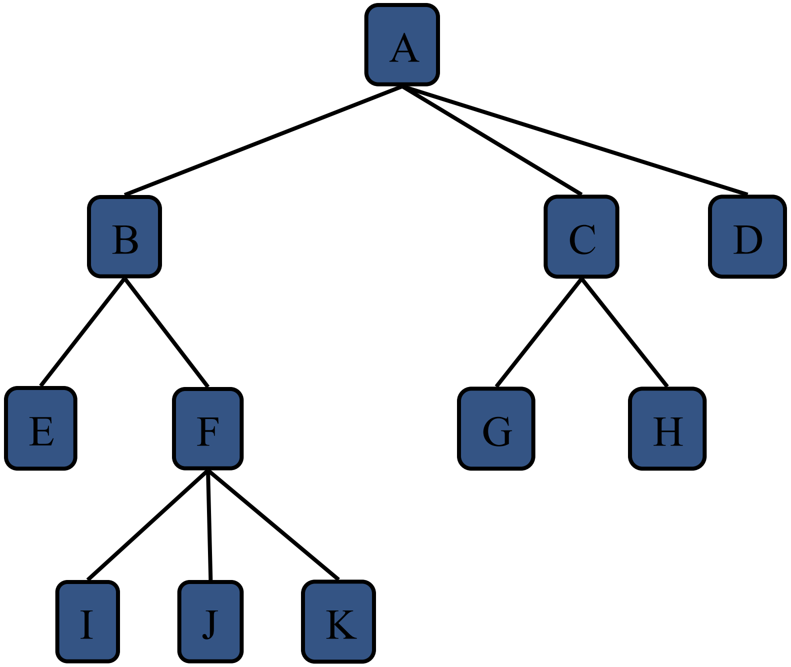
# Tree Terminology

- **Root**: a node without any parent (A)
- **Internal node**: a node with at least one child (A, B, C, F)
- **External node** (a.k.a. leaf ): a node without children (E, I, J, K, G, H, D)
- **Subtree**: tree consisting of a node and its descendants



# Tree Terminology

- **Ancestors** of a node: parent, grandparent, grand-grandparent, etc.
- **Depth** of a node: number of ancestors
- **Height** of a tree: maximum depth of any node (3)
- **Descendant** of a node: child, grandchild, grand-grandchild, etc.



# Tree ADT

- We use positions to define the tree ADT
- The positions in a tree are its nodes and neighboring positions satisfy the parent-child relationships

| **method** | **description** |
|---|---|
| root() | Return the tree's root; error if tree is empty |
| parent(v) | Return v's parent; error if v is a root |
| children(v) | Return v's children (an iterable collection of nodes) |
| isRoot(v) | Test whether v is a root |
| isExternal(v) | Test whether v is an external node |
| isInternal(v) | Test whether v is an internal node |

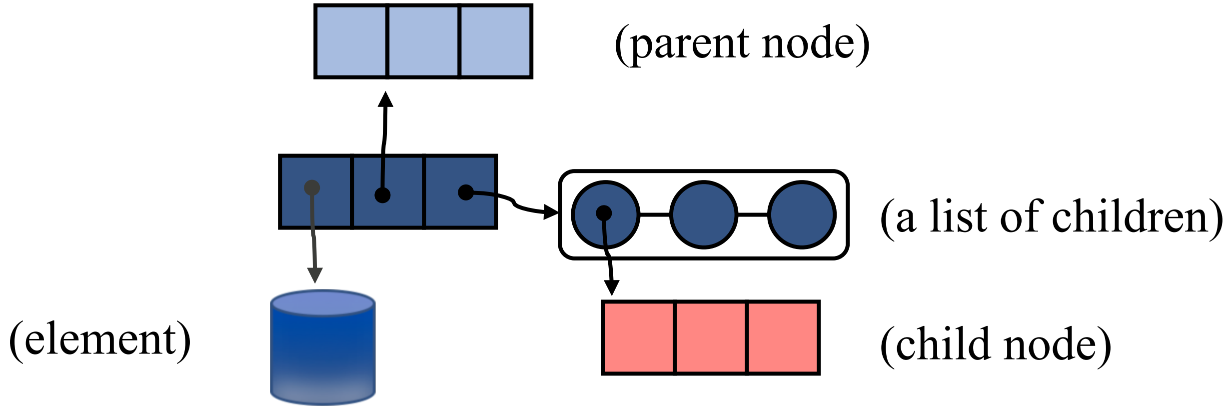
# Tree ADT

- Generic methods (not necessarily related to a tree structure):

| method | description |
|---|---|
| isEmpty() | Test whether the tree has any node or not |
| size() | Return the number of nodes in the tree |
| iterator() | Return an iterator of all the elements stored in the tree |
| positions() | Return an iterable collection of all the nodes of the tree |
| replace(v,e) | Replace with e and return the element stored at node v |

