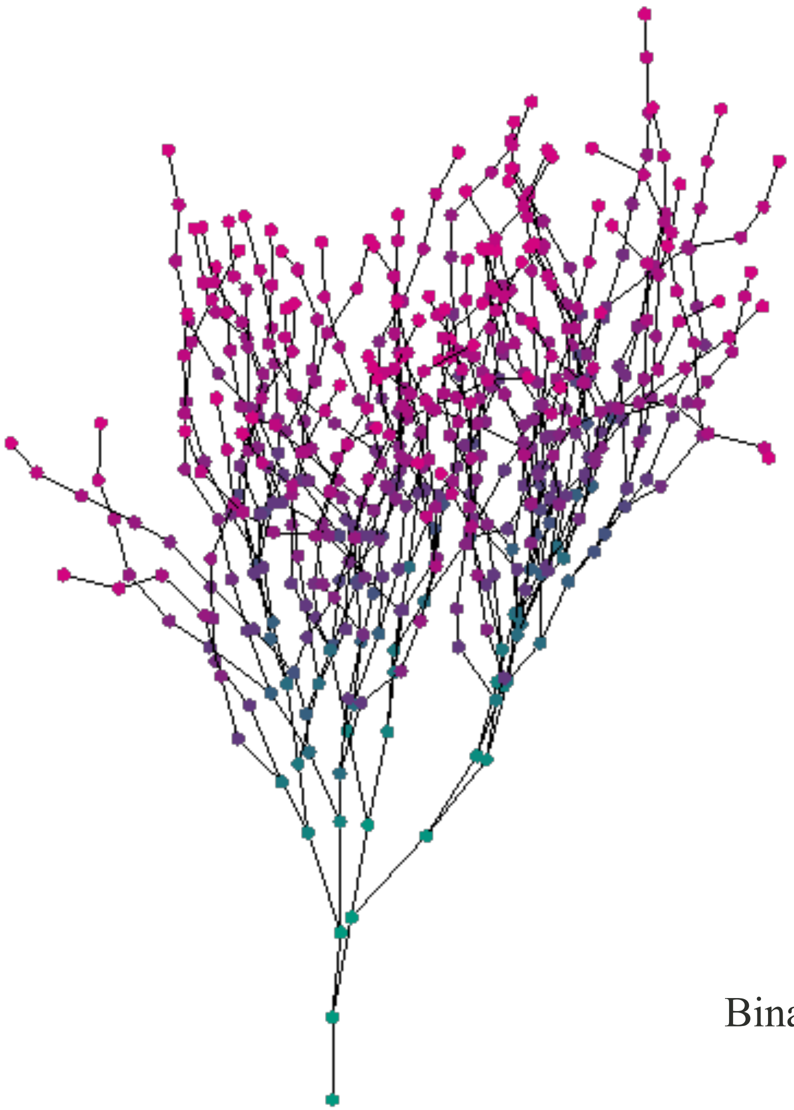
Fall 2019

Fang Yu

Software Security Lab.
Dept. Management Information Systems,
National Chengchi University

Data Structures
Lecture 11



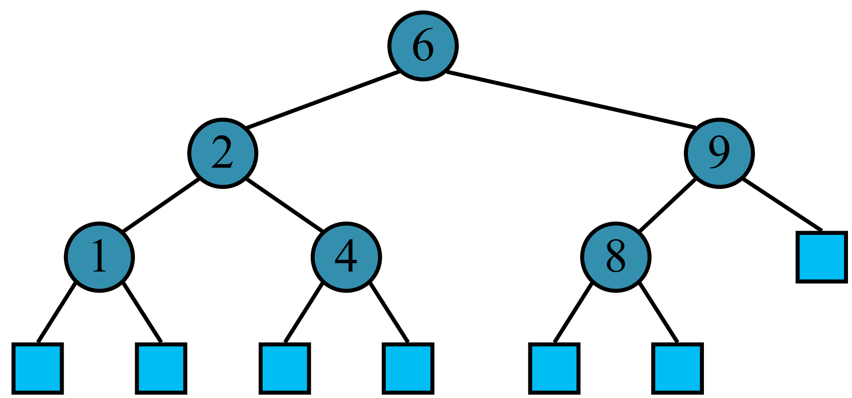
Search Trees

Binary Search Trees, AVL trees, and Splay Trees

Binary Search Trees

- A binary search tree is a binary tree storing keys (or key-value entries) at its internal nodes and satisfying the following property:
  - Let ***u***, ***v***, and ***w*** be three nodes such that ***u*** is in the left subtree of ***v*** and ***w*** is in the right subtree of ***v***. We have $key(u) \le key(v) \le key(w)$
- External nodes do not store items

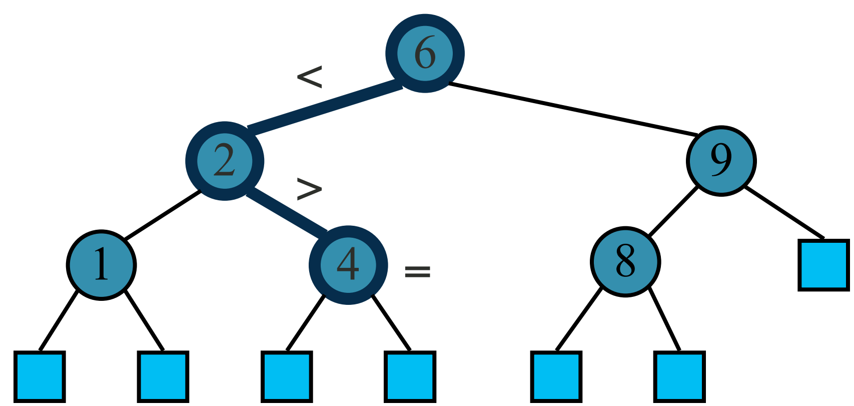
- An inorder traversal of a binary search trees visits the keys in an increasing order



Search

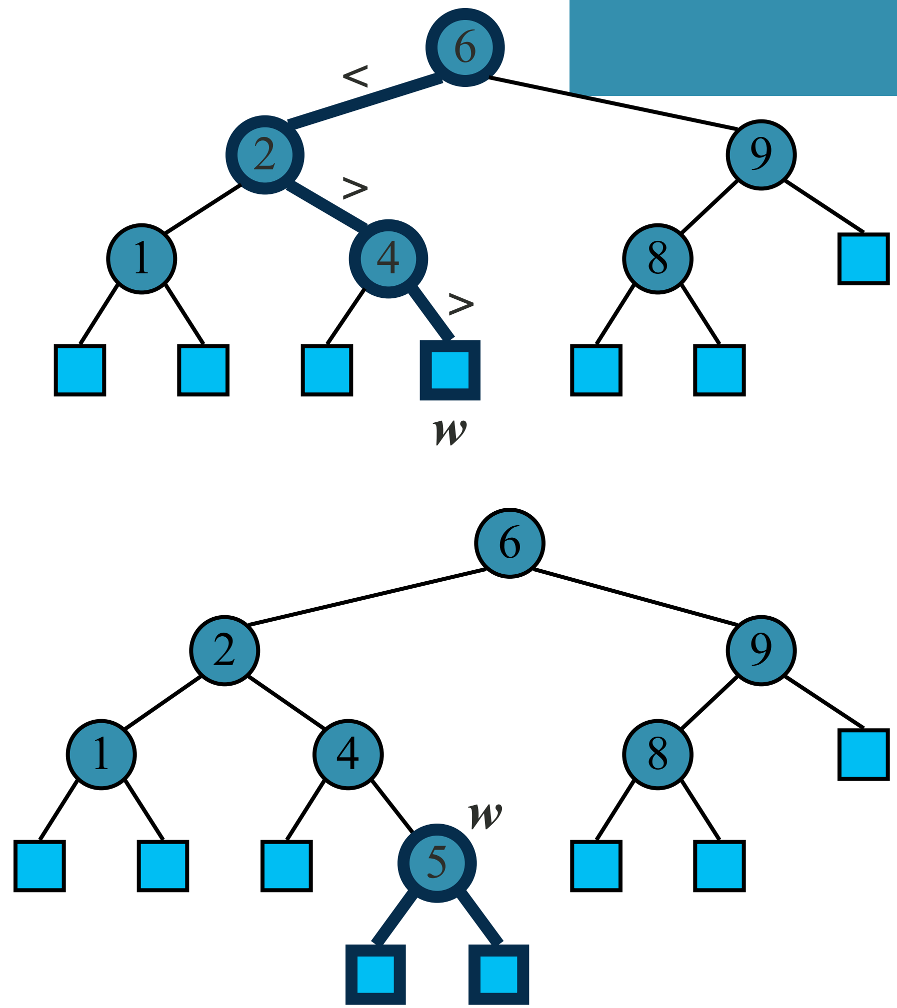
- To search for a key k, we trace a downward path starting at the root
- The next node visited depends on the comparison of k with the key of the current node
- If we reach a leaf, the key is not found
- Example: get(4):
  - Call TreeSearch(4,root)
- The algorithms for floorEntry and ceilingEntry are similar

Algorithm *TreeSearch*(k, v)
    if *T.isExternal* (v)
        return v
    if $k < key(v)$
        return *TreeSearch*(k, *T.left*(v))
    else if $k = key(v)$
        return v
    else $\{ k > key(v) \}$
        return *TreeSearch*(k, *T.right*(v))



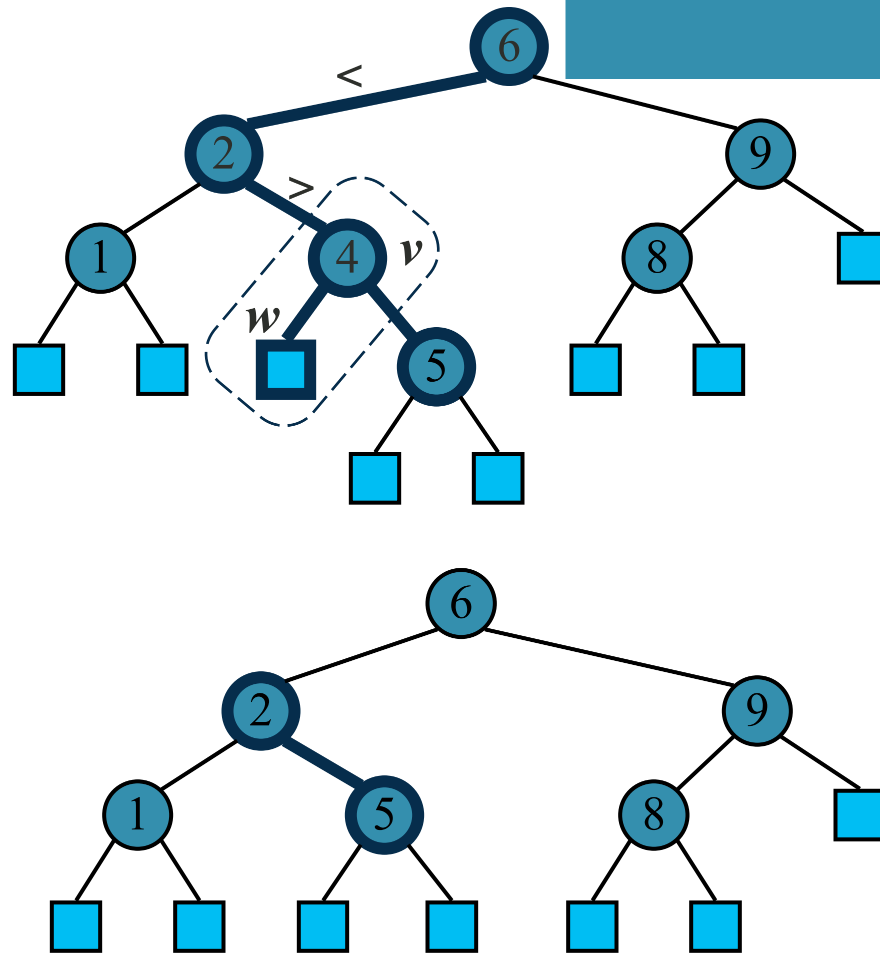
Insertion

- To perform operation put(k, o), we search for key k (using TreeSearch)
- Assume k is not already in the tree, and let w be the leaf reached by the search
- We insert k at node w and expand w into an internal node
- Example: insert 5



