CHAPTER 4
DISCOUNTED CASH FLOW VALUATION

Solutions to Questions and Problems

NOTE: All-end-of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The time line for the cash flows is:

   \[
   \begin{array}{c|c|c}
   \text{Year} & \text{Cash Flow} & \text{FV} \\
   \hline
   0 & $5,000 & \\
   10 & & \\
   \end{array}
   \]

   The simple interest per year is:

   \[
   $5,000 \times .08 = $400
   \]

   So, after 10 years, you will have:

   \[
   $400 \times 10 = $4,000 \text{ in interest.}
   \]

   The total balance will be $5,000 + 4,000 = $9,000

   With compound interest, we use the future value formula:

   \[
   FV = PV(1 + r)^t
   \]

   \[
   FV = $5,000(1.08)^{10} = $10,794.62
   \]

   The difference is:

   \[
   $10,794.62 - 9,000 = $1,794.62
   \]

2. To find the FV of a lump sum, we use:

   \[
   FV = PV(1 + r)^t
   \]

   \[
   a. \quad FV = $1,000(1.05)^{10} = $1,628.89
   \]
b.  

\[
\begin{array}{c}
0 \quad 10 \\
FV \quad 1,000
\end{array}
\]

\[FV = 1,000(1.10)^{10} = 2,593.74\]

c.  

\[
\begin{array}{c}
0 \quad 20 \\
FV \quad 1,000
\end{array}
\]

\[FV = 1,000(1.05)^{20} = 2,653.30\]

d. Because interest compounds on the interest already earned, the interest earned in part c is more than twice the interest earned in part a. With compound interest, future values grow exponentially.

3. To find the PV of a lump sum, we use:

\[PV = \frac{FV}{(1 + r)^t}\]

\[
\begin{array}{c}
0 \quad 6 \\
PV \quad 13,827
\end{array}
\]

\[PV = \frac{13,827}{(1.07)^6} = 9,213.51\]

\[
\begin{array}{c}
0 \quad 9 \\
PV \quad 43,852
\end{array}
\]

\[PV = \frac{43,852}{(1.15)^9} = 12,465.48\]

\[
\begin{array}{c}
0 \quad 18 \\
PV \quad 725,380
\end{array}
\]

\[PV = \frac{725,380}{(1.11)^{18}} = 110,854.15\]

\[
\begin{array}{c}
0 \quad 23 \\
PV \quad 590,710
\end{array}
\]

\[PV = \frac{590,710}{(1.18)^{23}} = 13,124.66\]

4. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[FV = PV(1 + r)^t\]

Solving for \(r\), we get:
\[ r = \left( \frac{FV}{PV} \right)^{1/t} - 1 \]

1. \[ FV = $307 = $242(1 + r)^4; \quad r = \left( \frac{307}{242} \right)^{1/4} - 1 = 6.13\% \]

2. \[ FV = $896 = $410(1 + r)^8; \quad r = \left( \frac{896}{410} \right)^{1/8} - 1 = 10.27\% \]

3. \[ FV = $162,181 = $51,700(1 + r)^{16}; \quad r = \left( \frac{162,181}{51,700} \right)^{1/16} - 1 = 7.41\% \]

4. \[ FV = $483,500 = $18,750(1 + r)^{27}; \quad r = \left( \frac{483,500}{18,750} \right)^{1/27} - 1 = 12.79\% \]

5. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( t \), we get:

\[ t = \frac{\ln(\text{FV} / \text{PV})}{\ln(1 + r)} \]

- \( t = \ln(1,284 / 625) / \ln 1.09 = 8.35 \) years

- \( t = \ln(4,341 / 810) / \ln 1.1 = 16.09 \) years

- \( t = \ln(402,662 / 18,400) / \ln 1.17 = 19.65 \) years
FV = $173,439 = $21,500(1.08)^t; \quad t = \ln($173,439 / $21,500) / \ln 1.08 = 27.13 \text{ years}

6. To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( t \), we get:

\[ t = \ln(FV / PV) / \ln(1 + r) \]

The length of time to double your money is:

\[ FV = $2 = $1(1.08)^t; \quad t = \ln 2 / \ln 1.08 = 9.01 \text{ years} \]

The length of time to quadruple your money is:

\[ FV = $4 = $1(1.08)^t; \quad t = \ln 4 / \ln 1.08 = 18.01 \text{ years} \]

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the difference in these answers is due to rounding). This is an important concept of time value of money.

7. The time line is:

To find the PV of a lump sum, we use:

\[ PV = FV / (1 + r)^t \]

\[ PV = \$630,000,000 / (1.071)^{20} = \$159,790,565.17 \]
8. The time line is:

![Time line diagram]

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV(1 + r)^t \]

Solving for \( r \), we get:

\[
\frac{FV}{PV} = (1 + r)^{\frac{1}{t}} - 1
\]

\[
r = \left( \frac{1,100,000}{1,680,000} \right)^{\frac{1}{3}} - 1 = -0.1317 \text{ or } -13.17\%
\]

Notice that the interest rate is negative. This occurs when the FV is less than the PV.

9. The time line is:

![Time line diagram]

A consol is a perpetuity. To find the PV of a perpetuity, we use the equation:

\[ PV = \frac{C}{r} \]

\[
PV = \frac{150}{0.046} = 3,260.87
\]

10. To find the future value with continuous compounding, we use the equation:

\[ FV = PVe^{rt} \]

\( a. \)

0 7

\[ S1,900 \quad FV \]

\[ FV = 1,900e^{0.12(7)} = 4,401.10 \]

\( b. \)

0 5

\[ S1,900 \quad FV \]

\[ FV = 1,900e^{0.10(5)} = 3,132.57 \]
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c. $1,900\times e^{0.05(12)} = $3,462.03

d. $1,900\times e^{0.07(10)} = $3,826.13

11. The time line is:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>$960</td>
<td>$840</td>
<td>$935</td>
<td>$1,350</td>
</tr>
</tbody>
</table>

To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

\[ PV = \frac{FV}{(1 + r)^t} \]

PV@10% = $960 / 1.10 + $840 / 1.10^2 + $935 / 1.10^3 + $1,350 / 1.10^4 = $3,191.49

PV@18% = $960 / 1.18 + $840 / 1.18^2 + $935 / 1.18^3 + $1,350 / 1.18^4 = $2,682.22

PV@24% = $960 / 1.24 + $840 / 1.24^2 + $935 / 1.24^3 + $1,350 / 1.24^4 = $2,381.91

12. The times lines are:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>$4,500</td>
<td>$4,500</td>
<td>$4,500</td>
<td>$4,500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0</th>
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<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>$7,000</td>
<td>$7,000</td>
<td>$7,000</td>
<td>$7,000</td>
</tr>
</tbody>
</table>

To find the PVA, we use the equation:

\[ PVA = C\left\{1 - \frac{1}{(1 + r)^t}\right\} / r \]

At a 5 percent interest rate:

X@5%: PVA = $4,500\left\{1 - \left(\frac{1}{1.05}\right)^9\right\} / .05 = $31,985.20

Y@5%: PVA = $7,000\left\{1 - \left(\frac{1}{1.05}\right)^7\right\} / .05 = $30,306.34
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And at a 22 percent interest rate:

X@22%: \[ PVA = 4,500 \left[ \frac{1 – (1/1.22)^9}{.22} \right] = 17,038.28 \]

Y@22%: \[ PVA = 7,000 \left[ \frac{1 – (1/1.22)^5}{.22} \right] = 20,045.48 \]

Notice that the PV of Cash flow X has a greater PV at a 5 percent interest rate, but a lower PV at a 22 percent interest rate. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger cash flows. At a higher interest rate, these bigger cash flows early are more important since the cost of waiting (the interest rate) is so much greater.

13. To find the PVA, we use the equation:

\[
PVA = C \left( \frac{1 – \left[ 1/(1 + r) \right]^t}{r} \right)
\]

\[\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
PV & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 \\
\end{array}\]

PVA@15 yrs: \[ PVA = 4,900 \left[ \frac{1 – (1/1.08)^{15}}{.08} \right] = 41,941.45 \]

\[\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
PV & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 \\
\end{array}\]

PVA@40 yrs: \[ PVA = 4,900 \left[ \frac{1 – (1/1.08)^{40}}{.08} \right] = 58,430.61 \]

\[\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
PV & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 \\
\end{array}\]

PVA@75 yrs: \[ PVA = 4,900 \left[ \frac{1 – (1/1.08)^{75}}{.08} \right] = 61,059.31 \]

To find the PV of a perpetuity, we use the equation:

\[
PV = \frac{C}{r}
\]

\[\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
PV & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 & 4,900 \\
\end{array}\]

PV = $4,900 / .08
PV = $61,250

Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75-year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only $190.69.