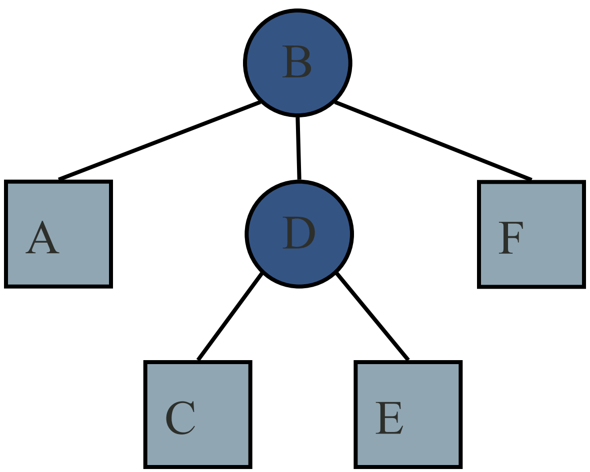
# A Linked Structure for Tree

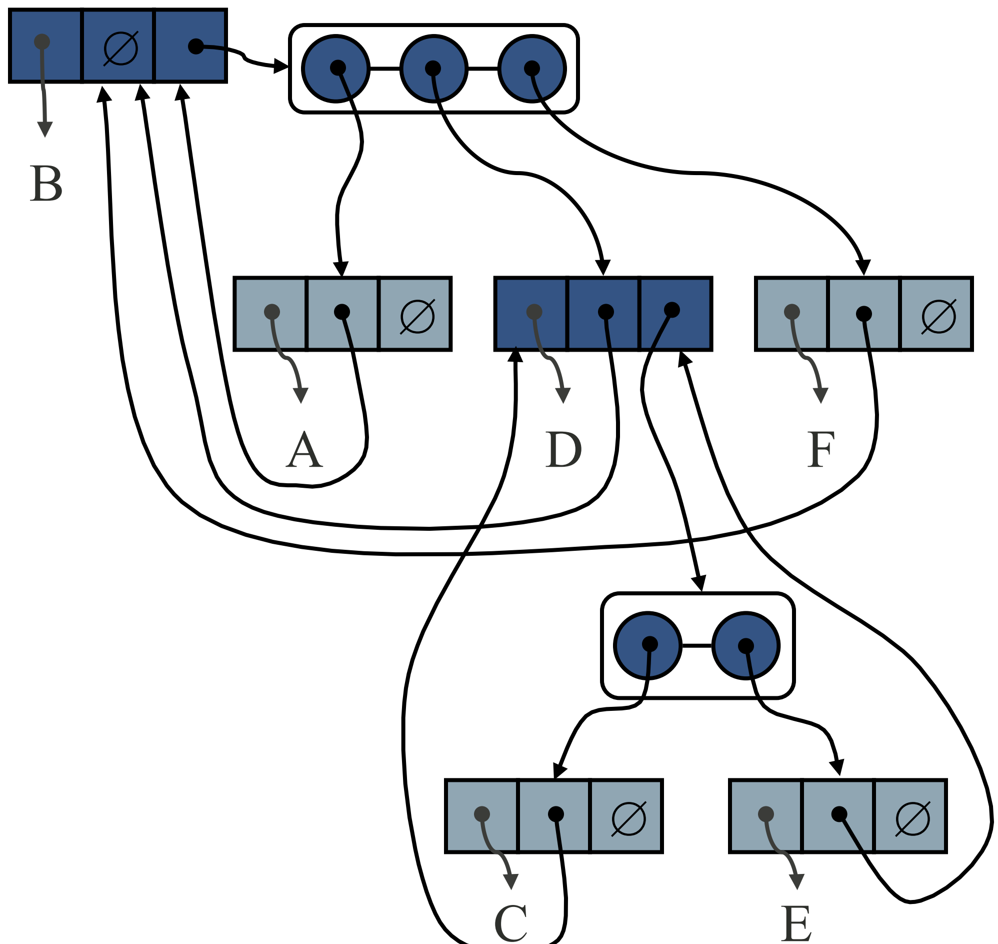
- A node is represented by an object storing
  - Element
  - A parent node
  - A sequence of children nodes



# A Linked Structure for Tree

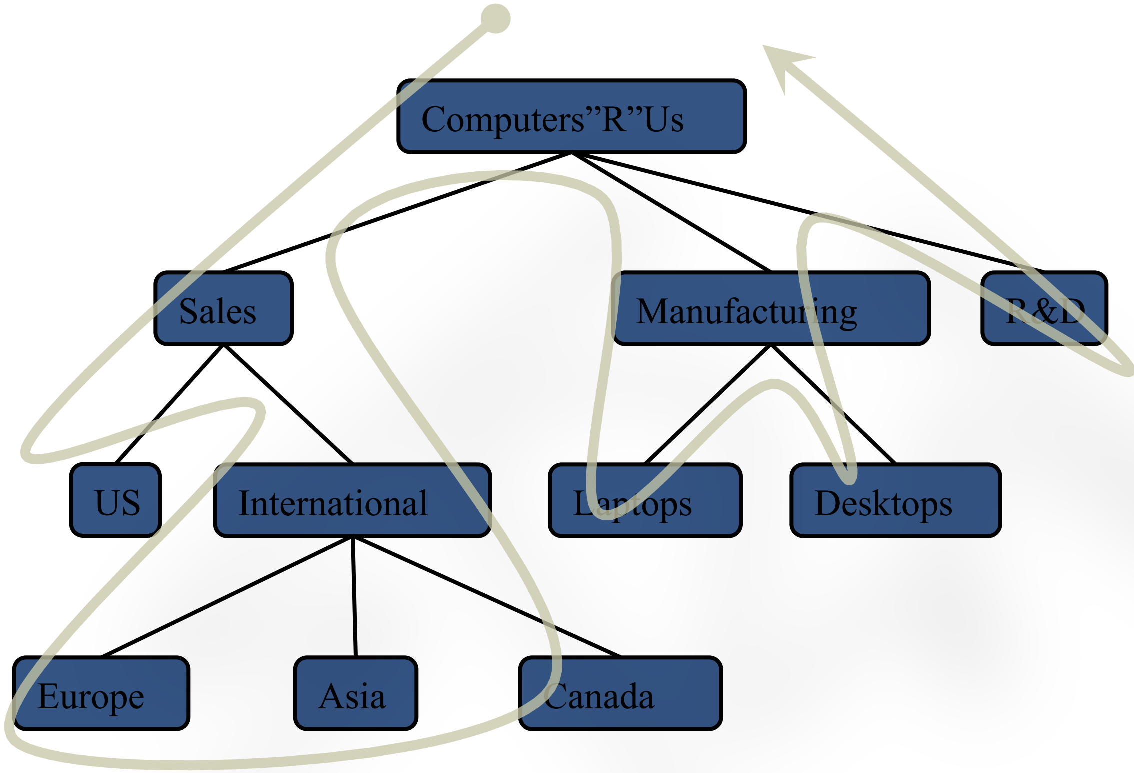
- Node objects implement the Position ADT