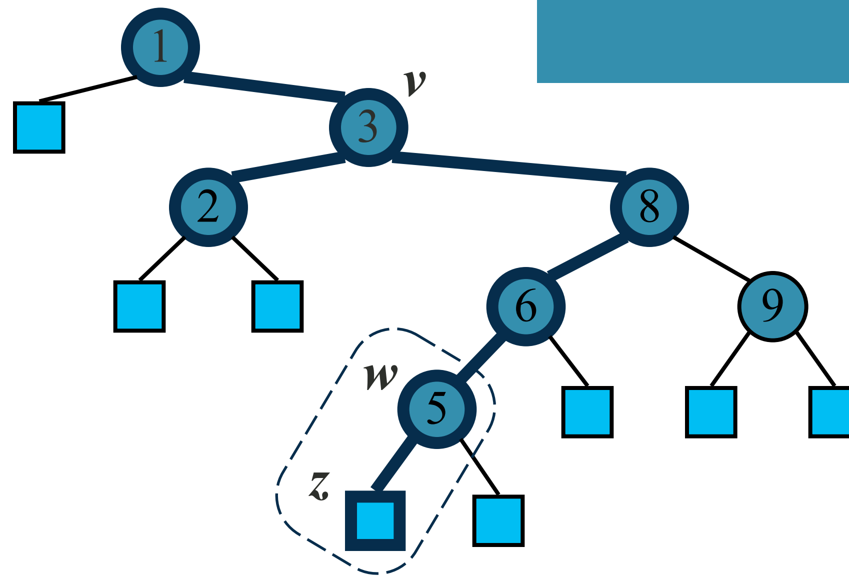
Deletion

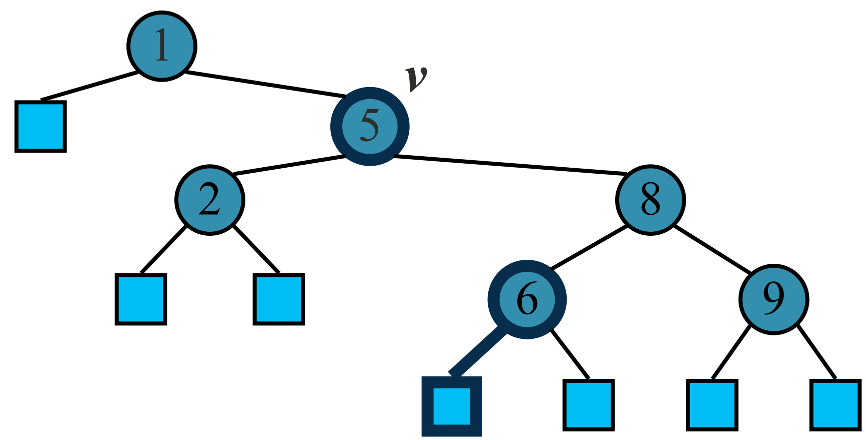
- To perform operation remove(***k***), we search for key ***k***
- Assume key ***k*** is in the tree, and let ***v*** be the node storing ***k***
- If node ***v*** has a leaf child ***w***, we remove ***v*** and ***w*** from the tree with operation removeExternal (***w***), which removes ***w*** and its parent
- Example: remove 4



Deletion (cont.)

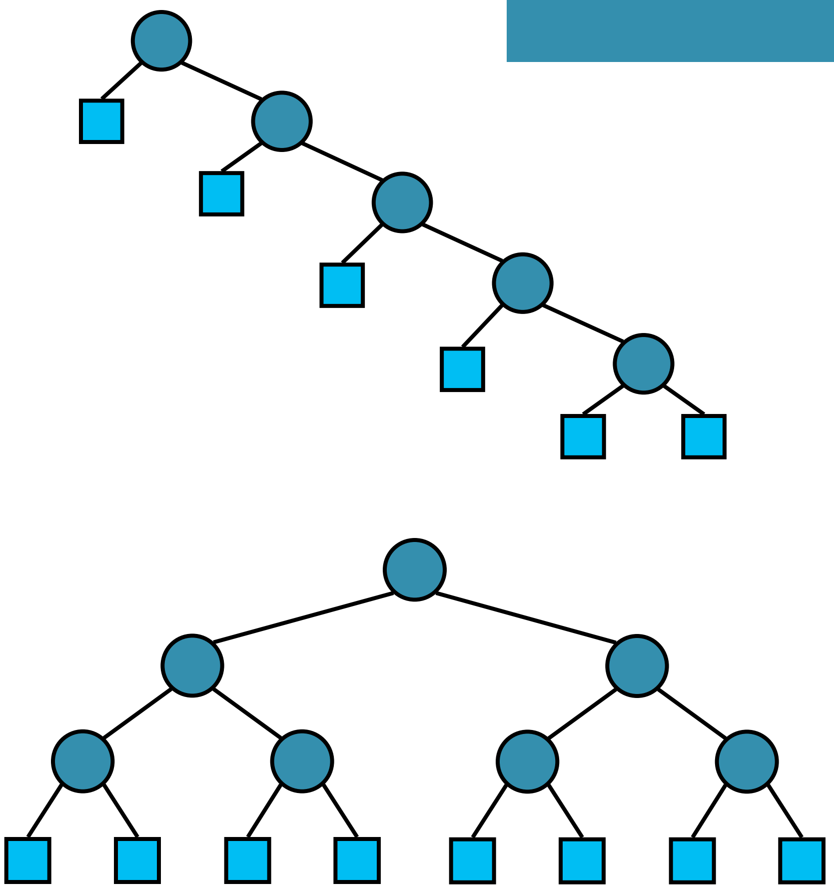
- We consider the case where the key ***k*** to be removed is stored at a node ***v*** whose children are both internal
  - we find the internal node ***w*** that follows ***v*** in an inorder traversal
  - we copy ***key***(***w***) into node ***v***
  - we remove node ***w*** and its left child ***z*** (which must be a leaf) by means of operation removeExternal(***z***)
- Example: remove 3





Performance

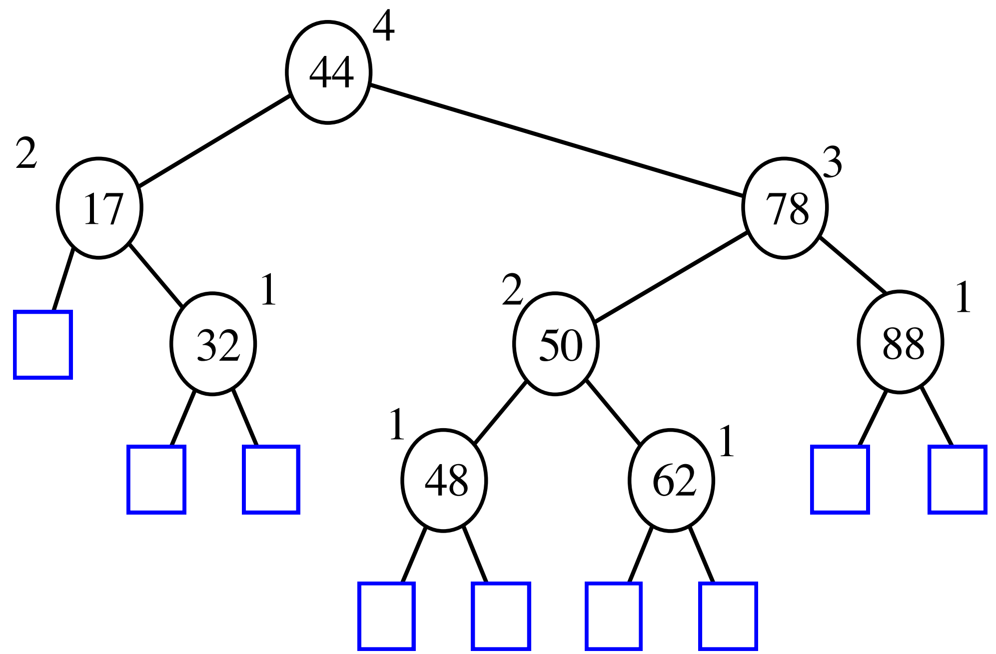
- Consider n ordered set items implemented by means of a binary search tree of height h
  - the space used is $O(n)$
  - methods get, put and remove take $O(h)$ time
- The height h is $O(n)$ in the worst case and $O(\log n)$ in the best case



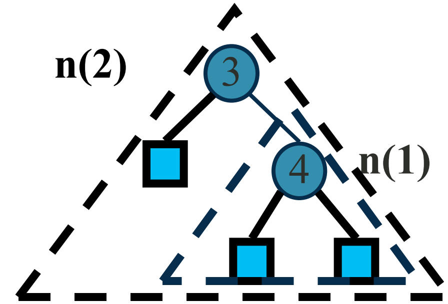
We want a balanced binary tree!