14. The time line is:
This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

\[ PV = \frac{C}{r} \]

\[ PV = \frac{15,000}{0.052} = 288,461.54 \]

To find the interest rate that equates the perpetuity cash flows with the PV of the cash flows, we can use the PV of a perpetuity equation:

\[ PV = \frac{C}{r} \]

\[ -320,000 = \frac{15,000}{r} \]

We can now solve for the interest rate as follows:

\[ r = \frac{15,000}{320,000} = 0.0469, \text{ or } 4.69\% \]

15. For discrete compounding, to find the EAR, we use the equation:

\[ EAR = \left[1 + \left(\frac{APR}{m}\right)\right]^m - 1 \]

\[ EAR = \left[1 + \left(\frac{0.07}{4}\right)\right]^4 - 1 = 0.0719, \text{ or } 7.19\% \]

\[ EAR = \left[1 + \left(\frac{0.16}{12}\right)\right]^{12} - 1 = 0.1723, \text{ or } 17.23\% \]

\[ EAR = \left[1 + \left(\frac{0.11}{365}\right)\right]^{365} - 1 = 0.1163, \text{ or } 11.63\% \]

To find the EAR with continuous compounding, we use the equation:

\[ EAR = e^r - 1 \]

\[ EAR = e^{0.12} - 1 = 0.1275, \text{ or } 12.75\% \]

16. Here, we are given the EAR and need to find the APR. Using the equation for discrete compounding:

\[ EAR = \left[1 + \left(\frac{APR}{m}\right)\right]^m - 1 \]

We can now solve for the APR. Doing so, we get:

\[ APR = \frac{m}{1 + \text{EAR}} - 1 \]

\[ APR = \frac{2}{1 + 0.0980} - 1 = 0.0957, \text{ or } 9.57\% \]

\[ APR = \frac{12}{1 + 0.1960} - 1 = 0.1803, \text{ or } 18.03\% \]
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\[ \text{EAR} = .0830 = [1 + \left( \frac{\text{APR}}{52} \right)]^{52} - 1 \quad \text{APR} = 52\left[ (1.0830)^{1/52} - 1 \right] = .0798, \text{ or } 7.98\% \]

Solving the continuous compounding EAR equation:

\[ \text{EAR} = e^{r} - 1 \]

We get:

\[ \text{APR} = \ln(1 + \text{EAR}) \]
\[ \text{APR} = \ln(1 + .1420) \]
\[ \text{APR} = .1328, \text{ or } 13.28\% \]

17. For discrete compounding, to find the EAR, we use the equation:

\[ \text{EAR} = \left[ 1 + \left( \frac{\text{APR}}{m} \right) \right]^m - 1 \]

So, for each bank, the EAR is:

First National: \[ \text{EAR} = \left[ 1 + \left( \frac{.1120}{12} \right) \right]^{12} - 1 = .1179, \text{ or } 11.79\% \]

First United: \[ \text{EAR} = \left[ 1 + \left( \frac{.1140}{2} \right) \right]^2 - 1 = .1172, \text{ or } 11.72\% \]

A higher APR does not necessarily mean the higher EAR. The number of compounding periods within a year will also affect the EAR.

18. The cost of a case of wine is 10 percent less than the cost of 12 individual bottles, so the cost of a case will be:

Cost of case = (12)(\$10)(1 - .10)
Cost of case = $108

Now, we need to find the interest rate. The cash flows are an annuity due, so:

\[ \text{PVA} = (1 + r) \left(C\left\{\frac{1 - \left[1/(1+r)^t\right]}{r}\right\} \right) / r \]
\[ $108 = (1 + r) \times $10\left(\frac{1 - \left[1/(1+r)^{12}\right]}{r}\right) \]

Solving for the interest rate, we get:

\[ r = .0198 \text{ or } 1.98\% \text{ per week} \]
So, the APR of this investment is:

\[
\text{APR} = .0198(52) \\
\text{APR} = 1.0277, \text{ or } 102.77\%
\]

And the EAR is:

\[
\text{EAR} = \left(1 + .0198\right)^{52} - 1 \\
\text{EAR} = 1.7668, \text{ or } 176.68\%
\]

The analysis appears to be correct. He really can earn about 177 percent buying wine by the case. The only question left is this: Can you really find a fine bottle of Bordeaux for $10?

19. The time line is:

\begin{center}
\begin{tabular}{cccccccc}
0 & 1 & \ldots & & & & \\
\$21,500 & $700 & $700 & $700 & $700 & $700 & $700 & $700 & $700
\end{tabular}
\end{center}

Here, we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

\[
\text{PVA} = C\left(1 - \frac{1}{(1 + r)^t}\right) / r \\
\$21,500 = \$700\left[1 - \frac{1}{(1/1.013)^t}\right] / .013
\]

Now, we solve for \( t \):

\[
1/1.013^t = 1 - \left[\frac{\$21,500(.013)}{\$700}\right] \\
1.013^t = 1/0.601 = 1.665 \\
t = \ln 1.665 / \ln 1.013 = 39.46 \text{ months}
\]

20. The time line is:

\begin{center}
\begin{tabular}{ccc}
0 & 1 & \\
\$3 & \$4
\end{tabular}
\end{center}

Here, we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

\[
\text{FV} = \text{PV}(1 + r) \\
\$4 = \$3(1 + r) \\
r = 4/3 - 1 = 33.33\% \text{ per week}
\]

The interest rate is 33.33\% per week. To find the APR, we multiply this rate by the number of weeks in a year, so:

\[
\text{APR} = (52)33.33\% = 1,733.33\%
\]
And using the equation to find the EAR:

\[
\text{EAR} = \left[1 + \left(\frac{\text{APR}}{m}\right)\right]^m - 1
\]

\[
\text{EAR} = \left[1 + .3333\right]^{52} - 1 = 313,916,515.69\%
\]

**Intermediate**

21. To find the FV of a lump sum with discrete compounding, we use:

\[
\text{FV} = \text{PV}(1 + r)^t
\]

\[\text{a.}\]

\[
\begin{array}{c|c|c}
0 & 6 & \text{FV} \\
$1,000 & & \\
\end{array}
\]

\[
\text{FV} = \$1,000(1.09)^6 = \$1,677.10
\]

\[\text{b.}\]

\[
\begin{array}{c|c|c}
0 & 12 & \text{FV} \\
$1,000 & & \\
\end{array}
\]

\[
\text{FV} = \$1,000(1 + .09/2)^{12} = \$1,695.88
\]

\[\text{c.}\]

\[
\begin{array}{c|c|c}
0 & 72 & \text{FV} \\
$1,000 & & \\
\end{array}
\]

\[
\text{FV} = \$1,000(1 + .09/12)^{72} = \$1,712.55
\]

\[\text{d.}\]

\[
\begin{array}{c|c|c}
0 & 6 & \text{FV} \\
$1,000 & & \\
\end{array}
\]

To find the future value with continuous compounding, we use the equation:

\[
\text{FV} = \text{PV}e^{rt}
\]

\[
\text{FV} = \$1,000e^{.09(6)} = \$1,716.01
\]

\[\text{e.}\]

The future value increases when the compounding period is shorter because interest is earned on previously accrued interest. The shorter the compounding period, the more frequently interest is earned, and the greater the future value, assuming the same stated interest rate.

22. The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest by First Simple Bank paid over 10 years will be:

\[.05(10) = .5\]
First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of $1, or:

\[(1 + r)^{10}\]

Setting the two equal, we get:

\[.05(10) = (1 + r)^{10} - 1\]

\[r = 1.5^{10} - 1 = .0414, \text{ or } 4.14\%\]

23. Although the stock and bond accounts have different interest rates, we can draw one time line, but we need to remember to apply different interest rates. The time line is:

```
0 1 ... 360 361 ... 660
Stock $800 $800 $800 $800 $800 ...
Bond $350 $350 $350 $350 $350 ...
```

We need to find the annuity payment in retirement. Our retirement savings ends at the same time the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

Stock account: \(FVA = $800 \left[\frac{1 - \left(1 + \frac{.11}{12}\right)^{360}}{\frac{.11}{12}}\right] = $2,243,615.79\)

Bond account: \(FVA = $350 \left[\frac{1 - \left(1 + \frac{.06}{12}\right)^{360}}{\frac{.06}{12}}\right] = $351,580.26\)

So, the total amount saved at retirement is:

\($2,243,615.79 + 351,580.26 = $2,595,196.05\)

Solving for the withdrawal amount in retirement using the PVA equation gives us:

\[PVA = $2,595,196.05 = C \left[1 - \frac{1}{\left(1 + \frac{.08}{12}\right)^{300}}\right] / \frac{.08}{12}\]

\[C = $2,595,196.06 / 129.5645 = $20,030.14 \text{ withdrawal per month}\]

24. The time line is:

```
0 4
$-1 $4
```

Since we are looking to quadruple our money, the PV and FV are irrelevant as long as the FV is four times as large as the PV. The number of periods is four, the number of quarters per year. So:

\[FV = $4 = $1 \left(1 + r\right)^{12(3)}\]

\[r = .4142, \text{ or } 41.42\%\]
25. Here, we need to find the interest rate for two possible investments. Each investment is a lump sum, so:

**G:**

\[
\begin{array}{c|c|c}
0 & -$65,000 & 6 \\
 & $125,000 & \\
\end{array}
\]

\[
PV = $65,000 = \frac{$125,000}{(1 + r)^6}
\]

\[
(1 + r)^6 = \frac{$125,000}{$65,000}
\]

\[
r = (1.92)^{\frac{1}{6}} - 1 = 0.1151, \text{ or } 11.51\%
\]

**H:**

\[
\begin{array}{c|c|c}
0 & -$65,000 & 10 \\
 & $185,000 & \\
\end{array}
\]

\[
PV = $65,000 = \frac{$185,000}{(1 + r)^10}
\]

\[
(1 + r)^{10} = \frac{$185,000}{$65,000}
\]

\[
r = (2.85)^{\frac{1}{10}} - 1 = 0.1103, \text{ or } 11.03\%
\]

26. This is a growing perpetuity. The present value of a growing perpetuity is:

\[
PV = \frac{C}{(r - g)}
\]

\[
PV = \frac{$175,000}{(0.10 - 0.035)}
\]

\[
PV = $2,692,307.69
\]

It is important to recognize that when dealing with annuities or perpetuities, the present value equation calculates the present value one period before the first payment. In this case, since the first payment is in two years, we have calculated the present value one year from now. To find the value today, we simply discount this value as a lump sum. Doing so, we find the value of the cash flow stream today is:

\[
PV = \frac{FV}{(1 + r)^t}
\]

\[
PV = \frac{$2,692,307.69}{(1 + 0.10)^1}
\]

\[
PV = $2,447,552.45
\]

27. The dividend payments are made quarterly, so we must use the quarterly interest rate. The quarterly interest rate is:

\[
\text{Quarterly rate} = \frac{\text{Stated rate}}{4}
\]

\[
\text{Quarterly rate} = 0.065 / 4
\]

\[
\text{Quarterly rate} = 0.01625
\]

The time line is:

\[
0 \quad 1 \quad \ldots \quad \infty
\]

\[
\begin{array}{cccccccccc}
\text{PV} & \$4.50 & \$4.50 & \$4.50 & \$4.50 & \$4.50 & \$4.50 & \$4.50 & \$4.50 & \$4.50
\end{array}
\]
Using the present value equation for a perpetuity, we find the value today of the dividends paid must be:

\[ PV = \frac{C}{r} \]

\[ PV = \frac{4.50}{.01625} \]

\[ PV = \$276.92 \]

28. The time line is:

\[ \begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \ldots & 25 \\
PV & \$6,500 & \$6,500 & \$6,500 & \$6,500 & \$6,500 & \$6,500 & \end{array} \]

We can use the PVA annuity equation to answer this question. The annuity has 23 payments, not 22 payments. Since there is a payment made in Year 3, the annuity actually begins in Year 2. So, the value of the annuity in Year 2 is:

\[ PVA = \frac{C\left(1 - \left[\frac{1}{1 + r}\right]^t\right)}{r} \]

\[ PVA = \frac{6,500\left(1 - \left[\frac{1}{1 + .07}\right]^{23}\right)}{.07} \]

\[ PVA = \$73,269.22 \]

This is the value of the annuity one period before the first payment, or Year 2. So, the value of the cash flows today is:

\[ PV = \frac{FV}{1 + r}^t \]

\[ PV = \frac{73,269.22}{(1 + .07)^2} \]

\[ PV = \$63,996.17 \]

29. The time line is:

\[ \begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \ldots & 20 \\
PV & \$650 & \$650 & \$650 & \$650 & \$650 & \$650 & \end{array} \]

We need to find the present value of an annuity. Using the PVA equation, and the 13 percent interest rate, we get:

\[ PVA = \frac{C\left(1 - \left[\frac{1}{1 + r}\right]^t\right)}{r} \]

\[ PVA = \frac{650\left(1 - \left[\frac{1}{1 + .13}\right]^{15}\right)}{.13} \]

\[ PVA = \$4,200.55 \]

This is the value of the annuity in Year 5, one period before the first payment. Finding the value of this amount today, we find:

\[ PV = \frac{FV}{1 + r}^t \]

\[ PV = \frac{4,200.55}{(1 + .11)^5} \]

\[ PV = \$2,492.82 \]
30. The amount borrowed is the value of the home times one minus the down payment, or:

\[ \text{Amount borrowed} = 550,000(1 - 0.20) \]
\[ \text{Amount borrowed} = 440,000 \]

The time line is:

\[ \begin{array}{ccccccccccc}
0 & 1 & \ldots & 360 \\
$440,000 & C & C & C & C & \ldots & C & C & C & C & C \\
\end{array} \]

The monthly payments with a balloon payment loan are calculated assuming a longer amortization schedule, in this case, 30 years. The payments based on a 30-year repayment schedule would be:

\[ PVA = \frac{440,000}{C} \left( 1 - \frac{1}{1 + (0.061/12)^{360}} \right) / (0.061/12) \]
\[ C = 2,666.38 \]

Now, at Year 8 (Month 96), we need to find the PV of the payments which have not been made. The time line is:

\[ \begin{array}{ccccccccccc}
96 & 97 & \ldots & 360 \\
PV & 2,666.38 & 2,666.38 & 2,666.38 & 2,666.38 & \ldots & 2,666.38 & 2,666.38 & 2,666.38 & 2,666.38 \\
\end{array} \]

The balloon payment will be:

\[ PVA = 2,666.38 \left( 1 - \frac{1}{1 + (0.061/12)^{22(12)}} \right) / (0.061/12) \]
\[ PVA = 386,994.11 \]

31. The time line is:

\[ \begin{array}{ccccccc}
0 & \ldots & 12 \\
$7,500 & \ldots & FV \\
\end{array} \]

Here, we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts. After the first six months, the balance will be:

\[ FV = 7,500 [1 + (0.024/12)]^6 = 7,590.45 \]

This is the balance in six months. The FV in another six months will be:

\[ FV = 7,590.45 [1 + (0.18/12)]^6 = 8,299.73 \]

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the FV. The interest accrued is:

\[ \text{Interest} = 8,299.73 - 7,500 = 799.73 \]
32. The time line is:

\[
\begin{array}{cccccccccc}
0 & 1 & \ldots & \infty \\
-2,500,000 & 227,000 & 227,000 & 227,000 & 227,000 & 227,000 & 227,000 & 227,000 & 227,000
\end{array}
\]

The company would be indifferent at the interest rate that makes the present value of the cash flows equal to the cost today. Since the cash flows are a perpetuity, we can use the PV of a perpetuity equation. Doing so, we find:

\[
PV = \frac{C}{r}
\]

\[
2,500,000 = \frac{227,000}{r}
\]

\[
r = \frac{227,000}{2,500,000} = .0908, \text{ or } 9.08\%
\]

33. The company will accept the project if the present value of the increased cash flows is greater than the cost. The cash flows are a growing perpetuity, so the present value is:

\[
PV = \frac{C}{(r-g)} - \frac{[1/(r-g)]}{[(1+g)/(1+r)]^t}
\]

\[
PV = 21,000 \left\{ \frac{1}{.10 - .04} - \frac{1}{.10 - .04} \times \frac{1+.04}{1+.10} \right\}^5
\]

\[
PV = 85,593.99
\]

The company should accept the project since the cost is less than the increased cash flows.

34. Since your salary grows at 4 percent per year, your salary next year will be:

Next year’s salary = $65,000 \times (1 + .04)

Next year’s salary = $67,600

This means your deposit next year will be:

Next year’s deposit = $67,600 \times (0.05)

Next year’s deposit = $3,380

Since your salary grows at 4 percent, you deposit will also grow at 4 percent. We can use the present value of a growing perpetuity equation to find the value of your deposits today. Doing so, we find:

\[
PV = \frac{C}{(r-g)} - \frac{[1/(r-g)]}{[(1+g)/(1+r)]^t}
\]

\[
PV = 3,380 \left\{ \frac{1}{.10 - .04} - \frac{1}{.10 - .04} \times \frac{1+.04}{1+.10} \right\}^{40}
\]

\[
PV = 50,357.59
\]

Now, we can find the future value of this lump sum in 40 years. We find:

\[
FV = PV(1+r)^t
\]

\[
FV = 50,357.59(1+.10)^{40}
\]

\[
FV = 2,279,147.23
\]