# Tree Traversal

- Visit all nodes in a tree
- Do some operations during the visit

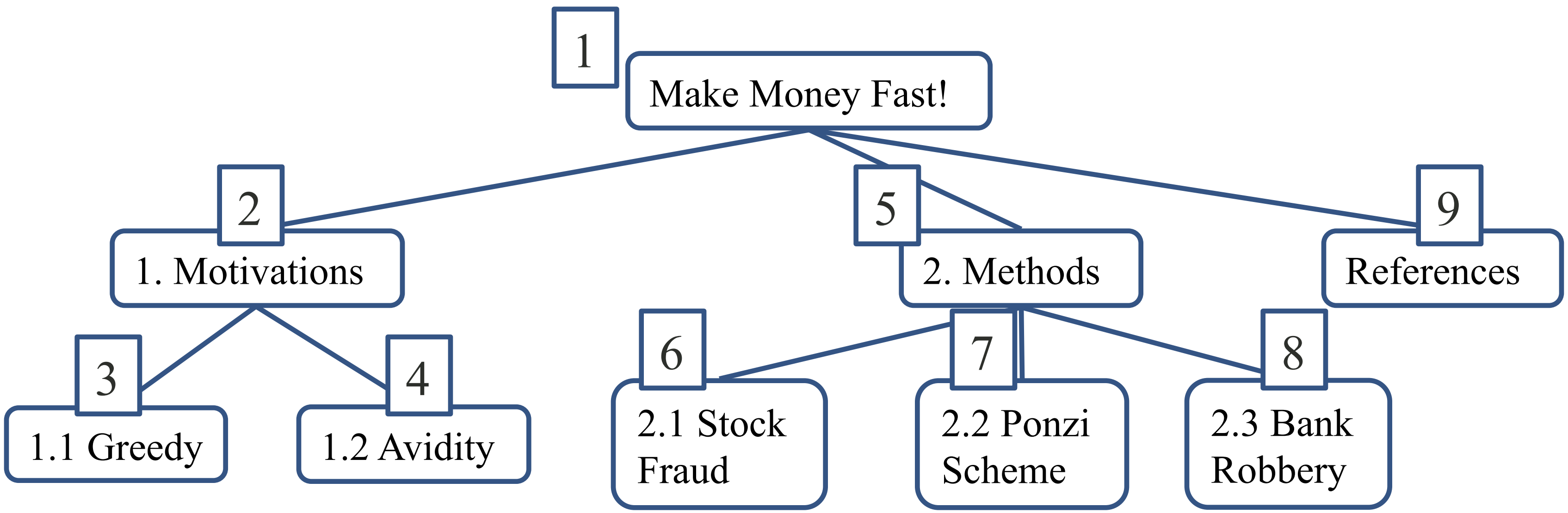


# Preorder Traversal

- A node is visited (so is the operation) before its descendants
- Application:
  - Print a structured document

**Algorithm** *preOrder*(*v*)
 *visit*(*v*)
 **for each** child *w* of *v*
  *preOrder* (*w*)

# Preorder Traversal



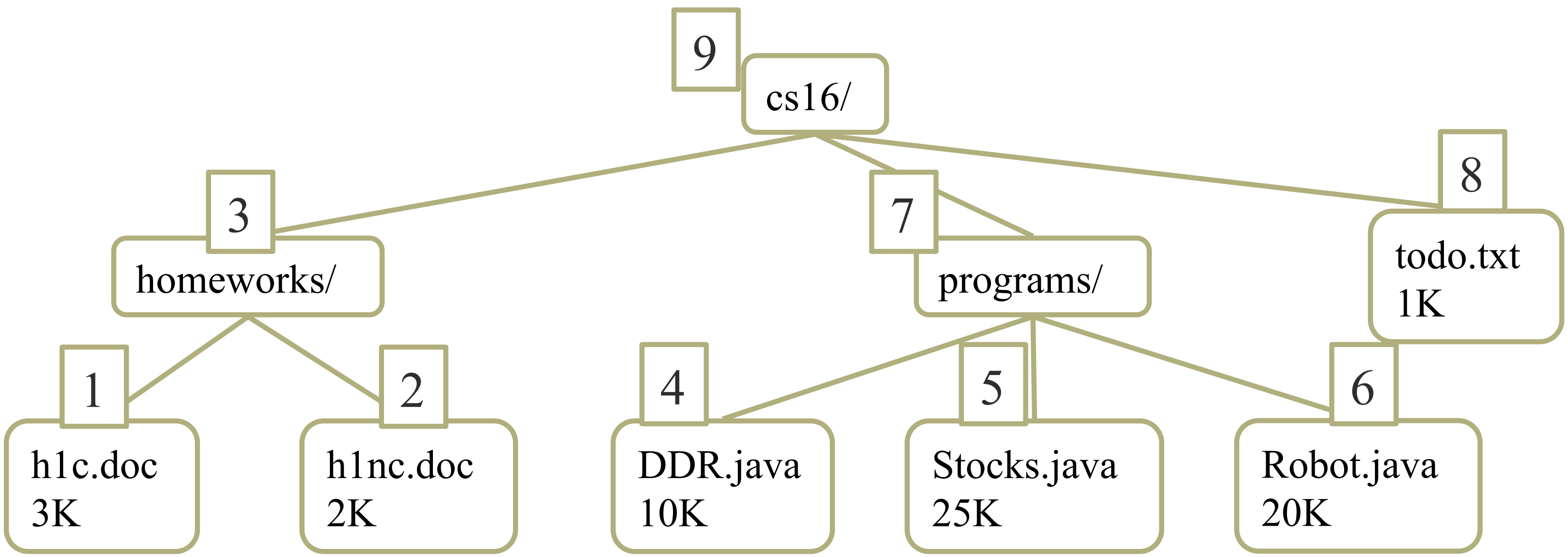

For your project, you can print a structured web site with its sub links using preorder traversal

# Postorder Traversal

- A node is visited after its descendants
- Application:
  - Compute space used by files in a directory and its subdirectories

```
Algorithm postOrder(v)
    for each child w of v
        postOrder (w)
    visit(v)
```

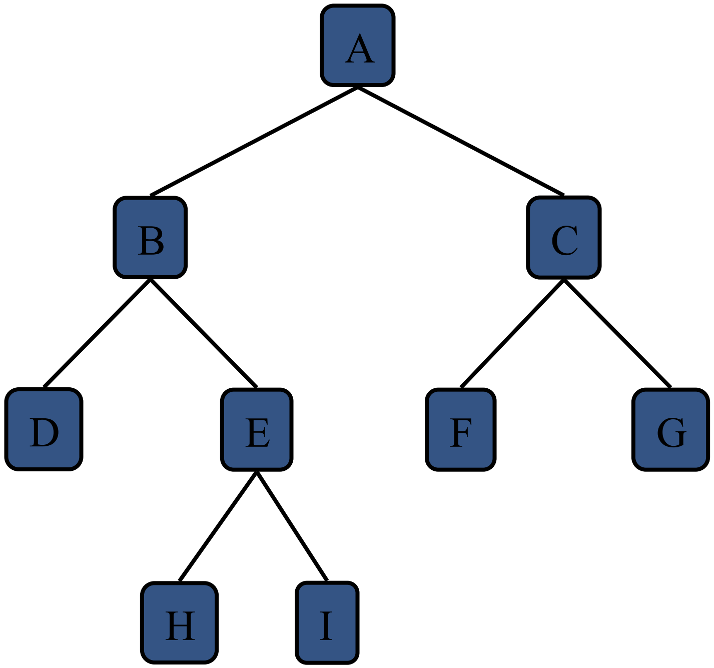
# Postorder Traversal




For your project, you can compute the score of a web site and its sub links
Using postorder traversal