AVL Tree Definition

- AVL trees are balanced
- An AVL Tree is a binary search tree such that for every internal node v of T, the heights of the children of v can differ by at most 1



An example of an AVL tree where the heights are shown next to the nodes:

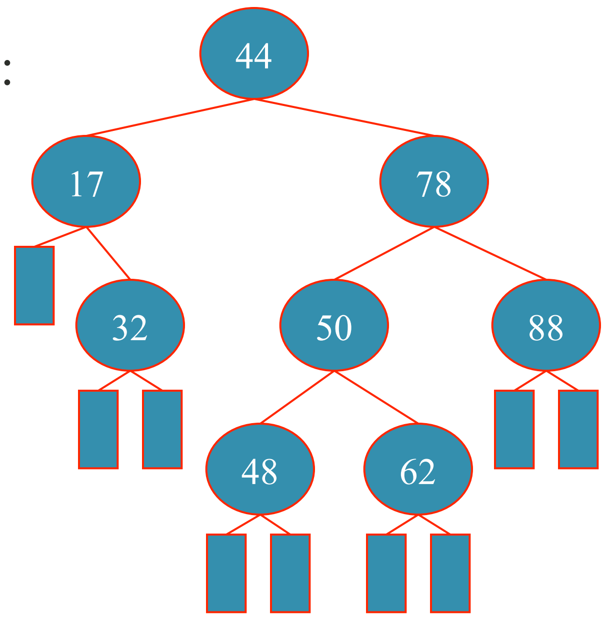


Height of an AVL Tree

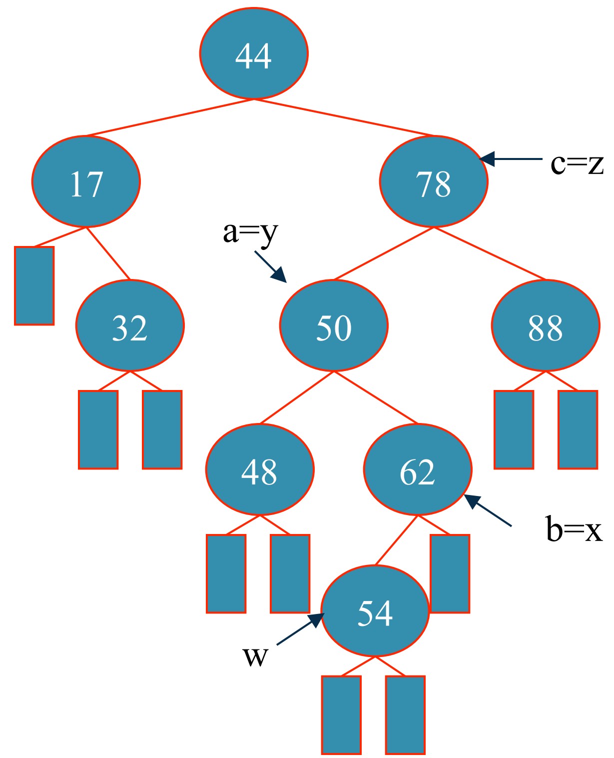
- Fact: The height of an AVL tree storing n keys is O(log n).
- Proof: Let us bound n(h): the minimum number of internal nodes of an AVL tree of height h.
- We easily see that $n(1) = 1$ and $n(2) = 2$
- For $n > 2$, an AVL tree of height h contains the root node, one AVL subtree of height n-1 and another of height n-2.
- That is, $n(h) = 1 + n(h-1) + n(h-2)$
- Knowing $n(h-1) > n(h-2)$, we get $n(h) > 2n(h-2)$. So
  $n(h) > 2n(h-2)$, $n(h) > 4n(h-4)$, $n(h) > 8n(n-6)$, … (by induction),
  $n(h) > 2^i n(h-2i)$
- Solving the base case we get: $n(h) > 2^{h/2-1}$
- Taking logarithms: $h < 2\log n(h) + 2$
- Thus the height of an AVL tree is O(log n)

Insertion

- Insertion is as in a binary search tree
- Always done by expanding an external node.
- Example:



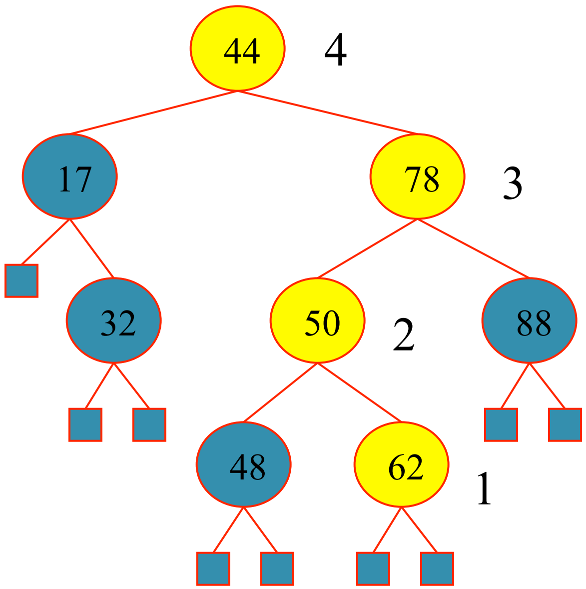
before insertion

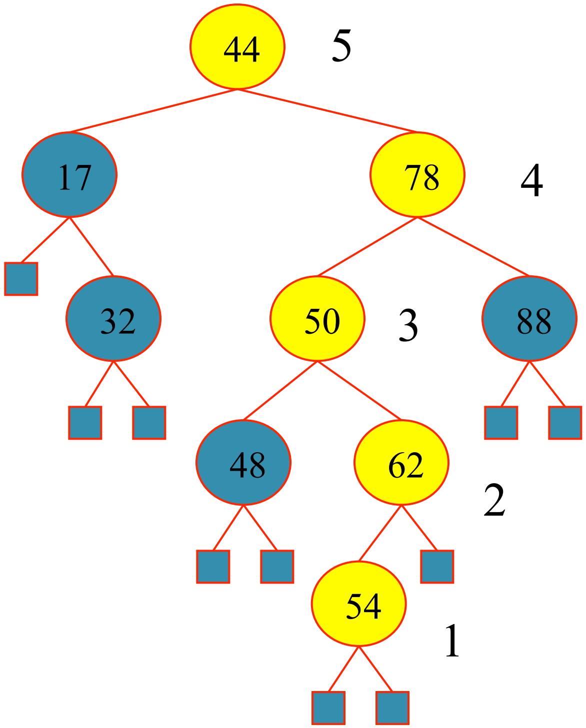


after insertion

After Insertion

- All nodes along the path increase their height by 1
- It may violate the AVL property





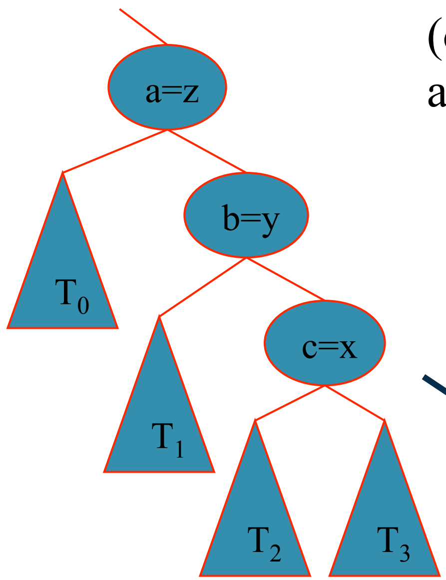
Search and repair

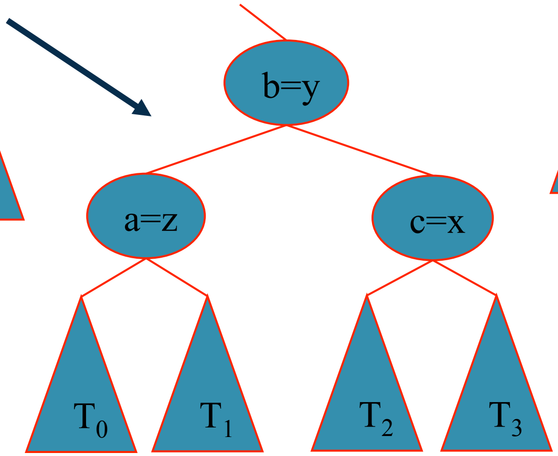
- Let z be the first violation node from the bottom along the path
- Let y be z'child with the higher height (y is 2 greater than its sibling)
- Let x be y's child with the higher height
- We rebalance z by calling **trinode restructuring** method

Trinode Restructuring

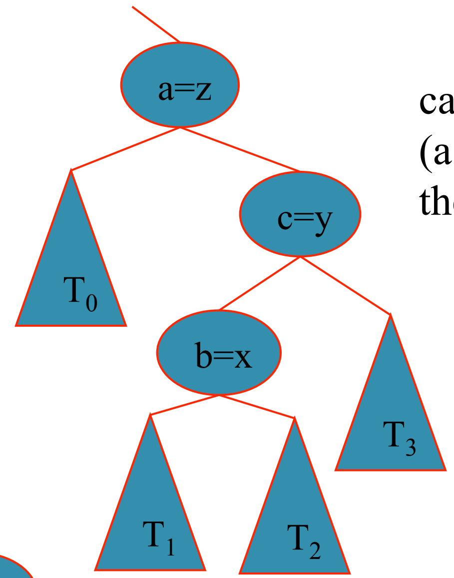
- let (a,b,c) be an inorder listing of x, y, z
- perform the rotations needed to make b the topmost node of the three

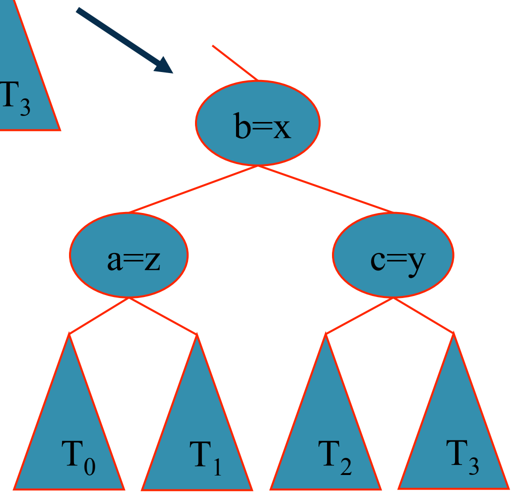
(other two cases are symmetrical)





case 1: single rotation
(a left rotation about a)

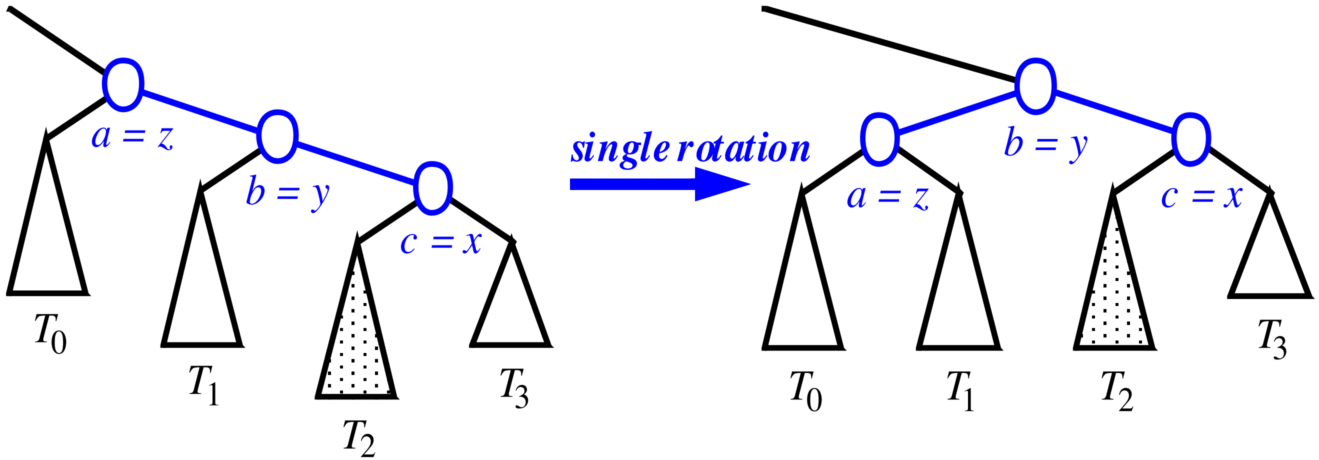


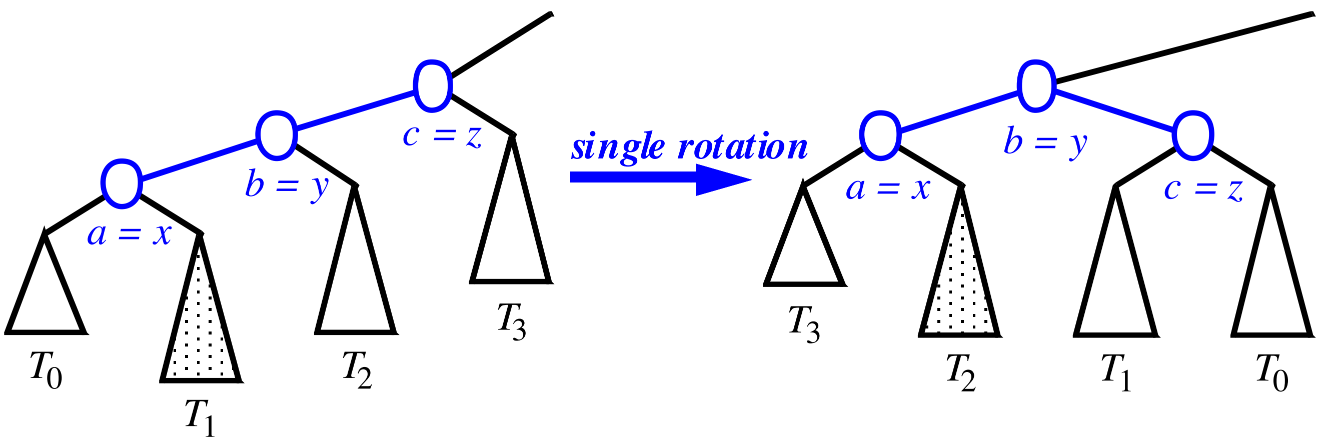


case 2: double rotation
(a right rotation about c, then a left rotation about a)