This is the value of your savings in 40 years.
35. The time line is:

```
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>$6,800</td>
<td>$6,800</td>
<td>$6,800</td>
<td>$6,800</td>
<td>$6,800</td>
<td>$6,800</td>
<td>$6,800</td>
<td>$6,800</td>
<td>$6,800</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

The relationship between the PVA and the interest rate is:

- PVA falls as $r$ increases, and PVA rises as $r$ decreases
- FVA rises as $r$ increases, and FVA falls as $r$ decreases

The present values of $6,800 per year for 15 years at the various interest rates given are:

- PVA@10% = $6,800 \left\{ \frac{1 - (1/1.10)^{15}}{.10} \right\} = $51,721.34
- PVA@5% = $6,800 \left\{ \frac{1 - (1/1.05)^{15}}{.05} \right\} = $70,581.67
- PVA@15% = $6,800 \left\{ \frac{1 - (1/1.15)^{15}}{.15} \right\} = $39,762.12

36. The time line is:

```
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$350,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Here, we are given the FVA, the interest rate, and the amount of the annuity. We need to solve for the number of payments. Using the FVA equation:

\[
FVA = $35,000 = $350 \left\{ \frac{[1 + (.10/12)]^t - 1}{.10/12} \right\}
\]

Solving for $t$, we get:

\[
1.00833^t = 1 + \left\{ \frac{($35,000) (.10/12)}{350} \right\}
\]

\[
t = \ln 1.83333 / \ln 1.00833 = 73.04 \text{ payments}
\]

37. The time line is:

```
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$65,000</td>
<td>$1,320</td>
<td>$1,320</td>
<td>$1,320</td>
<td>$1,320</td>
<td>$1,320</td>
<td>$1,320</td>
<td>$1,320</td>
<td>$1,320</td>
<td>$1,320</td>
<td></td>
</tr>
</tbody>
</table>
```

Here, we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

\[
PVA = $65,000 = $1,320 \left\{ \frac{[1 - (1/(1+r))]^{60}}{r} \right\}
\]
To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

\[ r = 0.672\% \]

The APR is the periodic interest rate times the number of periods in the year, so:

\[ \text{APR} = 12(0.672\%) = 8.07\% \]

38. The time line is:

\[
\begin{array}{cccccccccc}
0 & 1 & \ldots & 3 & 6 & 0 \\
\text{PV} & $950 & $950 & $950 & $950 & $950 & $950 & $950 & $950 & $950 \\
\end{array}
\]

The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the $950 monthly payments is:

\[
PVA = 950 \left[ 1 - \frac{1}{1 + (.053/12)} \right]^{360} / (.053/12) \approx 171,077.26
\]

The monthly payments of $950 will amount to a principal payment of $171,077.26. The amount of principal you will still owe is:

\[
$250,000 - 171,077.26 = $78,922.74
\]

This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal will be:

\[
\text{Balloon payment} = 78,922.74 \left[ 1 + (.053/12) \right]^{360} \approx 385,664.73
\]

39. The time line is:

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 \\
\text{PV} & $7,300 & $1,500 & ? & $2,700 & $2,900 \\
\text{FV} & & & & & &
\end{array}
\]

We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV of the cash flows we know are:

\[
\begin{align*}
\text{PV of Year 1 CF: } & = 1,500 / 1.08 = 1,388.89 \\
\text{PV of Year 3 CF: } & = 2,700 / 1.08^3 = 2,143.35 \\
\text{PV of Year 4 CF: } & = 2,900 / 1.08^4 = 2,131.59 \\
\end{align*}
\]
So, the PV of the missing CF is:

\[ \$7,300 \times (1.08)^{-1} - 1,388.89 - 2,143.35 - 2,131.59 = \$1,636.18 \]

The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount. The value of the missing CF is:

\[ \$1,636.18(1.08)^2 = \$1,908.44 \]

40. The time line is:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1M</td>
<td>$1.275M</td>
<td>$1.55M</td>
<td>$1.825M</td>
<td>$2.1M</td>
<td>$2.375M</td>
<td>$2.65M</td>
<td>$2.925M</td>
<td>$3.2M</td>
<td>$3.475M</td>
<td>$3.75M</td>
</tr>
</tbody>
</table>

To solve this problem, we simply need to find the PV of each lump sum and add them together. It is important to note that the first cash flow of $1 million occurs today, so we do not need to discount that cash flow. The PV of the lottery winnings is:

\[ \$1,000,000 + \frac{\$1,275,000}{1.09} + \frac{\$1,550,000}{1.09^2} + \frac{\$1,825,000}{1.09^3} + \frac{\$2,100,000}{1.09^4} + \frac{\$2,375,000}{1.09^5} + \frac{\$2,650,000}{1.09^6} + \frac{\$2,925,000}{1.09^7} + \frac{\$3,200,000}{1.09^8} + \frac{\$3,475,000}{1.09^9} + \frac{\$3,750,000}{1.09^{10}} = \$15,885,026.33 \]

41. Here, we are finding interest rate for an annuity cash flow. We are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. We should also note that the PV of the annuity is not the amount borrowed since we are making a down payment on the warehouse. The amount borrowed is:

\[ \text{Amount borrowed} = 0.80(\$4,500,000) = \$3,600,000 \]

The time line is:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>-$3,600,000</td>
<td>$27,500</td>
<td>$27,500</td>
<td>$27,500</td>
<td>$27,500</td>
</tr>
</tbody>
</table>

Using the PVA equation:

\[ \text{PVA} = \frac{\$3,600,000}{r} = \frac{\$27,500}{r} \left(1 - \frac{1}{(1 + r)^{360}} \right) \]

Unfortunately, this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

\[ r = 0.702\% \]

The APR is the monthly interest rate times the number of months in the year, so:

\[ \text{APR} = 12(0.702\%) = 8.43\% \]
And the EAR is:

\[ \text{EAR} = (1 + .00702)^{12} - 1 = .0876 \text{ or } 8.76\% \]

42. The time line is:

\[ \begin{array}{cccc}
0 & \text{PV} & 3 & \text{\$115,000} \\
\end{array} \]

The profit the firm earns is just the PV of the sales price minus the cost to produce the asset. We find the PV of the sales price as the PV of a lump sum:

\[ \text{PV} = \frac{\$115,000}{1.13^3} = \$79,700.77 \]

And the firm’s profit is:

\[ \text{Profit} = \$79,700.77 - 76,000 = \$3,700.77 \]

To find the interest rate at which the firm will break even, we need to find the interest rate using the PV (or FV) of a lump sum. Using the PV equation for a lump sum, we get:

\[ \begin{array}{cccc}
0 & \text{\$76,000} & 3 & \text{\$115,000} \\
\end{array} \]

\[ $76,000 = $115,000 / (1 + r)^3 \]

\[ r = (\frac{115,000}{76,000})^{\frac{1}{3}} - 1 = .1481 \text{ or } 14.81\% \]

43. The time line is:

\[ \begin{array}{cccc}
0 & \text{\$5,000} & 1 & \text{\ldots} & \text{5} & \text{\$5,000} & \text{6} & \text{\$5,000} & \text{\ldots} & \text{25} & \text{\$5,000} \\
\end{array} \]

We want to find the value of the cash flows today, so we will find the PV of the annuity, and then bring the lump sum PV back to today. The annuity has 20 payments, so the PV of the annuity is:

\[ \text{PVA} = \$5,000 \left\{ \frac{1 - (1/1.06)^{20}}{.06} \right\} = \$57,349.61 \]

Since this is an ordinary annuity equation, this is the PV one period before the first payment, so it is the PV at \( T = 5 \). To find the value today, we find the PV of this lump sum. The value today is:

\[ \text{PV} = \$57,349.61 / 1.06^5 = \$42,854.96 \]
44. The time line for the annuity is:

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
$1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 \\
\end{array}
\]

This question is asking for the present value of an annuity, but the interest rate changes during the life of the annuity. We need to find the present value of the cash flows for the last eight years first. The PV of these cash flows is:

\[
PVA_2 = \frac{1,500 \left[1 - \frac{1}{1 + \left(\frac{.06}{12}\right)^96}\right]}{\left(\frac{.06}{12}\right)} = 114,142.83
\]

Note that this is the PV of this annuity exactly seven years from today. Now, we can discount this lump sum to today as well as finding the PV of the annuity for the first 7 years. The value of this cash flow today is:

\[
PV = \frac{114,142.83}{\left(1 + \left(\frac{.12}{12}\right)^84\right)} + \frac{1,500 \left[1 - \frac{1}{1 + \left(\frac{.12}{12}\right)^84}\right]}{\left(\frac{.12}{12}\right)} = 134,455.36
\]

45. The time line for the annuity is:

\[
\begin{array}{cccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & \ldots & 180 \\
$1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & $1,500 & \ldots & $1,500 \\
\end{array}
\]

Here, we are trying to find the dollar amount invested today that will equal the FVA with a known interest rate, and payments. First, we need to determine how much we would have in the annuity account. Finding the FV of the annuity, we get:

\[
FVA = \frac{1,500 \left(1 - \frac{1}{\left[1 + \left(\frac{.087}{12}\right)\right]^{84}}\right)}{\left(\frac{.087}{12}\right)} = 552,490.07
\]