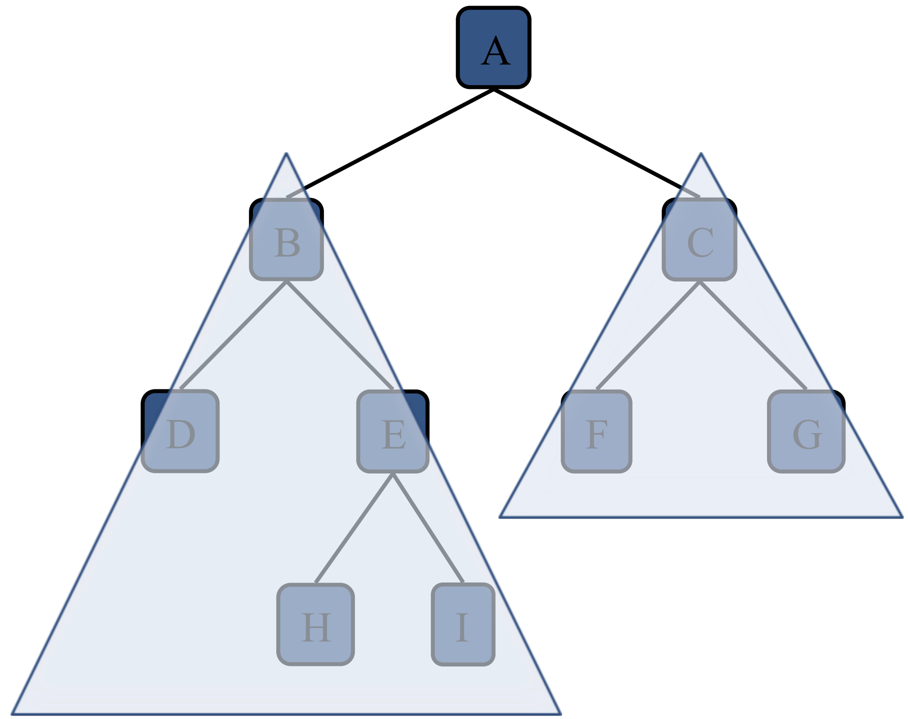
# Binary Tree

- A binary tree is a tree with the following properties:
  - Each internal node has at most two children
  - The children of a node are an ordered pair (left and right)
- We call the children of an internal node left child and right child



# Binary Tree

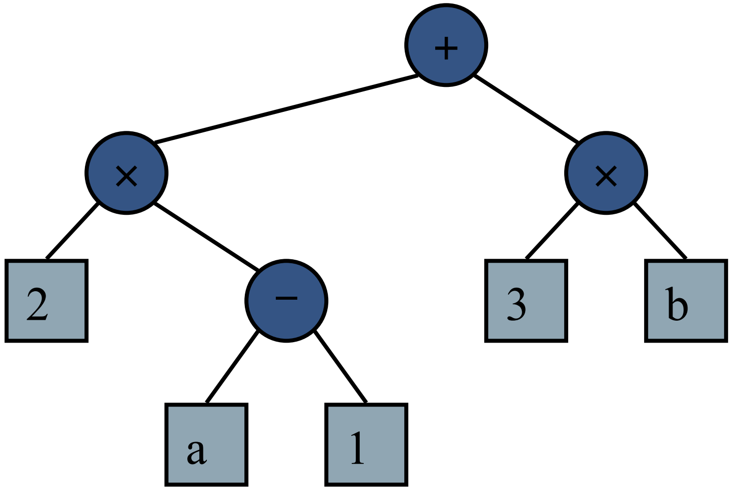
- Alternative recursive definition: a binary tree is either
  - a tree consisting of a single node, or
  - a tree whose root has an ordered pair of children, each of which is a binary tree



# Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - external nodes: operands
- Example: arithmetic expression tree for the expression:

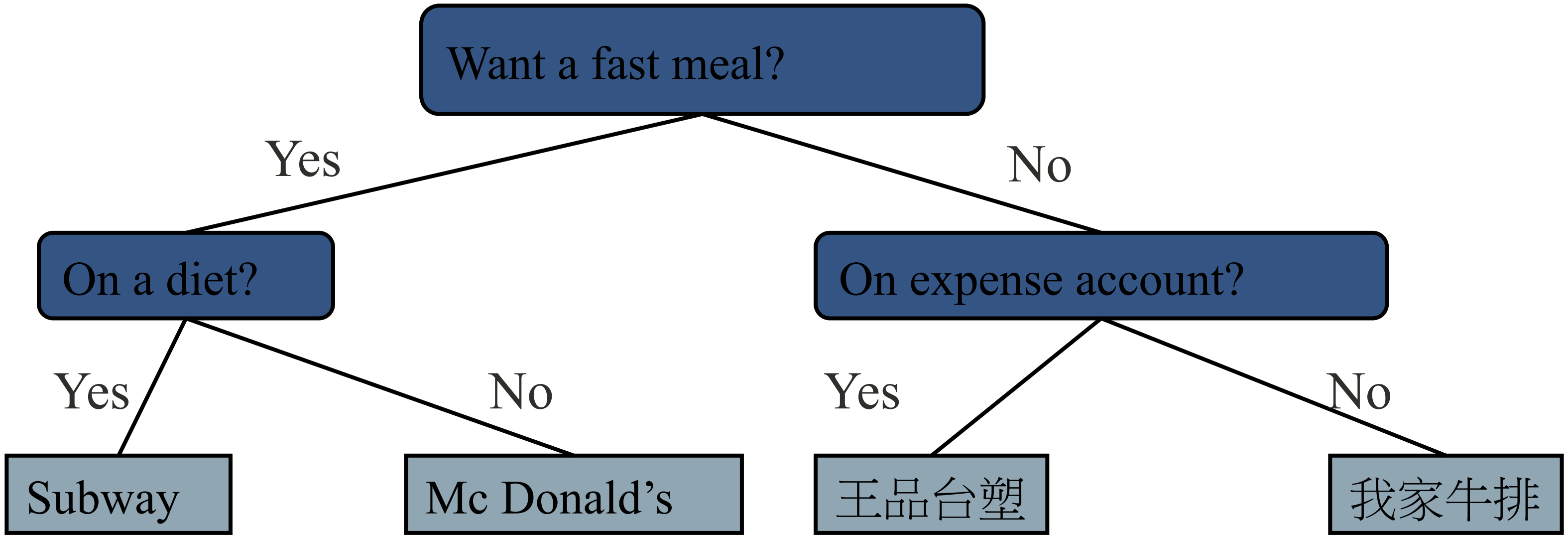
  $(2 \times (a - 1) + (3 \times b))$



# Decision Tree

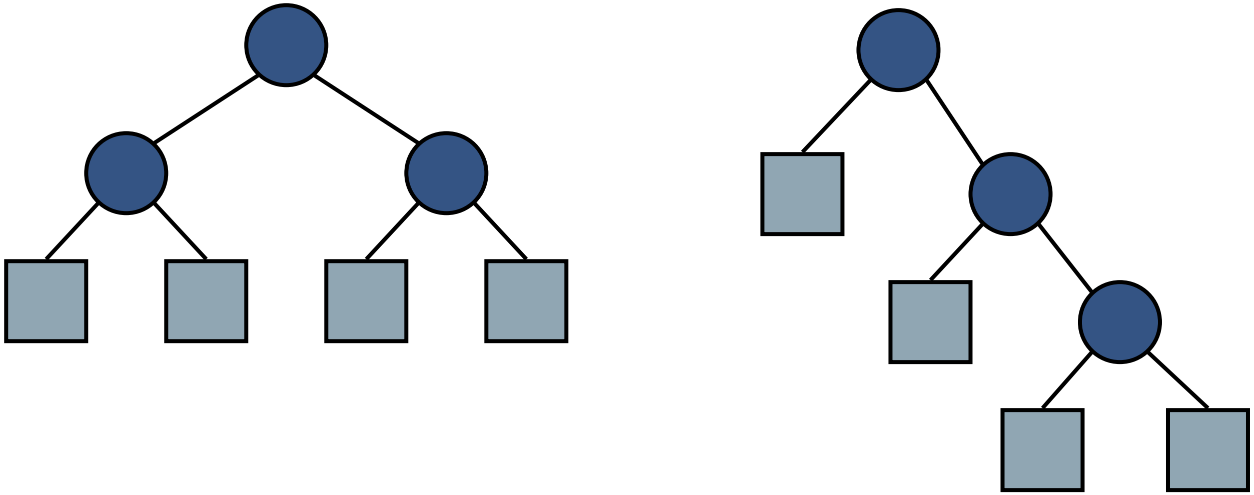
- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - external nodes: decisions

- Example: dining decision



# Proper Binary Trees

- Each internal node has exactly 2 children



# Proper Binary Trees

- n :number of total nodes
- e :number of external nodes
- i :number of internal nodes
- h :height (maximum

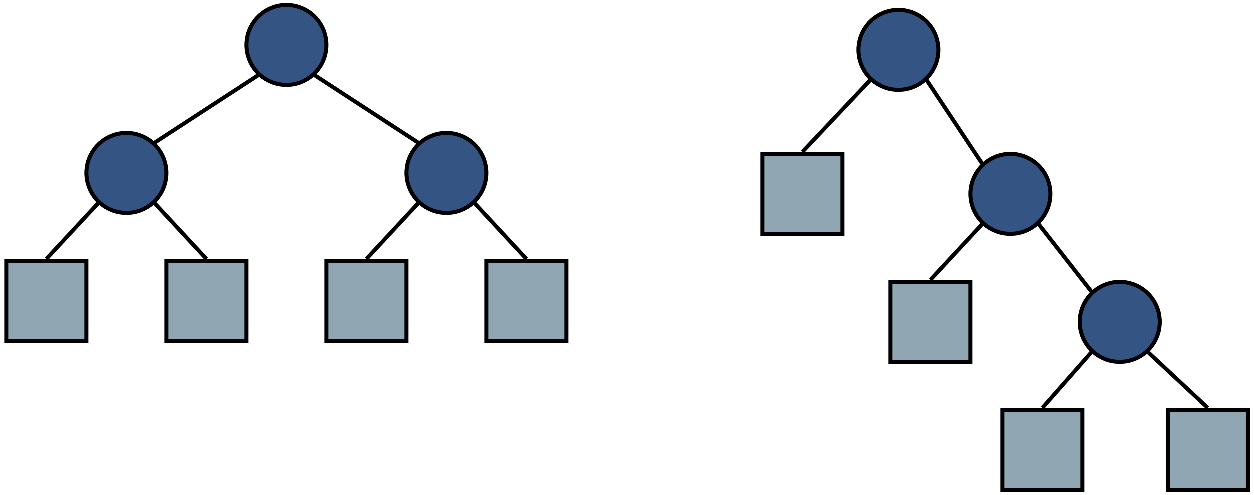
  depth of a node)

Properties:

1. $e = i + 1$
2. $n = 2e - 1$
3. $h \leq i$
4. $h \leq (n - 1)/2$
5. $e \leq 2^h$
6. $h \geq \log_2 e$
7. $h \geq \log_2 (n + 1) - 1$

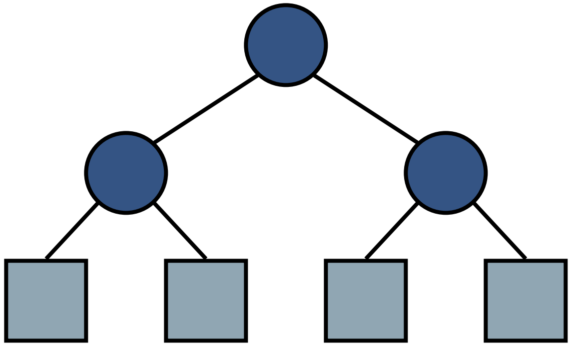
# Properties

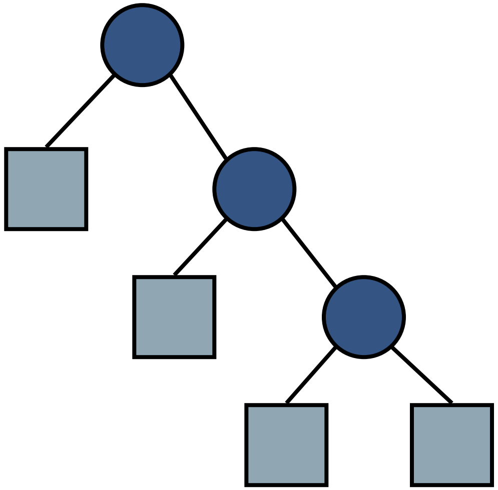
- 1. $e = i+1$
- 2. $n = e+i = 2e-1 = 2i+1$



# Properties

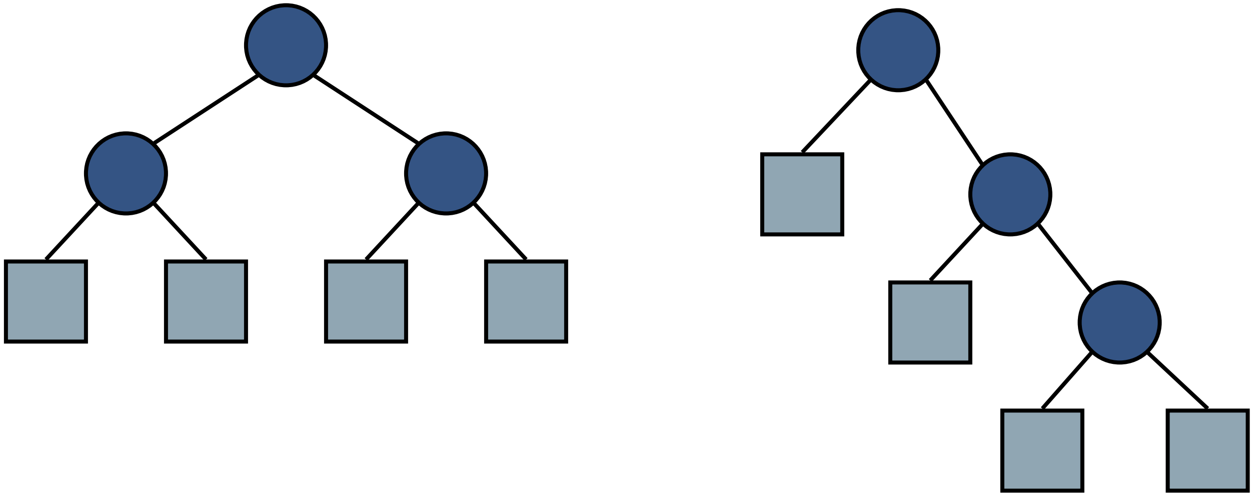
- 3. $h <= i$
- 4. $h <= (n-1)/2$





# Properties

- 5. $e <= 2^h$
- 6. $h >= \log_2 e$
- 7. $h >= \log_2 ((n+1)/2) = \log_2(n+1) - 1$