Restructuring (as Single Rotations)

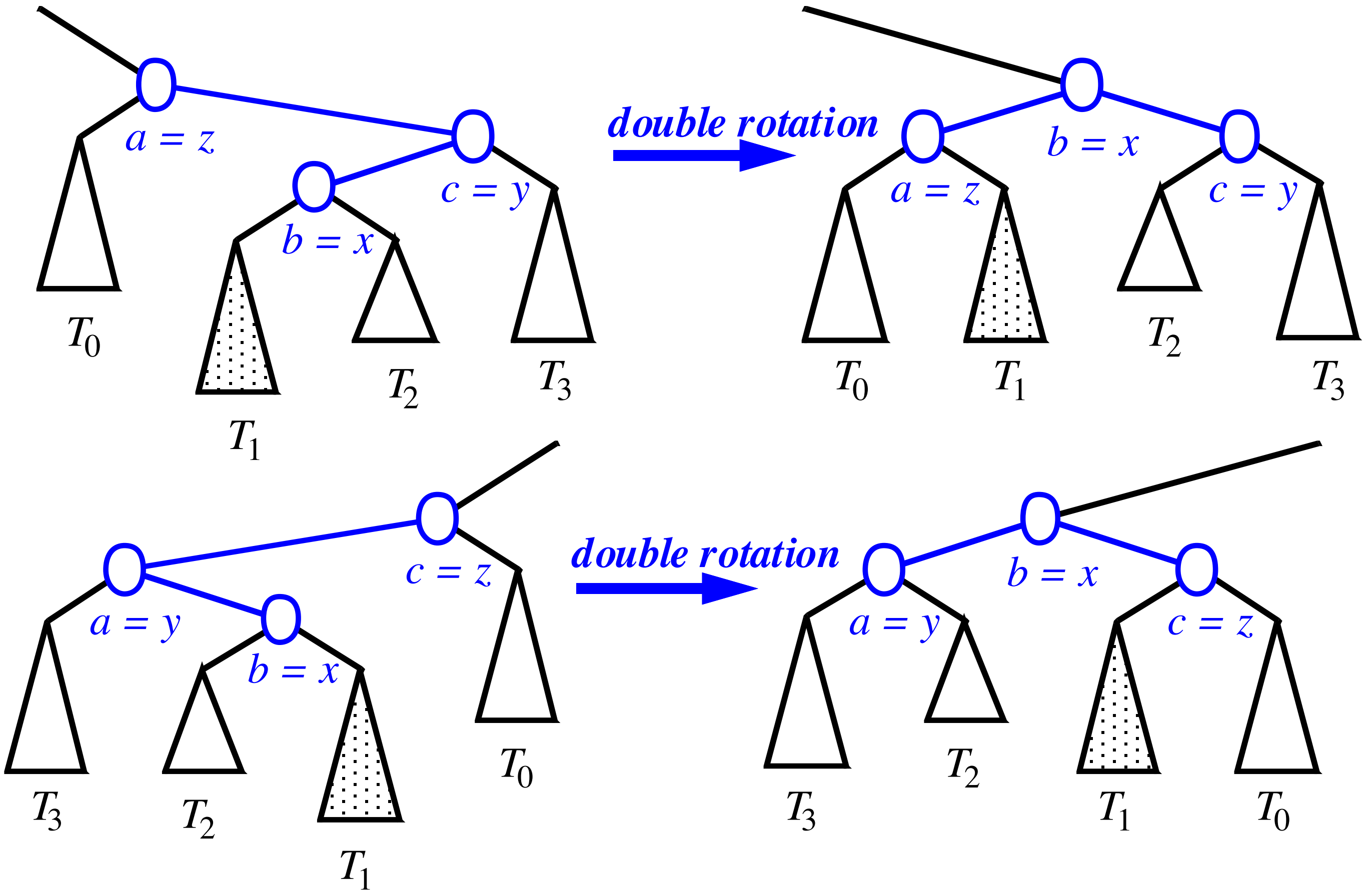
- Single Rotations:



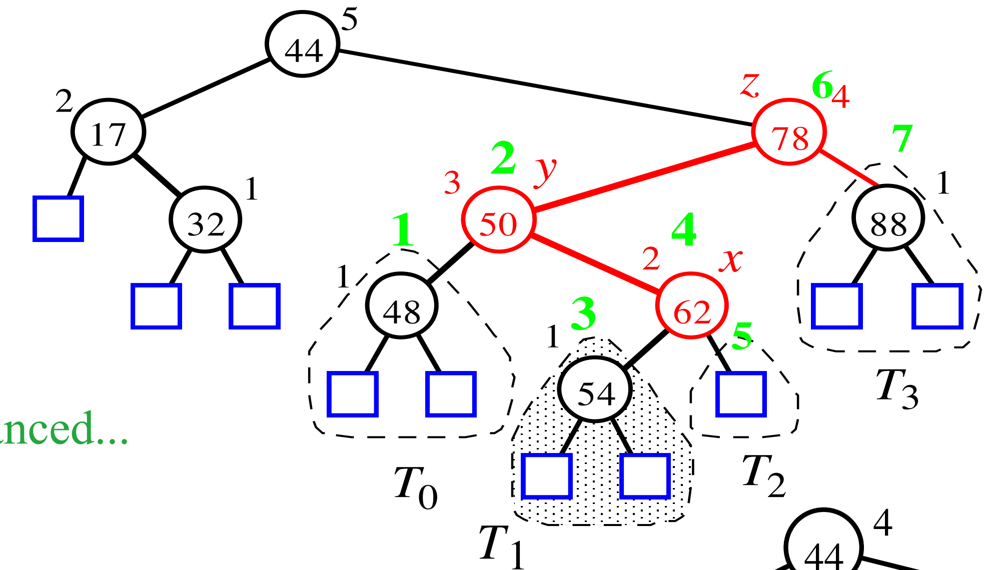


Restructuring (as Double Rotations)

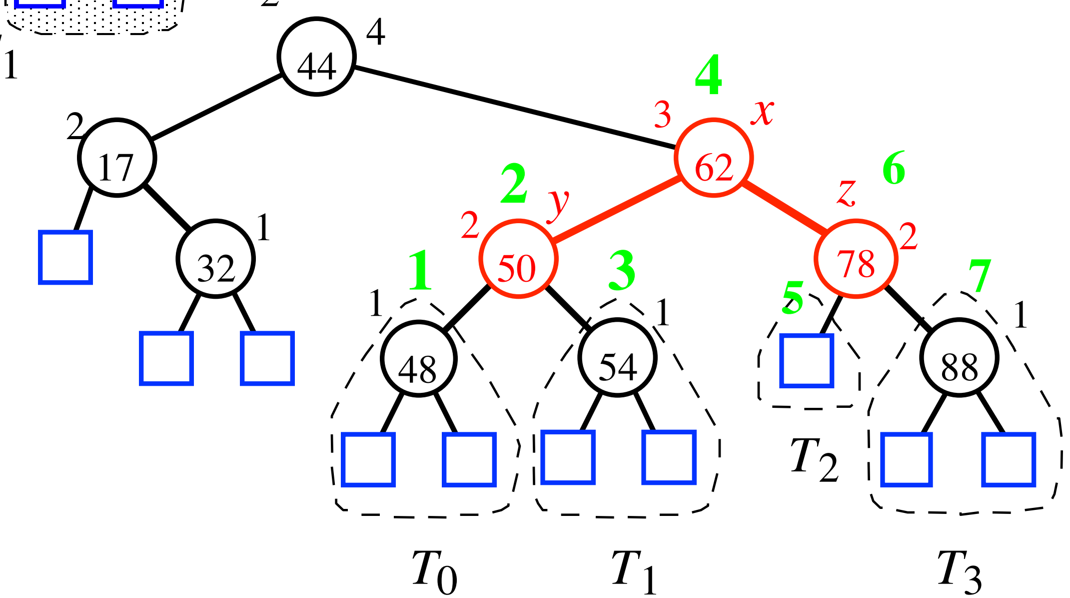
- double rotations:



Insertion Example, continued



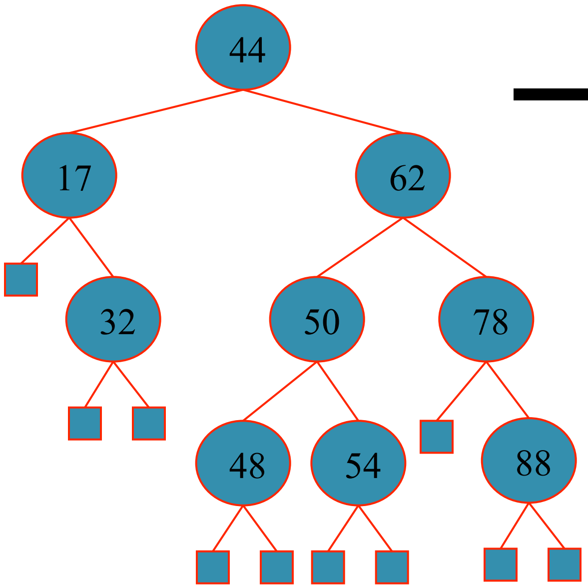
unbalanced...