Now, we need to find the PV of a lump sum that will give us the same FV. So, using the FV of a lump sum with continuous compounding, we get:

\[
FV = 552,490.07 = PVe^{.08(15)}
\]

\[
PV = \frac{552,490.07}{e^{.120}} = 166,406.81
\]

46. The time line is:

\[
\begin{array}{cccccccccccc}
0 & 1 & 7 & 14 & 15 & \ldots & \infty \\
\hline
PV & $2,500 & $2,500 & $2,500 & $2,500 & \ldots & $2,500 \\
\end{array}
\]

To find the value of the perpetuity at \(T = 7\), we first need to use the PV of a perpetuity equation. Using this equation we find:

\[
PV = \frac{2,500}{.061} = 40,983.61
\]
Remember that the PV of a perpetuity (and annuity) equations give the PV one period before the first payment, so, this is the value of the perpetuity at $t = 14$. To find the value at $t = 7$, we find the PV of this lump sum as:

$$PV = \frac{$40,983.61}{1.061^7} = $27,077.12$$

47. The time line is:

To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The interest rate for the cash flows of the loan is:

$$\text{PVA} = $26,000 = $2,513.33 \left\{ (1 - \frac{1}{(1 + r)^{12}}) \right\} / r \}$$

Again, we cannot solve this equation for $r$, so we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. Using a spreadsheet, we find:

$r = 2.361\%$ per month

So the APR is:

$$\text{APR} = 12(2.361\%) = 28.33\%$$

And the EAR is:

$$\text{EAR} = (1.02361)^{12} - 1 = 32.31\%$$

48. The time line is:

The cash flows in this problem are semiannual, so we need the effective semiannual rate. The interest rate given is the APR, so the monthly interest rate is:

$$\text{Monthly rate} = \frac{.12}{12} = .01$$

To get the semiannual interest rate, we can use the EAR equation, but instead of using 12 months as the exponent, we will use 6 months. The effective semiannual rate is:

$$\text{Semiannual rate} = (1.01)^6 - 1 = 6.15\%$$
We can now use this rate to find the PV of the annuity. The PV of the annuity is:

\[
PVA_{@ t=9} = \$5,300 \left[ \frac{1 - \frac{1}{1.0615^{10}}}{.0615} \right] = \$38,729.05
\]

Note, that this is the value one period (six months) before the first payment, so it is the value at \( t = 9 \). So, the value at the various times the questions asked for uses this value 9 years from now.

\[
PV_{@ t=5} = \frac{\$38,729.05}{1.0615^8} = \$24,022.10
\]

Note, that you can also calculate this present value (as well as the remaining present values) using the number of years. To do this, you need the EAR. The EAR is:

\[
EAR = (1 + .01)^{12} - 1 = 12.68\%
\]

So, we can find the PV at \( t = 5 \) using the following method as well:

\[
PV_{@ t=5} = \frac{\$38,729.05}{1.1268^4} = \$24,022.10
\]

The value of the annuity at the other times in the problem is:

\[
PV_{@ t=3} = \frac{\$38,729.05}{1.0615^{12}} = \$18,918.99
\]

\[
PV_{@ t=3} = \frac{\$38,729.05}{1.1268^6} = \$18,918.99
\]

\[
PV_{@ t=0} = \frac{\$38,729.05}{1.0615^{18}} = \$13,222.95
\]

\[
PV_{@ t=0} = \frac{\$38,729.05}{1.1268^9} = \$13,222.95
\]

49. a. The time line for the ordinary annuity is:

- PV $20,000 $20,000 $20,000 $20,000 $20,000

If the payments are in the form of an ordinary annuity, the present value will be:

\[
PVA = C \left( \frac{1 - \left[ \frac{1}{(1 + r)^t} \right]}{r} \right)
\]

\[
PVA = $20,000 \left[ \frac{1 - \left[ \frac{1}{(1 + .07)^5} \right]}{.07} \right]
\]

\[
PVA = $82,003.95
\]

The time line for the annuity due is:

- PV $20,000 $20,000 $20,000 $20,000 $20,000

If the payments are an annuity due, the present value will be:

\[
PVA_{due} = (1 + r) PVA
\]

\[
PVA_{due} = (1 + .07)$82,003.95
\]

\[
PVA_{due} = $87,744.23
\]
b. The time line for the ordinary annuity is:

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
FV & $20,000 & $20,000 & $20,000 & $20,000 & $20,000 \\
\end{array}
\]

We can find the future value of the ordinary annuity as:

\[
FVA = C\left\{\frac{(1 + r)^t - 1}{r}\right\}
\]

\[
FVA = $20,000\left\{\frac{(1 + .07)^5 - 1}{.07}\right\}
\]

\[
FVA = $115,014.78
\]

The time line for the annuity due is:

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
FV & $20,000 & $20,000 & $20,000 & $20,000 & $20,000 \\
\end{array}
\]

If the payments are an annuity due, the future value will be:

\[
FVA_{due} = (1 + r) FVA
\]

\[
FVA_{due} = (1 + .07)$115,014.78
\]

\[
FVA_{due} = $123,065.81
\]

c. Assuming a positive interest rate, the present value of an annuity due will always be larger than the present value of an ordinary annuity. Each cash flow in an annuity due is received one period earlier, which means there is one period less to discount each cash flow. Assuming a positive interest rate, the future value of an ordinary due will always higher than the future value of an ordinary annuity. Since each cash flow is made one period sooner, each cash flow receives one extra period of compounding.

50. The time line is:

\[
\begin{array}{ccccccc}
0 & 1 & \cdots & 59 & 60 \\
C & C & C & C & C & C & C \\
\end{array}
\]

We need to use the PVA due equation, that is:

\[
PVA_{due} = (1 + r) PVA
\]

Using this equation:

\[
PVA_{due} = $73,000 = \left[1 + (.0645/12)\right] \times C\left\{1 - 1 / \left[1 + (.0645/12)\right]^{60}\right\} / (.0645/12)
\]

\[
C = $1,418.99
\]

Notice, to find the payment for the PVA due we simply compound the payment for an ordinary annuity forward one period.

\textit{Challenge}
51. The time line is:

\[
\begin{array}{cccccccc}
0 & 1 & \cdots & 23 & 24 \\
\text{\$2,300} & C & C & C & C & C & C & C \\
\end{array}
\]

The monthly interest rate is the annual interest rate divided by 12, or:

Monthly interest rate = \( \frac{.104}{12} \)

Now we can set the present value of the lease payments equal to the cost of the equipment, or $2,300. The lease payments are in the form of an annuity due, so:

\[
PVA_{\text{due}} = (1 + r) \left( C \left\{ 1 - \frac{1}{(1 + r)^t} \right\} / r \right)
\]

\[
$2,300 = (1 + .00867) \left( \left\{ 1 - \frac{1}{(1 + .00867)^{24}} \right\} / .00867 \right)
\]

\[
C = \$105.64
\]

52. The time line is:

\[
\begin{array}{cccccccc}
0 & 1 & \cdots & 15 & 16 & 17 & 18 & 19 & 20 \\
C & C & C & C & C & C & C & C & C \\
\text{\$45,000} & \text{\$45,000} & \text{\$45,000} & \text{\$45,000} & \text{\$45,000} & \text{\$45,000} & \text{\$45,000} & \text{\$45,000} & \text{\$45,000} \\
\end{array}
\]

First, we will calculate the present value of the college expenses for each child. The expenses are an annuity, so the present value of the college expenses is:

\[
PVA = C \left\{ 1 - \frac{1}{(1 + r)^t} \right\} / r
\]

\[
PVA = \$45,000 \left\{ 1 - \frac{1}{(1 + .075)^4} \right\} / .075
\]

\[
PVA = \$150,719.68
\]

This is the cost of each child’s college expenses one year before they enter college. So, the cost of the oldest child’s college expenses today will be:

\[
PV = \frac{FV}{(1 + r)^t}
\]

\[
PV = \frac{\$150,719.68}{(1 + .075)^{14}}
\]

\[
PV = \$54,758.49
\]

And the cost of the youngest child’s college expenses today will be:

\[
PV = \frac{FV}{(1 + r)^t}
\]

\[
PV = \frac{\$150,719.68}{(1 + .075)^{16}}
\]

\[
PV = \$47,384.31
\]

Therefore, the total cost today of your children’s college expenses is:

\[
\text{Cost today} = \$54,758.49 + 47,384.31
\]
Cost today = $102,142.80

This is the present value of your annual savings, which are an annuity. So, the amount you must save each year will be:

\[
PVA = C \left( \frac{1 - \left[ \frac{1}{1 + r} \right]^t}{r} \right)
\]

\[\$102,142.80 = C \left( \frac{1 - \left[ \frac{1}{1 + .075} \right]^{15}}{.075} \right)\]

\[C = \$11,571.48\]