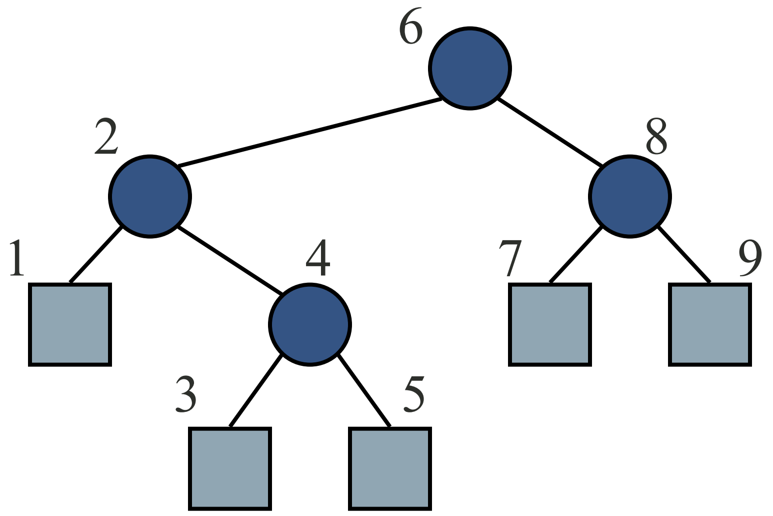
# BinaryTree ADT

- The BinaryTree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT
- Additional methods:
  - position left(p)
  - position right(p)
  - boolean hasLeft(p)
  - boolean hasRight(p)
- Update methods may be defined by data structures implementing the BinaryTree ADT

# Inorder Traversal

- A node is visited after its left subtree and before its right subtree

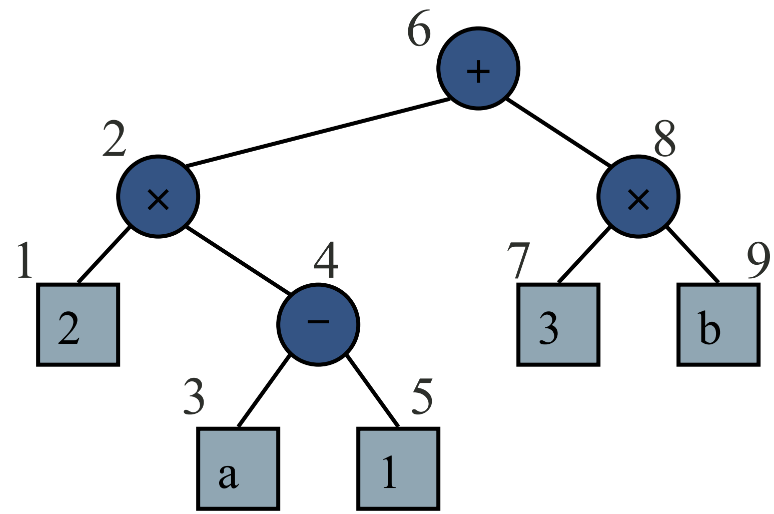
```
Algorithm inOrder(v)
    if hasLeft (v)
        inOrder (left (v))
    visit(v)
    if hasRight (v)
        inOrder (right (v))
```



# Print Arithmetic Expressions

- Specialization of an inorder traversal
  - print operand or operator when visiting node
  - print "(" before traversing left subtree
  - print ")" after traversing right subtree

```
Algorithm printExpression(v)
    if hasLeft (v)
        print("(")
        printExpression (left(v))
    print(v.element ())
    if hasRight (v)
        printExpression (right(v))
        print (")")
```