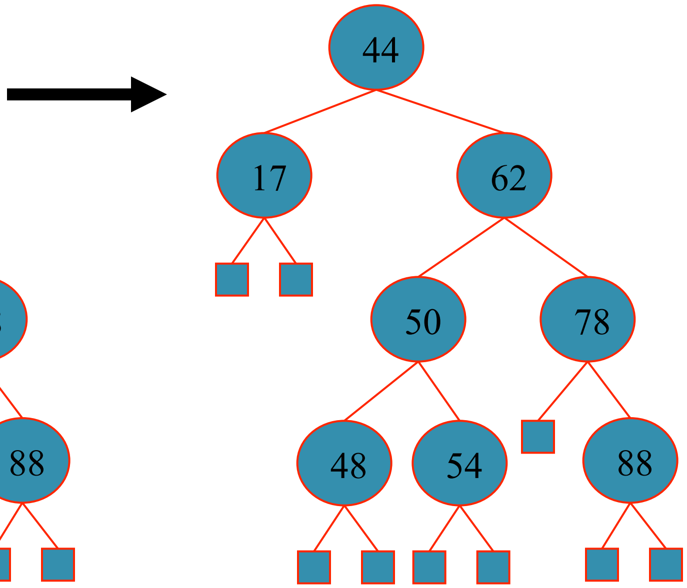
...balanced

Removal

- Removal begins as in a binary search tree, which means the node removed will become an empty external node. Its parent, w, may cause an unbalance.
- Example:



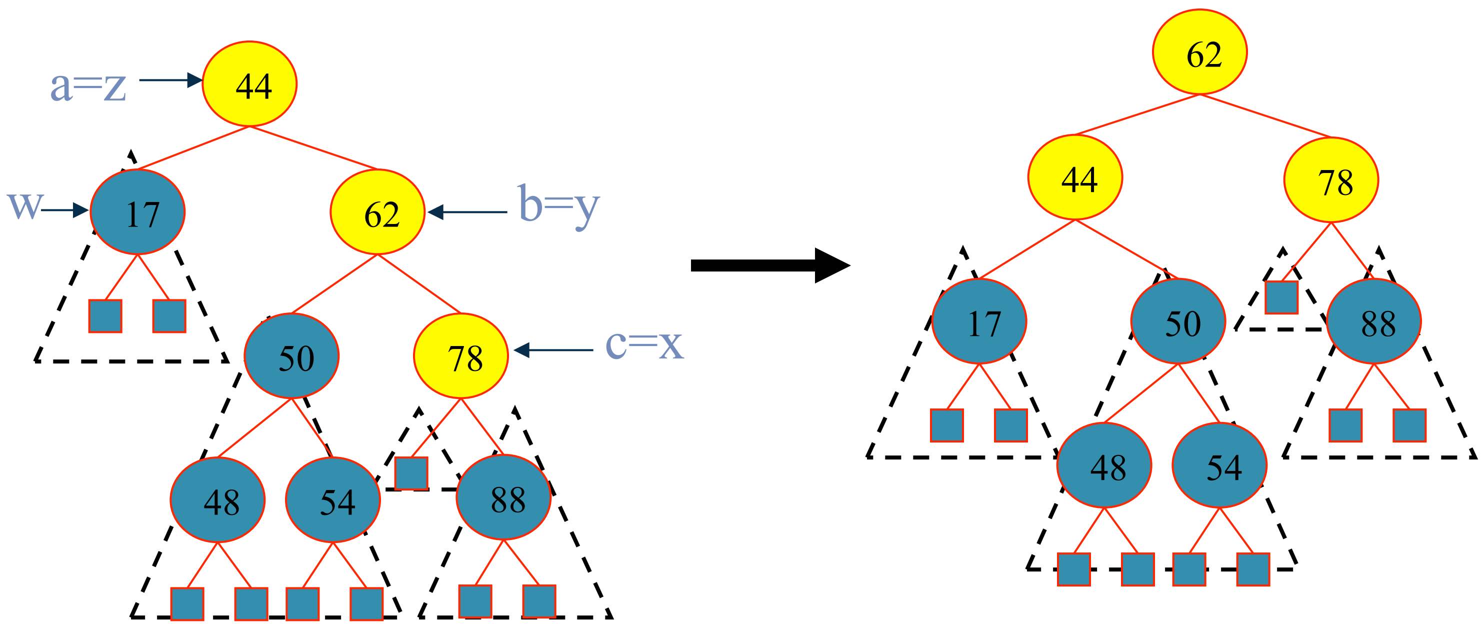
before deletion of 32



after deletion

Rebalancing after a Removal

- Let z be the first unbalanced node encountered while travelling up the tree from w. Also, let y be the child of z with the larger height, and let x be the child of y with the larger height
- As this restructuring may upset the balance of another node higher in the tree, we must continue checking for balance until the root of T is reached



AVL Tree Performance

- a single restructure takes O(1) time
  - using a linked-structure binary tree
- get takes O(log n) time
  - height of tree is O(log n), no restructures needed
- put takes O(log n) time
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)
- remove takes O(log n) time
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)

Splay Tree

- a splay tree is a binary search tree where a node is splayed after it is accessed (for a search or update)
- deepest internal node accessed is splayed
- splay: move the node to the root
- splaying costs O(h), where h is height of the tree – which is still O(n) worst-case
  - O(h) rotations, each of which is O(1)

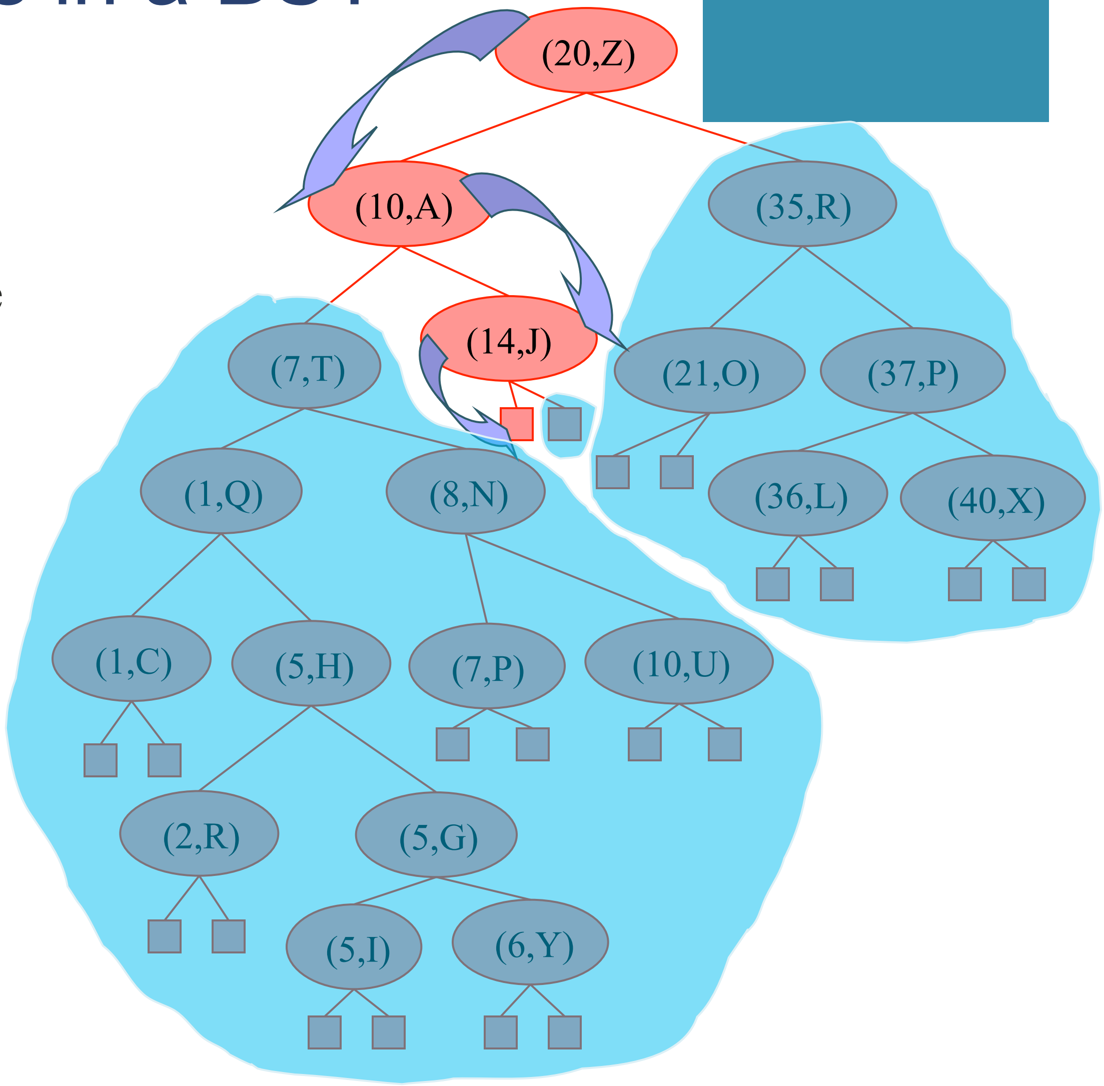
Splay Tree

- which nodes are splayed after each operation?

method	**splay node**
get(k)	if key found, use that node if key not found, use parent of ending external node
put(k,v)	use the new node containing the entry inserted
remove(k)	use the parent of the internal node that was actually removed from the tree (the parent of the node that the removed item was swapped with)

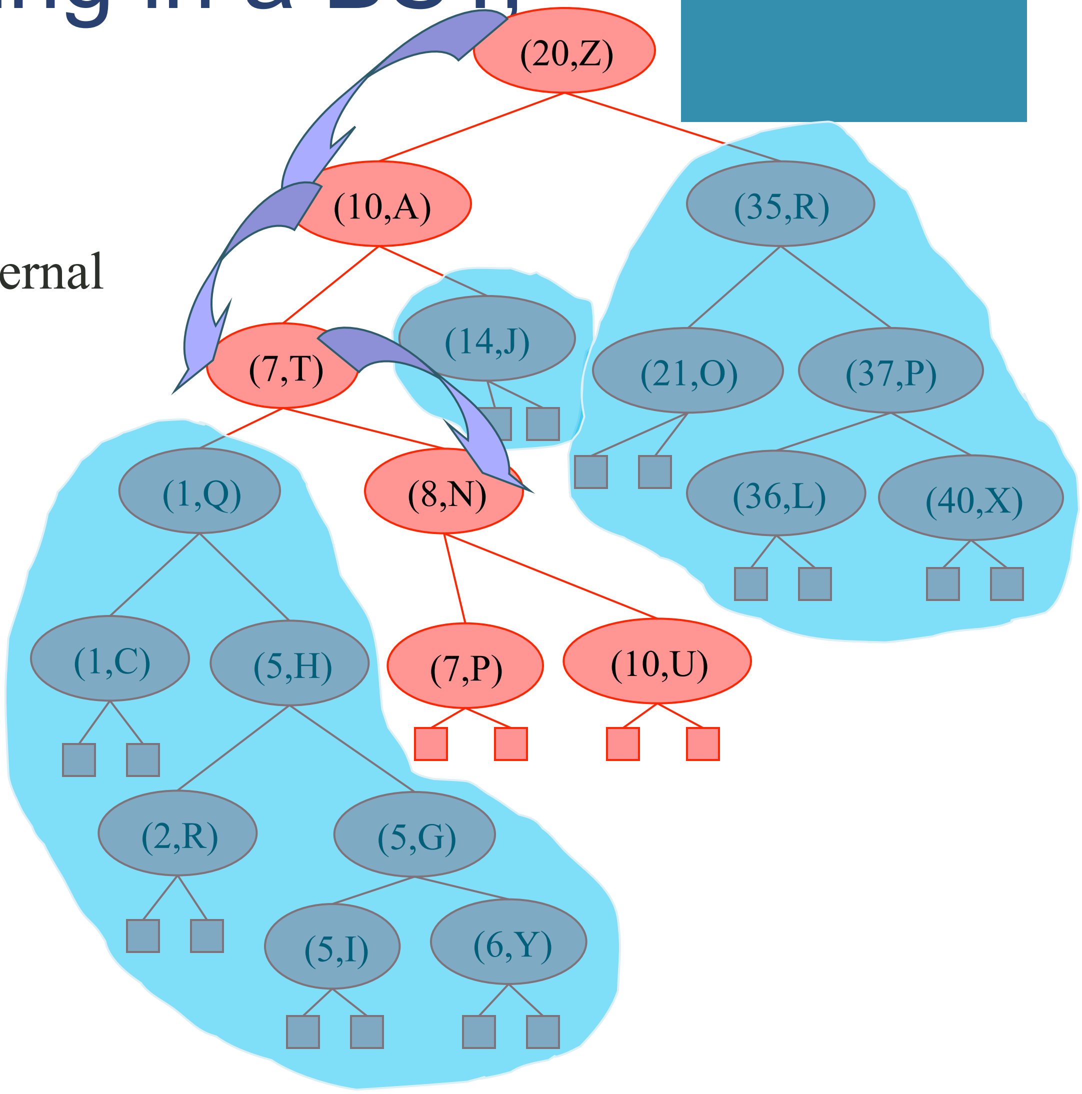
Searching in a Splay Tree: Starts the Same as in a BST