53. The salary is a growing annuity, so we use the equation for the present value of a growing annuity. The salary growth rate is 3.5 percent and the discount rate is 9 percent, so the value of the salary offer today is:

\[
PV = C \left[ \frac{1}{(r - g)} - \frac{1}{(r - g)} \times \left( \frac{1 + g}{1 + r} \right)^t \right]
\]

\[PV = \$55,000 \left[ \frac{1}{.09 - .035} - \frac{1}{.09 - .035} \times \left( \frac{1 + .035}{1 + .09} \right)^{25} \right]\]

\[PV = \$725,939.59\]

The yearly bonuses are 10 percent of the annual salary. This means that next year’s bonus will be:

Next year’s bonus = .10($55,000)

Next year’s bonus = $5,500

Since the salary grows at 3.5 percent, the bonus will grow at 3.5 percent as well. Using the growing annuity equation, with a 3.5 percent growth rate and a 12 percent discount rate, the present value of the annual bonuses is:

\[
PV = C \left[ \frac{1}{(r - g)} - \frac{1}{(r - g)} \times \left( \frac{1 + g}{1 + r} \right)^t \right]
\]

\[PV = \$5,500 \left[ \frac{1}{.09 - .035} - \frac{1}{.09 - .035} \times \left( \frac{1 + .035}{1 + .09} \right)^{25} \right]\]

\[PV = \$72,593.96\]

Notice the present value of the bonus is 10 percent of the present value of the salary. The present value of the bonus will always be the same percentage of the present value of the salary as the bonus percentage. So, the total value of the offer is:

\[PV = PV\text{(Salary)} + PV\text{(Bonus)} + \text{Bonus paid today}\]

\[PV = \$725,939.59 + 72,593.96 + 10,000\]

\[PV = \$808,533.55\]

54. Here, we need to compare two options. In order to do so, we must get the value of the two cash flow streams to the same time, so we will find the value of each today. We must also make sure to use the aftertax cash flows, since it is more relevant. For Option A, the aftertax cash flows are:

Aftertax cash flows = Pretax cash flows(1 – tax rate)

Aftertax cash flows = $250,000(1 – .28)

Aftertax cash flows = $180,000

So, the cash flows are:

\[
\begin{array}{cccccc}
0 & 1 & \cdots & 30 & 31 \\
\end{array}
\]
The aftertax cash flows from Option A are in the form of an annuity due, so the present value of the cash flow today is:

\[ PVA_{\text{due}} = (1 + \frac{r}{1 + \frac{1}{(1 + r)^t}})C \left\{ 1 - \left[ \frac{1}{1 + \frac{1}{(1 + r)^t}} \right] \right\} \]

\[ PVA_{\text{due}} = (1 + .07)180,000 \left\{ 1 - \left[ \frac{1}{1 + .07} \right]^{31} \right\} / .07 \]

\[ PVA_{\text{due}} = 2,413,627.41 \]

For Option B, the aftertax cash flows are:

Aftertax cash flows = Pretax cash flows \( (1 - \text{tax rate}) \)

Aftertax cash flows = 200,000 \( (1 - .28) \)

Aftertax cash flows = $144,000

The cash flows are:

\[
\begin{array}{cccccccccc}
0 & 1 & \ldots & 29 & 30 \\
PV & $530,000 & $144,000 & $144,000 & $144,000 & $144,000 & $144,000 & $144,000 & $144,000 & $144,000 \\
\end{array}
\]

The aftertax cash flows from Option B are an ordinary annuity, plus the cash flow today, so the present value is:

\[ PV = C \left\{ 1 - \left[ \frac{1}{1 + \frac{1}{(1 + r)^t}} \right] \right\} + CF_0 \]

\[ PV = 144,000 \left\{ 1 - \left[ \frac{1}{1 + .07} \right]^{30} \right\} / .07 \) + $530,000 \]

\[ PV = 2,316,901.93 \]

You should choose Option A because it has a higher present value on an aftertax basis.

55. We need to find the first payment into the retirement account. The present value of the desired amount at retirement is:

\[ PV = \frac{FV}{(1 + r)^t} \]

\[ PV = \frac{2,000,000}{(1 + .09)^{30}} \]

\[ PV = 150,742.27 \]

This is the value today. Since the savings are in the form of a growing annuity, we can use the growing annuity equation and solve for the payment. Doing so, we get:

\[ PV = C \left\{ \frac{1}{(1 + (r - g))} - \left[ \frac{1}{(1 + r)} \right] \right\} \times \left[ \frac{(1 + g)(1 + r)}{(1 + g)} \right] \]

\[ 150,742.27 = C \left\{ \frac{1}{(1 + .09 - .03)} - \left[ \frac{1}{(1 + .09)} \right] \right\} \times \left[ \frac{(1 + .03)(1 + .09)}{(1 + .03)} \right] \]

\[ C = 11,069.69 \]
This is the amount you need to save next year. So, the percentage of your salary is:

Percentage of salary = $11,069.69/$70,000
Percentage of salary = .1581 or 15.81%

Note that this is the percentage of your salary you must save each year. Since your salary is increasing at 3 percent, and the savings are increasing at 3 percent, the percentage of salary will remain constant.

56. Since she put $1,000 down, the amount borrowed will be:

Amount borrowed = $30,000 – 1,000
Amount borrowed = $29,000

So, the monthly payments will be:

\[ PVA = C \left( \frac{1 - \left( \frac{1}{1 + r} \right)^t}{r} \right) \]
\[ $29,000 = C \left[ \frac{1 - \left( \frac{1}{1 + .072/12} \right)^{60}}{.072/12} \right] \]
\[ C = $576.98 \]

The amount remaining on the loan is the present value of the remaining payments. Since the first payment was made on October 1, 2009, and she made a payment on October 1, 2011, there are 35 payments remaining, with the first payment due immediately. So, we can find the present value of the remaining 34 payments after November 1, 2011, and add the payment made on this date. So the remaining principal owed on the loan is:

\[ PV = C \left( \frac{1 - \left( \frac{1}{1 + r} \right)^t}{r} \right) + C_0 \]
\[ PV = $576.98 \left[ \frac{1 - \left( \frac{1}{1 + .072/12} \right)^{34}}{.072/12} \right] \]
\[ C = $17,697.79 \]

She must also pay a one percent prepayment penalty and the payment due on November 1, 2011, so the total amount of the payment is:

Total payment = Balloon amount(1 + Prepayment penalty) + Current payment
Total payment = $17,697.79(1 + .01) + $576.98
Total payment = $18,451.74

57. The time line is:

\[ \begin{array}{cccccccccc}
0 & 1 & \ldots & 120 & \ldots & 360 & 361 & \ldots & 600 \\
-2,100 & \ldots & -2,100 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
$320,000 & C & C & C & \ldots & \ldots & \ldots & \ldots & $1,000,000 \\
\end{array} \]

The cash flows for this problem occur monthly, and the interest rate given is the EAR. Since the cash flows occur monthly, we must get the effective monthly rate. One way to do this is to find the APR based on monthly compounding, and then divide by 12. So, the pre-retirement APR is:

\[ \text{EAR} = .11 = \left[ 1 + \left( \frac{\text{APR}}{12} \right) \right]^{12} - 1; \quad \text{APR} = 12[(1.11)^{1/12} - 1] = 10.48\% \]

And the post-retirement APR is:
First, we will calculate how much he needs at retirement. The amount needed at retirement is the PV of the monthly spending plus the PV of the inheritance. The PV of these two cash flows is:

\[
PVA = $23,000 \frac{1 - [1 / (1 + 0.0772/12)^{12(20)}]}{0.0772/12} = $2,807,787.80
\]

\[
PV = \frac{$1,000,000}{(1 + 0.08)^{20}} = $214,548.21
\]

So, at retirement, he needs:

\[
$2,807,787.80 + 214,548.21 = $3,022,336.00
\]

He will be saving $2,100 per month for the next 10 years until he purchases the cabin. The value of his savings after 10 years will be:

\[
FVA = $2,100 \left[ \frac{1 - [1 + (0.1048/12)]^{12(10)}}{0.1048/12} \right] = $442,239.69
\]

After he purchases the cabin, the amount he will have left is:

\[
$442,239.69 - 320,000 = $122,239.69
\]

He still has 20 years until retirement. When he is ready to retire, this amount will have grown to:

\[
FV = $122,239.69 \left[ 1 + (0.1048/12) \right]^{12(20)} = $985,534.47
\]

So, when he is ready to retire, based on his current savings, he will be short:

\[
$3,022,336.00 - 985,534.47 = $2,036,801.54
\]

This amount is the FV of the monthly savings he must make between years 10 and 30. So, finding the annuity payment using the FVA equation, we find his monthly savings will need to be:

\[
FVA = $2,036,801.54 = C \left[ \frac{1 - [1 + (0.1048/12)]^{12(20)}}{0.1048/12} \right]
\]

\[
C = $2,519.10
\]