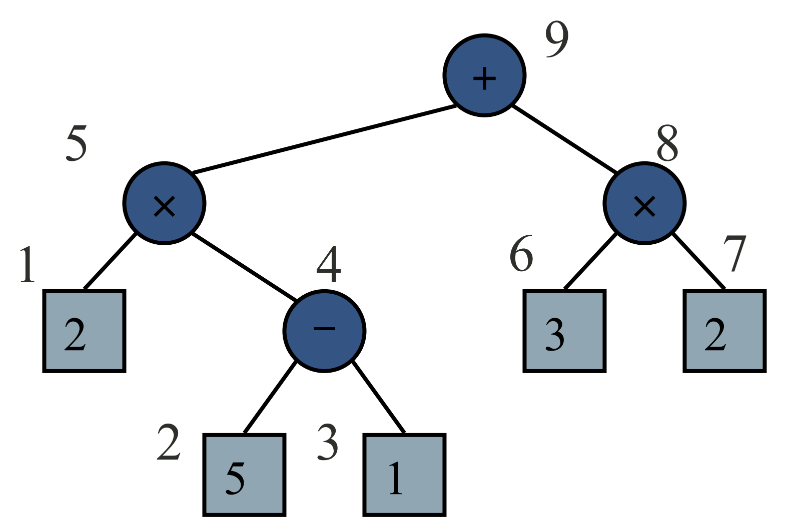



$((2 \times (a - 1)) + (3 \times b))$

# Evaluate Arithmetic Expressions

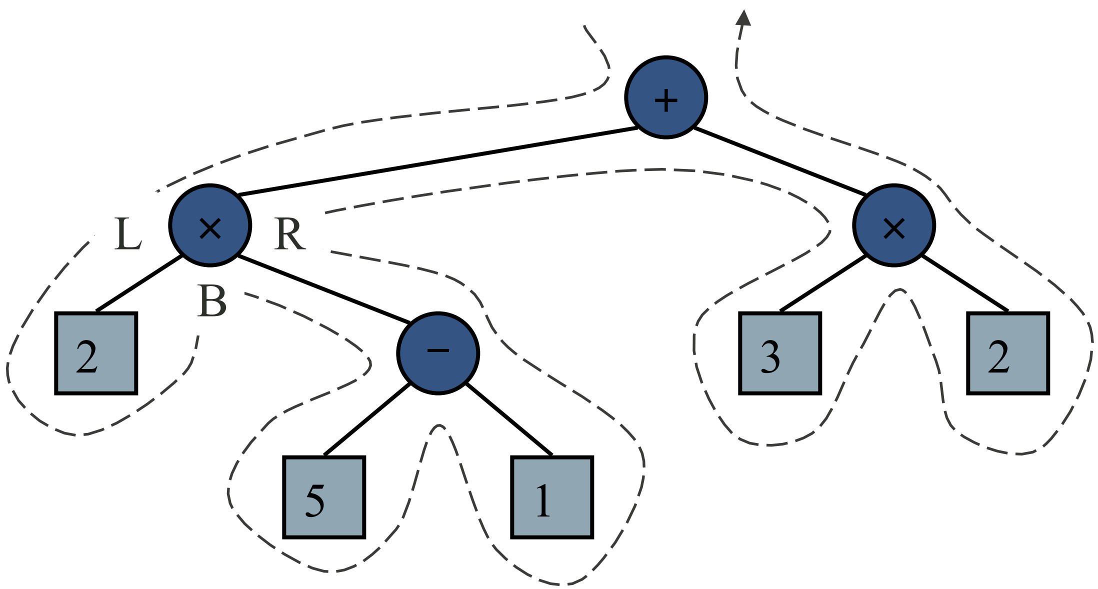
- Specialization of a postorder traversal
  - recursive method returning the value of a subtree
  - when visiting an internal node, combine the values of the subtrees

**Algorithm** *evalExpr*($v$)
  **if** *isExternal* ($v$)
    **return** *v.element* ()
  **else**
    $x \leftarrow$ *evalExpr*(*leftChild* ($v$))
    $y \leftarrow$ *evalExpr*(*rightChild* ($v$))
    $\Diamond \leftarrow$ operator stored at $v$
    **return** $x \Diamond y$



# Euler Tour Traversal

- Generic traversal of a binary tree
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)



# A template method pattern

- A generic computation mechanism
- Specialized for an application by redefining the visit actions

**Algorithm** eularTour(T,v)
Perform the action for visiting node v on the left
**If** v has a left child u in T **then**
    eularTour(T, u)
Perform the action for visiting node v from below
**If** v has a right child w in T **then**
    eularTour(T, w)
Perform the action for visiting node v on the right

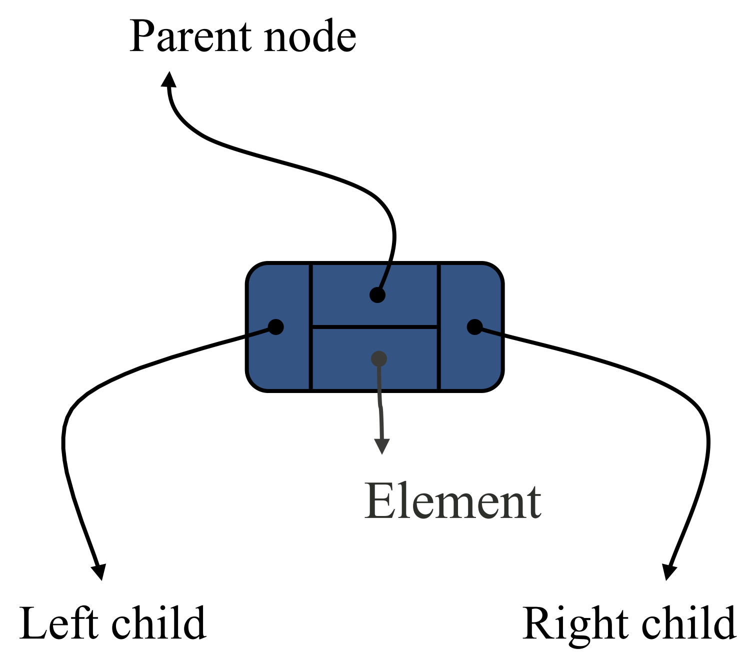
# An application of EularTour

- printExpression
  - On the left action: print (
  - From below action: print v
  - On the right action: print )

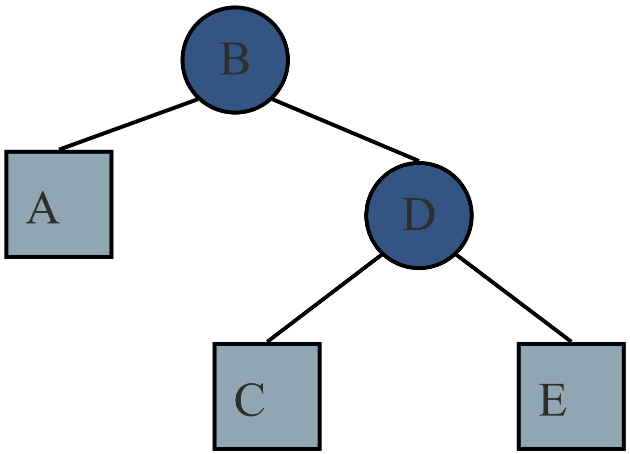
**Algorithm** printExpression(T,v)
if T.isInternal(v) then print “(”
**If** v has a left child u in T **then**
    printExpression(T, u)
print(v)
**If** v has a right child w in T **then**
    printExpression(T, w)
if T.isInternal(v) then print “)”