- Search proceeds down the tree to found item or an external node.
- Example: Search for the item with key 11.



Example Searching in a BST, continued

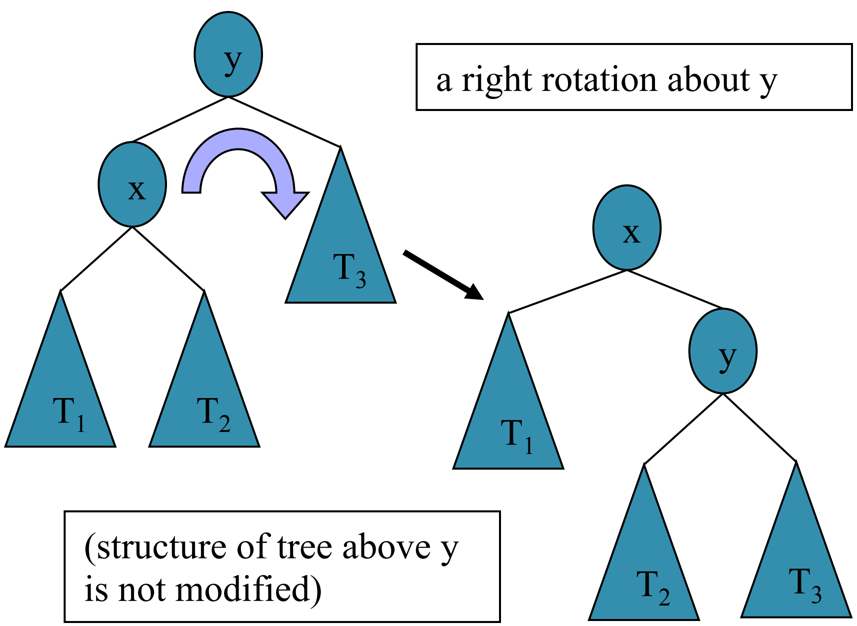
- search for key 8, ends at an internal node.



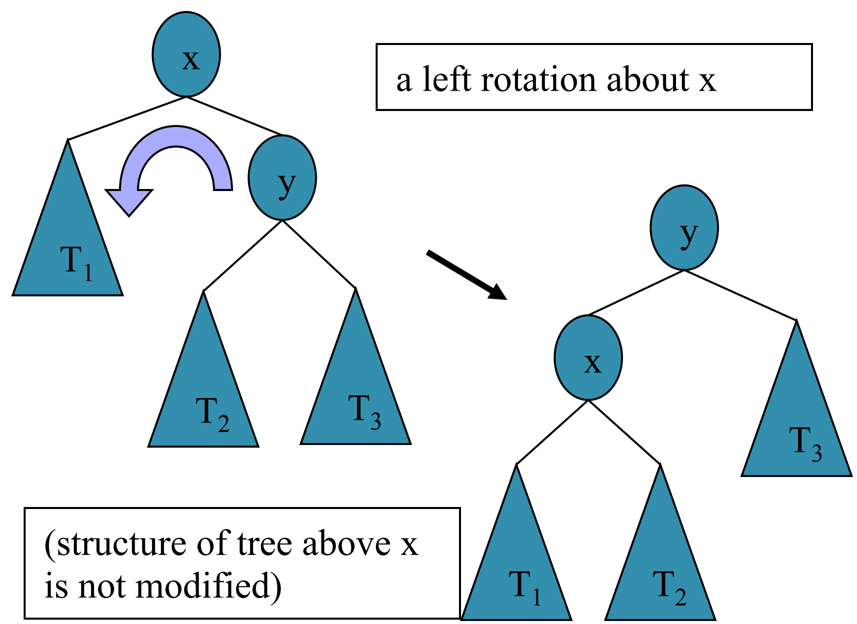
Splay Trees do Rotations after Every Operation (Even Search)

- new operation: ***splay***
  - splaying moves a node to the root using rotations

- right rotation
  - makes the left child *x* of a node *y* into *y*'s parent; *y* becomes the right child of *x*

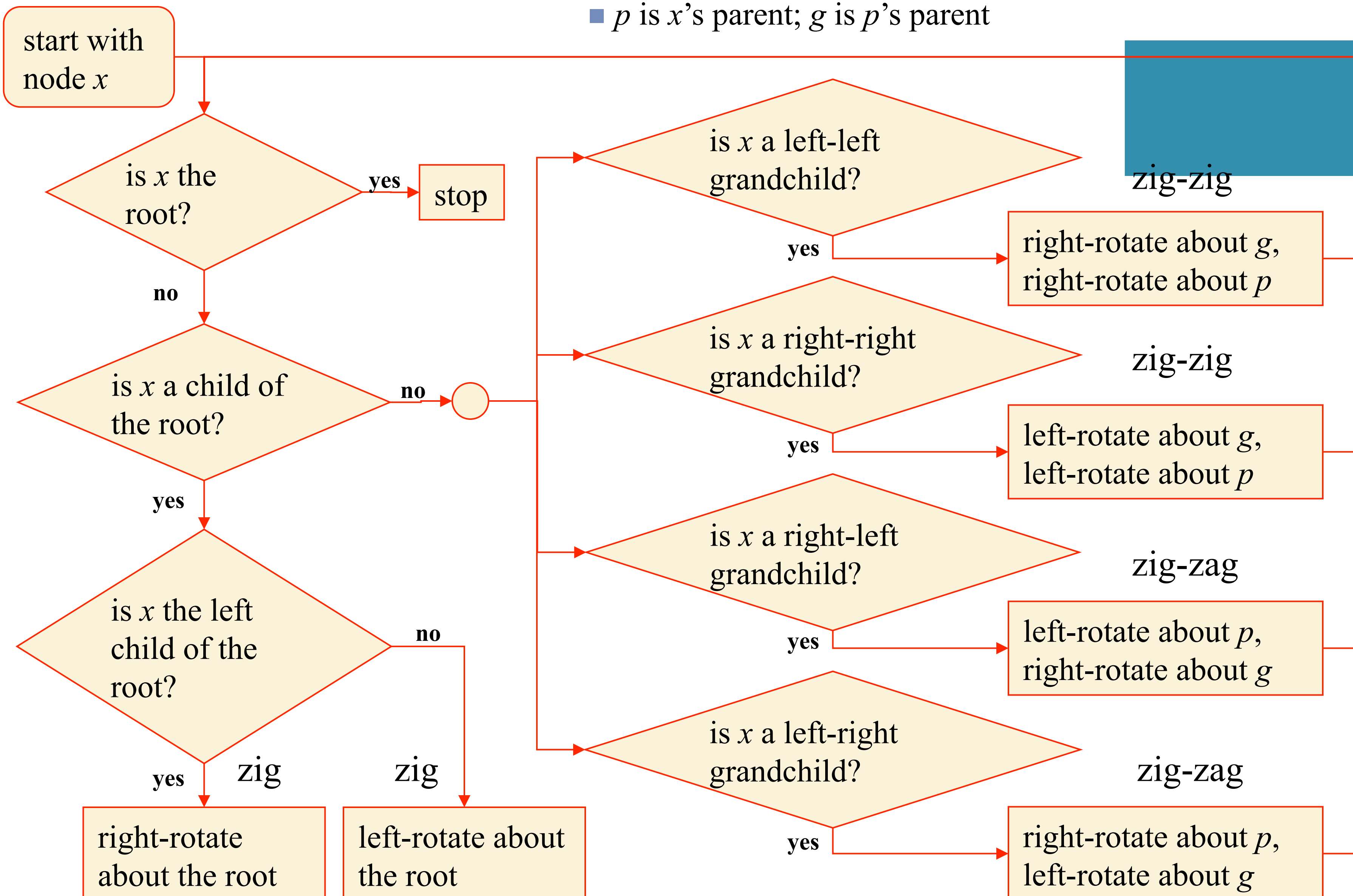


- left rotation
  - makes the right child *y* of a node *x* into *x*'s parent; *x* becomes the left child of *y*