58. To answer this question, we should find the PV of both options, and compare them. Since we are purchasing the car, the lowest PV is the best option. The PV of the leasing is simply the PV of the lease payments, plus the $1,500. The interest rate we would use for the leasing option is the same as the interest rate of the loan. The PV of leasing is:

\[
PV = $1,500 + $405 \frac{1 - [1 / (1 + 0.06/12)^{36}]}{0.06/12} = $14,812.76
\]
The PV of purchasing the car is the current price of the car minus the PV of the resale price. The PV of the resale price is:

\[
PV = \frac{-20,000}{1 + (0.06/12)}^{12(3)} = 16,712.90
\]

The PV of the decision to purchase is:

\[
$31,000 - 16,712.90 = 14,287.10
\]

In this case, it is cheaper to buy the car than lease it since the PV of the leasing cash flows is lower. To find the breakeven resale price, we need to find the resale price that makes the PV of the two options the same. In other words, the PV of the decision to buy should be:

\[
$31,000 - PV\text{ of resale price} = 14,812.76
\]

PV of resale price = 16,187.24

The resale price that would make the PV of the lease versus buy decision equal is the FV of this value, so:

Breakeven resale price = $16,187.24\[1 + (0.06/12)]^{12(3)} = 19,370.95

59. To find the quarterly salary for the player, we first need to find the PV of the current contract. The cash flows for the contract are annual, and we are given a daily interest rate. We need to find the EAR so the interest compounding is the same as the timing of the cash flows. The EAR is:

\[
EAR = \left[1 + \frac{0.05}{365}\right]^{365} - 1 = 5.13\%
\]

The PV of the current contract offer is the sum of the PV of the cash flows. So, the PV is:

\[
PV = 8,500,000 + \frac{3,900,000}{1.0513} + \frac{4,600,000}{1.0513^2} + \frac{5,300,000}{1.0513^3} + \frac{5,800,000}{1.0513^4} + \frac{6,400,000}{1.0513^5} + \frac{7,300,000}{1.0513^6} + \frac{8,500,000}{1.0513^7} + \frac{9,100,000}{1.0513^8} + \frac{9,900,000}{1.0513^9} + \frac{10,800,000}{1.0513^{10}}
\]

PV = $36,075,085.12

The player wants the contract increased in value by $1,500,000, so the PV of the new contract will be:

\[
PV = 36,075,085.12 + 1,500,000 = 37,575,085.12
\]

The player has also requested a signing bonus payable today in the amount of $10 million. We can simply subtract this amount from the PV of the new contract. The remaining amount will be the PV of the future quarterly paychecks.

\[
37,575,085.12 - 10,000,000 = 27,575,085.12
\]
To find the quarterly payments, first realize that the interest rate we need is the effective quarterly rate. Using the daily interest rate, we can find the quarterly interest rate using the EAR equation, with the number of days being 91.25, the number of days in a quarter (365 / 4). The effective quarterly rate is:

$$\text{Effective quarterly rate} = (1 + (.05/365))^{91.25} - 1 = .01258 \text{ or } 1.258\%$$

Now, we have the interest rate, the length of the annuity, and the PV. Using the PVA equation and solving for the payment, we get:

$$\text{PVA} = 27,575,085.12 = C\left[\frac{1 - (1/1.01258)^{24}}{.01258}\right]$$

$$C = 1,338,243.52$$

60. The time line for the cash flows is:

\[
\begin{array}{ccc}
0 & 1 \\
-17,000 & 20,000 \\
\end{array}
\]

To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The cash flows of the loan are the $20,000 you must repay in one year, and the $17,200 you borrow today. The interest rate of the loan is:

$$\frac{20,000}{17,000} - 1 = .1765 \text{ or } 17.65\%$$

Because of the discount, you only get the use of $17,000, and the interest you pay on that amount is 17.65%, not 15%.

61. The time line is:

\[
\begin{array}{cccccccc}
-24 & -23 & \ldots & -12 & -11 & \ldots & 0 & 1 & \ldots & 60 \\
3,083.33 & 3,083.33 & \ldots & 3,250 & & \ldots & 3,250 & 3,583.33 & \ldots & 3,583.33 \\
150,000 & & & & & & & & \end{array}
\]

Here, we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate, so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:

$$\text{APR} = 12\left[(1.09)^{1/12} - 1\right] = 8.65\%$$

To find the value today of the back pay from two years ago, we will find the FV of the annuity (salary), and then find the FV of the lump sum value of the salary. Doing so gives us:

$$\text{FV} = (37,000/12) \left[\frac{[1 + (.0865/12)]^{12} - 1}{(.0865/12)}\right] (1 + .09) = 41,967.73$$
Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way.

Now, we need to find the value today of last year’s back pay:

\[ FVA = \left( \frac{39,000}{12} \right) \left[ \left( 1 + \frac{.0865}{12} \right)^{12} - 1 \right] / \left( \frac{.0865}{12} \right) = 40,583.72 \]

Next, we find the value today of the five year’s future salary:

\[ PVA = \left( \frac{43,000}{12} \right) \left[ \left( 1 - \frac{1}{\left[ 1 + \frac{.0865}{12} \right]^{12(5)}} \right) / \left( \frac{.0865}{12} \right) \right] = 174,046.93 \]

The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

\[ Award = 41,967.73 + 40,583.72 + 174,046.93 + 150,000 + 25,000 \]

\[ Award = 431,598.39 \]

As the plaintiff, you would prefer a lower interest rate. In this problem, we are calculating both the PV and FV of annuities. A lower interest rate will decrease the FVA, but increase the PVA. So, by a lower interest rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time, this is the more important cash flow to the plaintiff.

62. Again, to find the interest rate of a loan, we need to look at the cash flows of the loan. Since this loan is in the form of a lump sum, the amount you will repay is the FV of the principal amount, which will be:

\[ Loan \text{ repayment amount} = 10,000(1.08) = 10,800 \]

The amount you will receive today is the principal amount of the loan times one minus the points.

\[ Amount \text{ received} = 10,000(1 - .03) = 9,700 \]

So, the time line is:

\[ 0 \quad 9 \]

\[ -$9,700 \quad $10,800 \]

Now, we simply find the interest rate for this PV and FV.

\[ 10,800 = 9,700(1 + r) \]

\[ r = \frac{10,800}{9,700} - 1 = .1134, \text{ or } 11.34\% \]

With a quoted interest rate of 11 percent and two points, the EAR is:

\[ Loan \text{ repayment amount} = 10,000(1.11) = 11,100 \]

\[ Amount \text{ received} = 10,000(1 - .02) = 9,800 \]

\[ 11,100 = 9,800(1 + r) \]
\[ r = \left( \frac{\$11,100}{\$9,800} \right) - 1 = 0.1327, \text{ or } 13.27\% \]

The effective rate is not affected by the loan amount, since it drops out when solving for \( r \).

63. First, we will find the APR and EAR for the loan with the refundable fee. Remember, we need to use the actual cash flows of the loan to find the interest rate. With the $2,400 application fee, you will need to borrow $202,400 to have $200,000 after deducting the fee. The time line is:

\[
\begin{array}{cccccccccc}
0 & 1 & \ldots & 360 \\
\$202,400 & C & C & C & C & C & C & C & C & C
\end{array}
\]

Solving for the payment under these circumstances, we get:

\[ PVA = \$202,400 = C \left\{ \frac{1 - 1/(1.004417)^{360}}{0.004417} \right\} \]

where \(0.004417 = 0.053/12\)

\[ C = \$1,123.94 \]

We can now use this amount in the PVA equation with the original amount we wished to borrow, $200,000.

\[
\begin{array}{cccccccccc}
0 & 1 & \ldots & 360 \\
\$200,000 & C & C & C & C & C & C & C & C & C & C & C
\end{array}
\]

Solving for \( r \), we find:

\[ PVA = \$200,000 = \$1,123.94 \left\{ \frac{1 - [1 / (1 + r)^{360}]}{r} \right\} \]

Solving for \( r \) with a spreadsheet, on a financial calculator, or by trial and error, gives:

\[ r = 0.4506\% \text{ per month} \]

\[ APR = 12(0.4506\%) = 5.41\% \]

\[ EAR = (1 + 0.004506)^{12} - 1 = 0.0554, \text{ or } 5.54\% \]

With the nonrefundable fee, the APR of the loan is simply the quoted APR since the fee is not considered part of the loan. So:

\[ APR = 5.30\% \]

\[ EAR = [1 + (0.053/12)]^{12} - 1 = 0.0543, \text{ or } 5.43\% \]
64. The time line is:

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & \ldots & 36 \\
-1,000 & 45.64 & 45.64 & 45.64 & 45.64 & 45.64 & 45.64 & \ldots & 45.64 \\
\end{array}
\]

Be careful of interest rate quotations. The actual interest rate of a loan is determined by the cash flows. Here, we are told that the PV of the loan is $1,000, and the payments are $45.64 per month for three years, so the interest rate on the loan is:

\[
PVA = $1,000 = 45.64 \left\{ 1 - \left[ 1 / (1 + r) \right]^{36} \right\} / r
\]

Solving for $r$ with a spreadsheet, on a financial calculator, or by trial and error, gives:

$r = 2.98\%$ per month

APR = 12(2.98\%) = 35.71\%

EAR = \((1 + 0.0298)^{12} - 1\) = .4218, or 42.18\%

It’s called add-on interest because the interest amount of the loan is added to the principal amount of the loan before the loan payments are calculated.