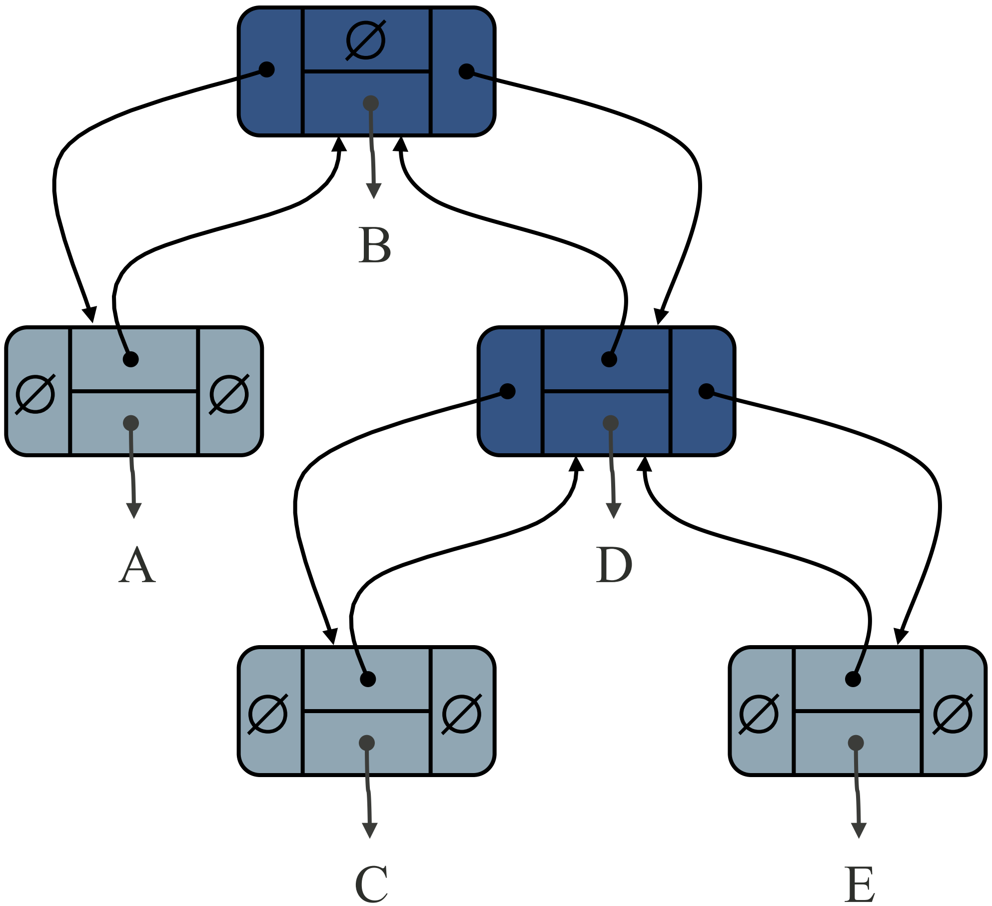
# A Linked Structure for Binary Trees

- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node



# A Linked Structure for Binary Trees

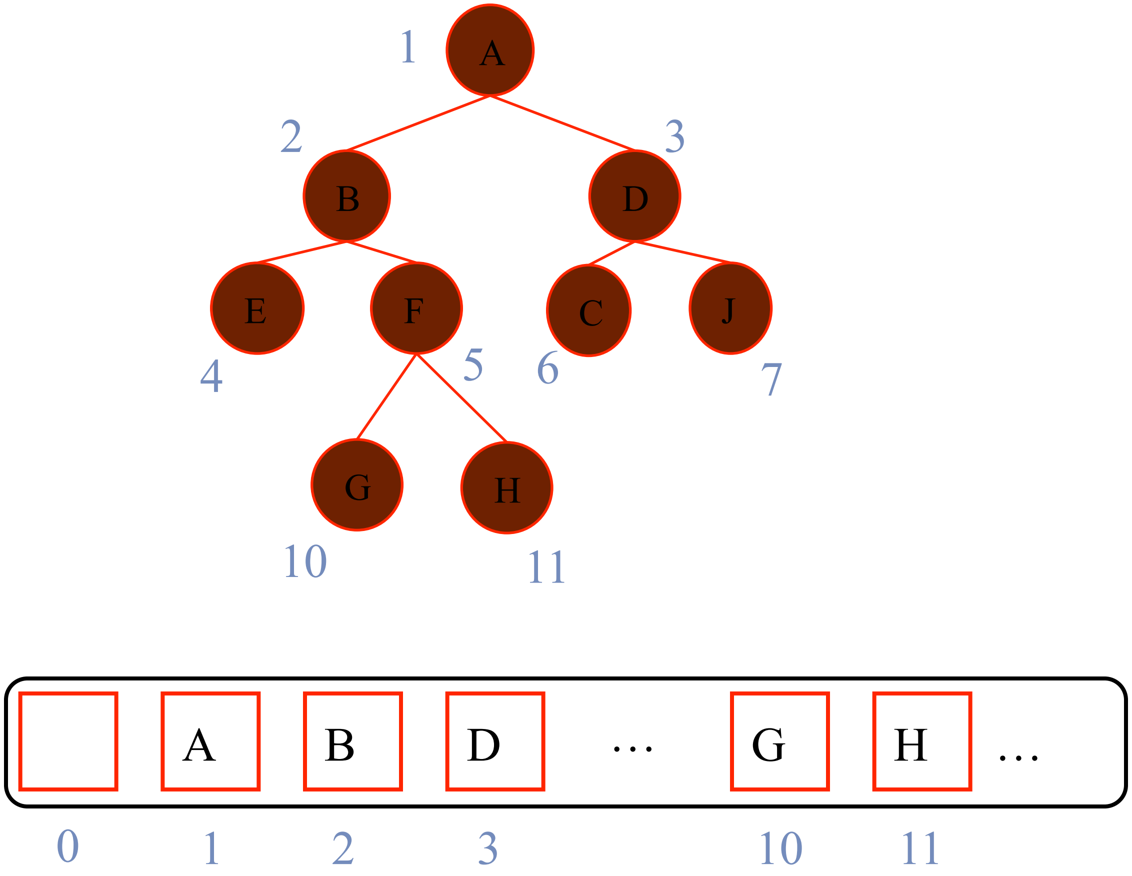


# An Array-Based Representation

- Nodes are stored in an array A
- Node v is stored at A[rank(v)]
  - rank(root) = 1
  - Left in even: if node is the left child of parent(node), rank(node) = 2 · rank(parent(node))
  - Right in odd: if node is the right child of parent(node), rank(node) = 2 · rank(parent(node)) + 1
- A[0] is always empty
- A[i] is empty if there is no node in the ith position
- The array size N is $2^{(h+1)}$

# An Array-Based Representation



# HW 6 (Due on 11/16)

Compute the score of a ***website***!

- Construct a tree and its nodes according to a given website
  - An element (referred by a node) represents one web page and has three fields: (name, url, score)
- Given a keyword and its weight, compute the score of each node
  - Score = number of appearance * weight
  - The score of a node = the score of the content of its url + the scores of its children
  - This can be done by a postorder traversal of a tree
- Output the hierarchy of the website (with names and scores) using parentheses
  - This can be done by an eular tour

# An example input

You will be given a website like:

- Fang Yu, http://www3.nccu.edu.tw/~yuf
  - Publication, http://www3.nccu.edu.tw/~yuf/publication.htm
  - Tool, http://www3.nccu.edu.tw/~yuf/research_projects_and_tools.htm
    - Stranger, http://www.cs.ucsb.edu/~vlab/stranger/
    - OpenWAVES, http://www.openwaves.net
  - Biography, http://www3.nccu.edu.tw/~yuf/short_biography.htm
    - Vlab, http://www.cs.ucsb.edu/~vlab
  - Course, http://www3.nccu.edu.tw/~yuf/course.htm

# An example output

Given a set of keywords, (Yu,1.2), (Fang, 1.8) you shall output something like

```
( Fang Yu, 56.6
    (Publication, 18)
    (Tool, 15.6
        (Stranger, 2.6)
        (OpenWAVES, 8.8)
    )
    (Biography, 9.2
        (Vlab, 3.2)
    )
    (Course, 4.8)
)
```

**Fang Yu, 56.6** indicates that the sum of the score in the content of the given url (http://www3.nccu.edu.tw/~yuf) and its sub links

# Coming Up…

- We will have a programming test in the lab on Nov. 6
- The project proposal is due on Nov. 9
- We will talk about heap (some kind of a tree) on Nov. 9.
  - Read Chapter 8