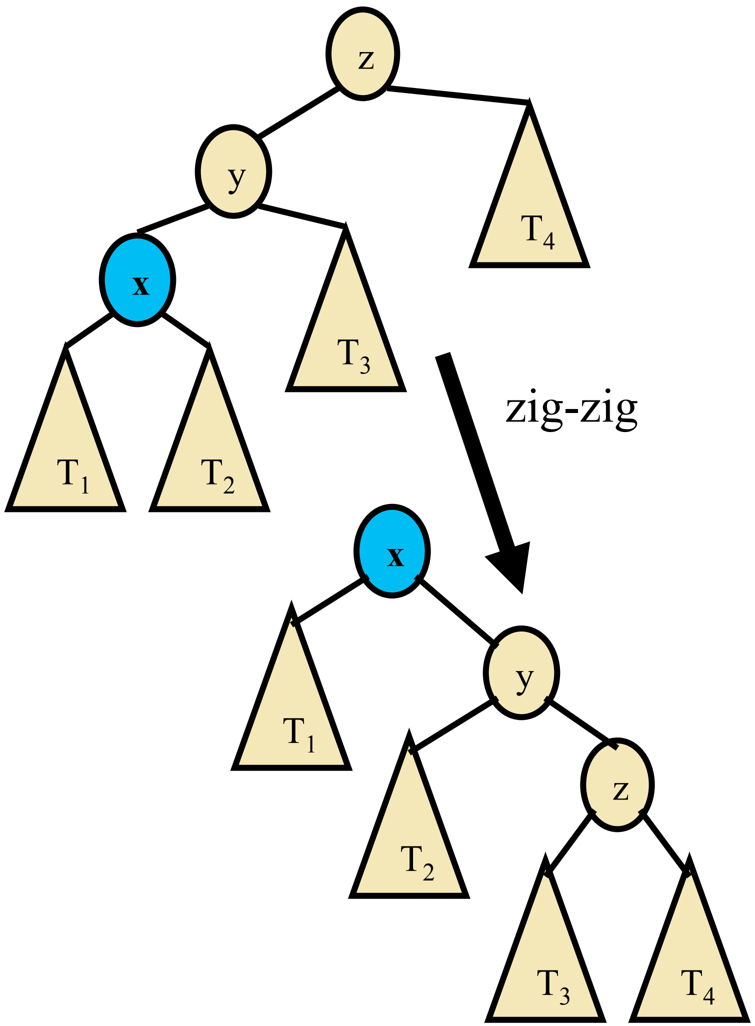


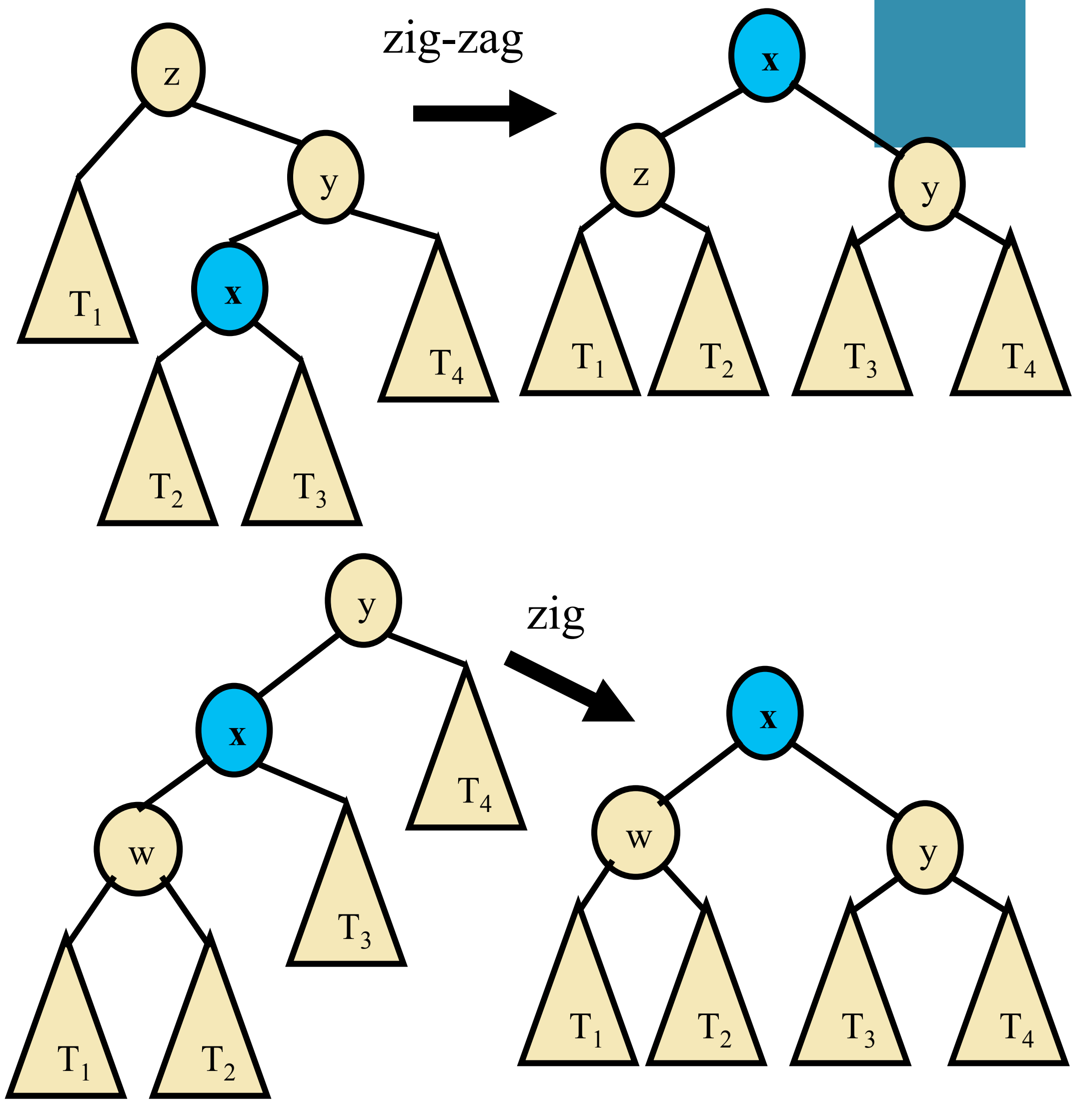
Splaying:

- "x is a left-left grandchild" means x is a left child of its parent, which is itself a left child of its parent
- p is x's parent; g is p's parent



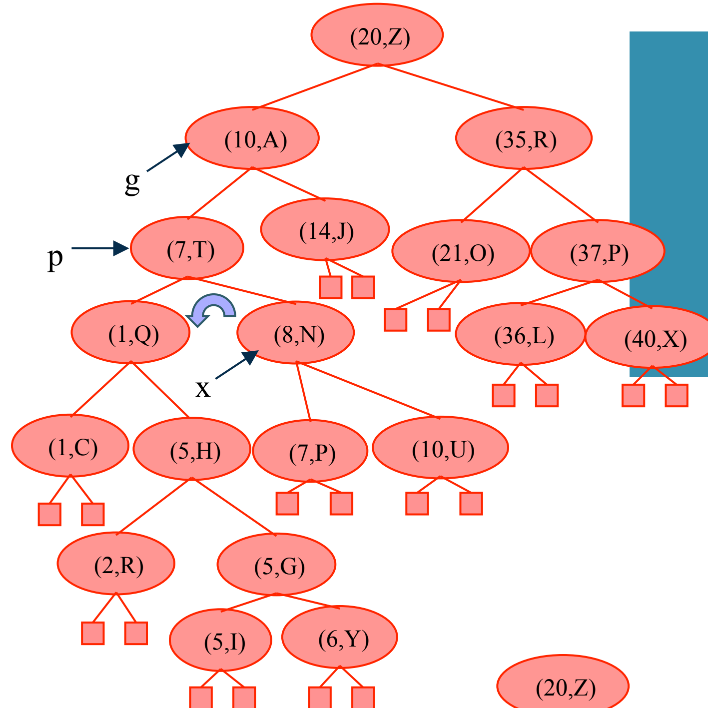
Visualizing the Splaying Cases



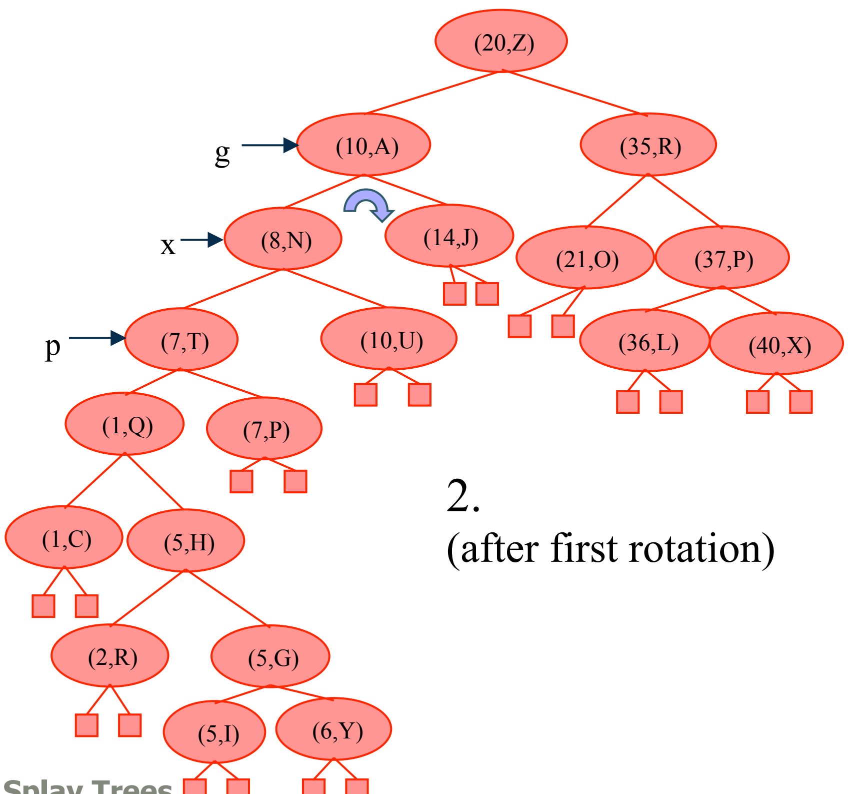


Splaying Example

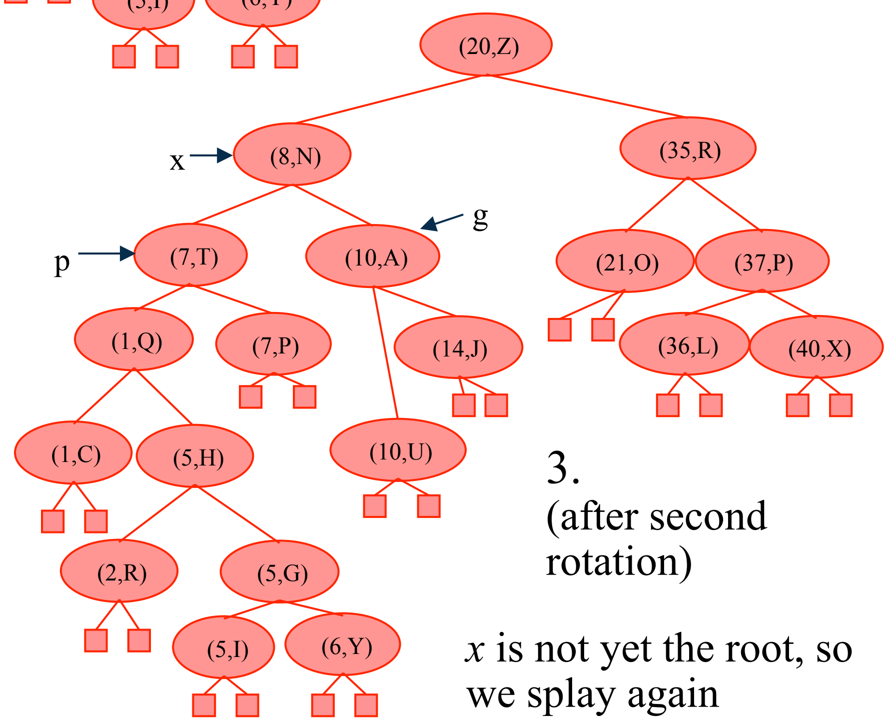
- let x = (8,N)
  - x is the right child of its parent, which is the left child of the grandparent
  - left-rotate around p, then right-rotate around g



1.
(before rotating)



2.
(after first rotation)

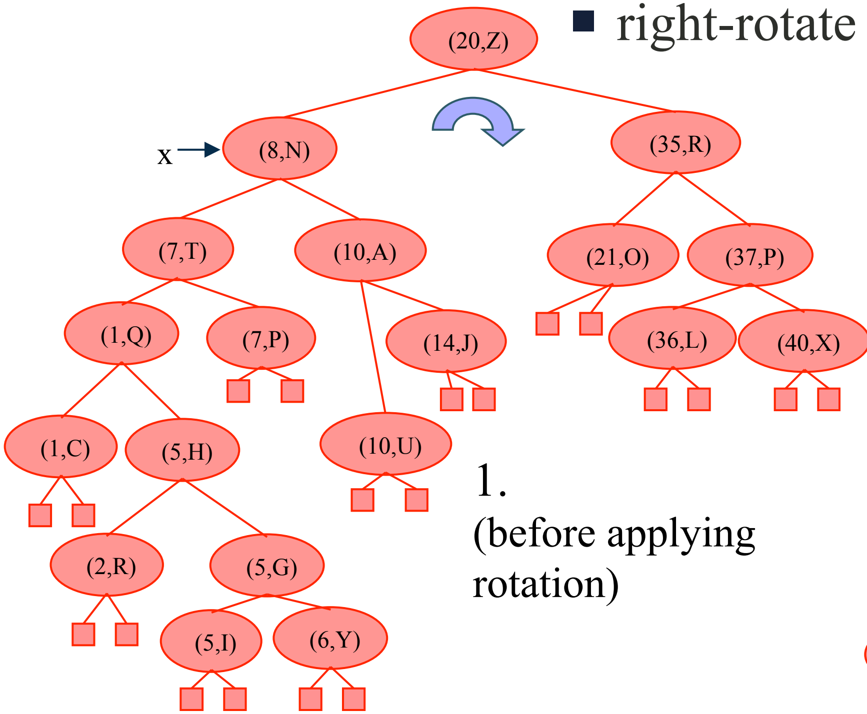


3.
(after second rotation)

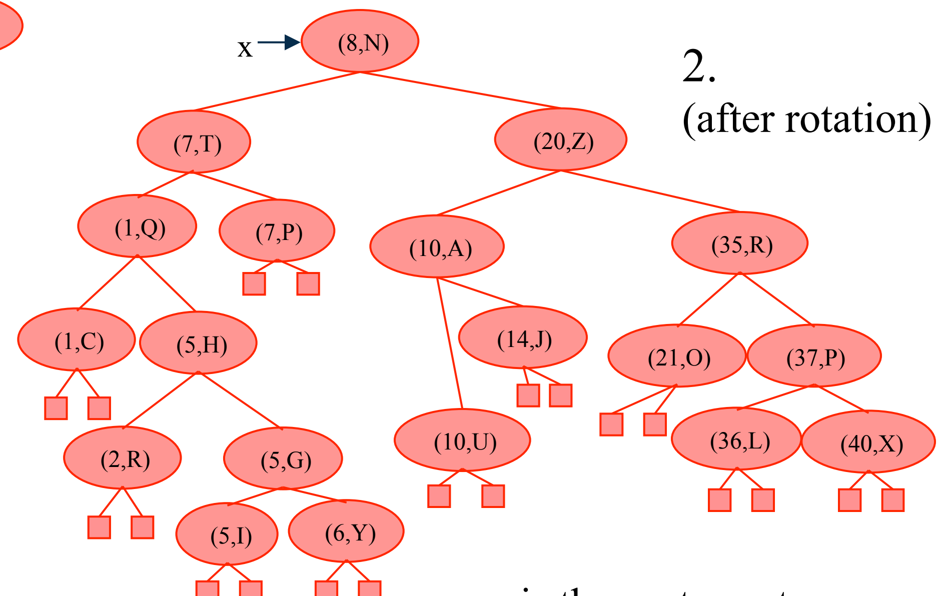
x is not yet the root, so we splay again

Splaying Example, Continued

- now x is the left child of the root
  - right-rotate around root



1.
(before applying rotation)



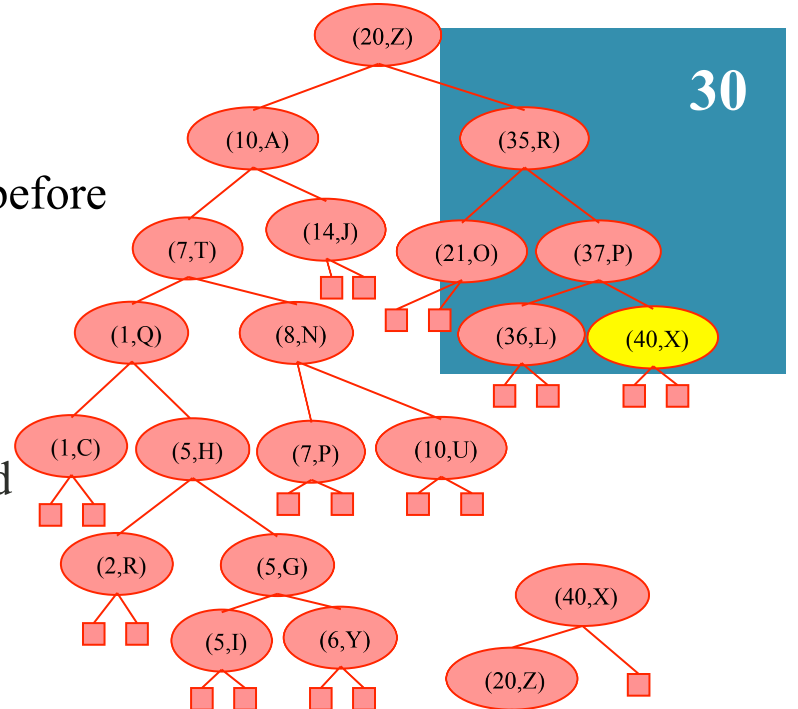
2.
(after rotation)

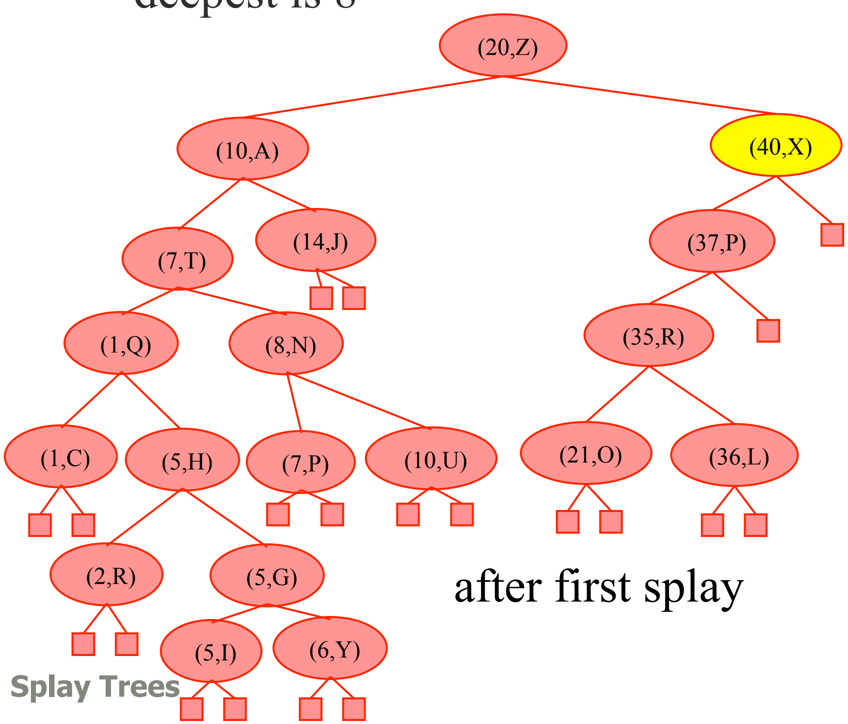
x is the root, so stop

Example Result of Splaying

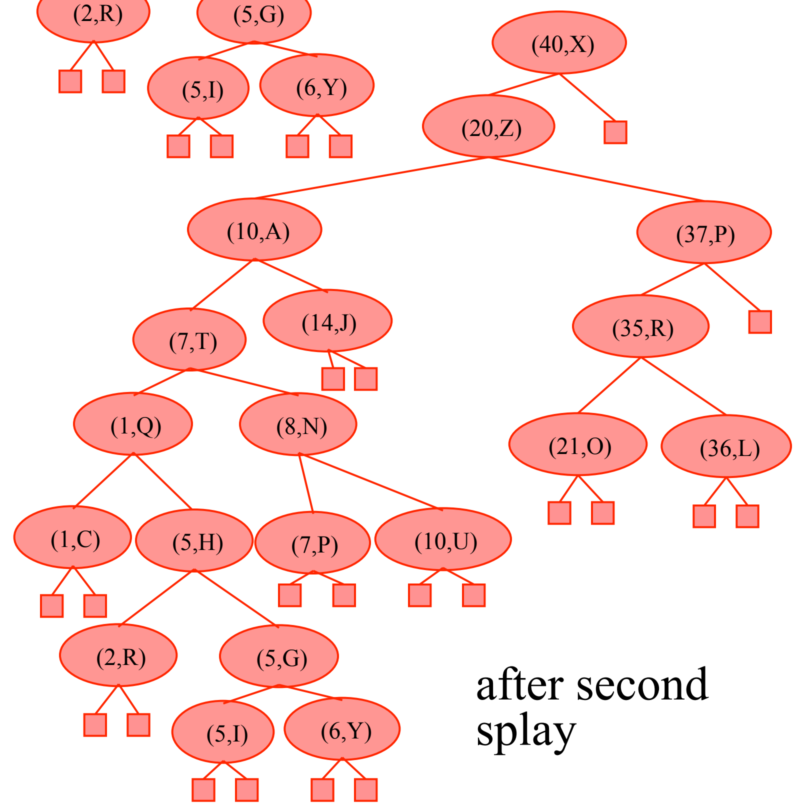
- tree might not be more balanced
- e.g. splay (40,X)
  - before, the depth of the shallowest leaf is 3 and the deepest is 7
  - after, the depth of shallowest leaf is 1 and deepest is 8

before





after first splay



after second splay

Performance of Splay Trees

- Amortized cost of any splay operation is $O(\log n)$
- This implies that splay trees can actually adapt to perform searches on frequently-requested items much faster than $O(\log n)$ in some cases

Project Demo on Jan. 9

Beat Google:

- Stage 1 : Rank web pages by keywords
- Stage 2 : Rank web sites by keywords
- Stage 3 : Re-rank google web sites by keywords
- Stage 4 : Derive relative keywords by top-ranked web sites
- Stage 5: *Webrize* your search engine
- Stage 6: Mobilize your search engine

Project Demo

- Location: The MIS 5F PC classroom
- Each team gives 8 minutes PPT presentation focusing on the project interests, key ideas, and achievements + 7 minutes system demo
- In the demo, each team needs to **run your system** to show how it works and how it achieves the requirement for each stage. I will also check your source code.
- BONUS: Students who successfully challenge other team's system may get extra points.

Project Hints

- How to call google?
- How to find the reference links?
- How to encode Chinese?

HW10 (Due on Dec. 26)

Use Google and get the links!

- Get a keyword from user
- Return the urls listed in the search result
- Save the results in a hash table (we will discuss it in the next lecture)

- After this HW, you can step to the forth stage of the project
- You can apply the same technique to other search engines

Coming Up

- Recap: Binary Search Trees
  - TB Chapter 10
- Maps and Hash tables
  - TB Chapter 9 and 10