65. We will calculate the number of periods necessary to repay the balance with no fee first. We simply need to use the PVA equation and solve for the number of payments.

Without fee and annual rate = 18.6\%:

\[
PVA = $10,000 = 200 \left\{ 1 - \left[ 1 / (1.0155)^t \right] \right\} / .0155
\]

Solving for $t$, we get:

\[
t = \ln\{1 / [1 - ($10,000/$200)(.0155)]\} / \ln(1.0155)
\]

$t = 96.98$ months

Without fee and annual rate = 8.2\%:

\[
PVA = $10,000 = 200 \left\{ 1 - \left[ 1 / (1.006833)^t \right] \right\} / .006833
\]

Solving for $t$, we get:

\[
t = \ln\{1 / [1 - ($10,000/$200)(.006833)]\} / \ln(1.006833)
\]

$t = 61.39$ months
Note that we do not need to calculate the time necessary to repay your current credit card with a fee since no fee will be incurred. The time to repay the new card with a transfer fee is:

With fee and annual rate = 8.20%:

\[ PVA = \frac{\$10,200}{.006833} \left[ 1 - \left(1 - \frac{1}{1.006833}\right)^{t} \right] \]

where .006833 = .082/12

Solving for \( t \), we get:

\[ t = \ln\left\{ \frac{1}{1 - (\frac{\$10,200}{\$200})(.006833)} \right\} / \ln(1.006833) \]

\[ t = \ln 1.53492 / \ln 1.006833 \]

\[ t = 62.92 \text{ months} \]

66. We need to find the FV of the premiums to compare with the cash payment promised at age 65. We have to find the value of the premiums at year 6 first since the interest rate changes at that time. So:

\[ FV_1 = \$500(1.11)^{5} = \$842.53 \]

\[ FV_2 = \$600(1.11)^{4} = \$910.84 \]

\[ FV_3 = \$700(1.11)^{3} = \$957.34 \]

\[ FV_4 = \$800(1.11)^{2} = \$985.68 \]

\[ FV_5 = \$900(1.11)^{1} = \$999.00 \]

Value at year six = $842.53 + 910.84 + 957.34 + 985.68 + 999.00 + 1,000.00 = $5,695.39

Finding the FV of this lump sum at the child’s 65th birthday:

\[ FV = \$5,695.39(1.07)^{59} = \$308,437.08 \]

The policy is not worth buying; the future value of the policy is $308,437.08, but the policy contract will pay off $275,000. The premiums are worth $33,437.08 more than the policy payoff.

Note, we could also compare the PV of the two cash flows. The PV of the premiums is:

\[ PV = \frac{\$500}{1.11} + \frac{\$600}{1.11^2} + \frac{\$700}{1.11^3} + \frac{\$800}{1.11^4} + \frac{\$900}{1.11^5} + \frac{\$1,000}{1.11^6} = \$3,044.99 \]

And the value today of the $275,000 at age 65 is:

\[ PV = \frac{\$275,000}{1.07^{59}} = \$5,077.97 \]

\[ PV = \frac{\$5,077.97}{1.11^6} = \$2,714.89 \]

The premiums still have the higher cash flow. At time zero, the difference is $330.10. Whenever you are comparing two or more cash flow streams, the cash flow with the highest value at one time will have the highest value at any other time.

Here is a question for you: Suppose you invest $330.10, the difference in the cash flows at time zero, for six years at an 11 percent interest rate, and then for 59 years at a seven percent interest rate. How
much will it be worth? Without doing calculations, you know it will be worth $33,437.08, the
difference in the cash flows at time 65!

67. Since the payments occur at six month intervals, we need to get the effective six-month interest rate. We
can calculate the daily interest rate since we have an APR compounded daily, so the effective
six-month interest rate is:

\[
\text{Effective six-month rate} = (1 + \text{Daily rate})^{180} - 1
\]
\[
\text{Effective six-month rate} = (1 + 0.09/360)^{180} - 1
\]
\[
\text{Effective six-month rate} = 0.0460 \text{ or } 4.60\%
\]

Now, we can use the PVA equation to find the present value of the semi-annual payments. Doing so,
we find:

\[
PVA = C \left( \frac{1 - \left(1/(1 + r)\right)^t}{r} \right)
\]
\[
PVA = 1,250,000 \left( \frac{1 - \left(1/(1 + 0.0460)\right)^{40}}{0.0460} \right)
\]
\[
PVA = 22,670,253.86
\]

This is the value six months from today, which is one period (six months) prior to the first payment. So, the
value today is:

\[
PV = \frac{22,670,253.86}{1 + 0.0460}
\]
\[
PV = 21,672,827.50
\]

This means the total value of the lottery winnings today is:

Value of winnings today = $21,672,827.50 + 2,500,000
Value of winnings today = $24,172,827.50

You should not take the offer since the value of the offer is less than the present value of the payments.

68. Here, we need to find the interest rate that makes the PVA, the college costs, equal to the FVA, the
savings. The PV of the college costs is:

\[
PVA = 25,000 \left[ \frac{1 - \left(1/ (1 + r)\right)^4}{r} \right]
\]

And the FV of the savings is:

\[
FVA = 11,000 \left[ \frac{(1 + r)^6 - 1}{r} \right]
\]

Setting these two equations equal to each other, we get:

\[
25,000 \left[ \frac{1 - \left(1/ (1 + r)\right)^4}{r} \right] = 11,000 \left[ \frac{(1 + r)^6 - 1}{r} \right]
\]

Reducing the equation gives us:

\[
(1 + r)^6 - 4.40(1 + r)^4 + 44.00 = 0
\]

Now, we need to find the root of this equation. We can solve using trial and error, a root-solving calculator routine, or a spreadsheet. Using a spreadsheet, we find:

\[
r = 8.54\%
\]
69. The time line is:

Here, we need to find the interest rate that makes us indifferent between an annuity and a perpetuity. To solve this problem, we need to find the PV of the two options and set them equal to each other. The PV of the perpetuity is:

\[ PV = \frac{-15,000}{r} \]

And the PV of the annuity is:

\[ PVA = \frac{26,000 \left\{ 1 - \left[ \frac{1}{1 + r} \right]^{10} \right\} }{r} \]

Setting them equal and solving for \( r \), we get:

\[ \frac{-15,000}{26,000} = 1 - \left[ \frac{1}{1 + r} \right]^{10} \]

\[ .5769^{1/10} = 1 / (1 + r) \]

\[ r = .0898, \text{ or } 8.98\% \]

70. The time line is:

The cash flows in this problem occur every two years, so we need to find the effective two year rate. One way to find the effective two year rate is to use an equation similar to the EAR, except use the number of days in two years as the exponent. (We use the number of days in two years since it is daily compounding; if monthly compounding was assumed, we would use the number of months in two years.) So, the effective two-year interest rate is:

\[ \text{Effective 2-year rate} = \left[ 1 + \left( \frac{.13}{365} \right) \right]^{365(2)} - 1 = 29.69\% \]

We can use this interest rate to find the PV of the perpetuity. Doing so, we find:

\[ PV = \frac{30,000}{.2969} = $101,054.32 \]
Solutions Manual

This is an important point: Remember that the PV equation for a perpetuity (and an ordinary annuity) tells you the PV one period before the first cash flow. In this problem, since the cash flows are two years apart, we have found the value of the perpetuity one period (two years) before the first payment, which is one year ago. We need to compound this value for one year to find the value today. The value of the cash flows today is:

\[ PV = \$101,054.32 (1 + \frac{.13}{365})^{365} = \$115,080.86 \]

The second part of the question assumes the perpetuity cash flows begin in four years. In this case, when we use the PV of a perpetuity equation, we find the value of the perpetuity two years from today. So, the value of these cash flows today is:

\[ PV = \frac{\$101,054.32}{(1 + \frac{.13}{365})^{2(365)}} = \$77,921.70 \]

71. To solve for the PVA due:

\[
PVA = \frac{C}{(1 + r)} + \frac{C}{(1 + r)^2} + \ldots + \frac{C}{(1 + r)^t}
\]

\[
PVA_{\text{due}} = C + \frac{C}{(1 + r)} + \ldots + \frac{C}{(1 + r)^{t-1}}
\]

\[
PVA_{\text{due}} = (1 + r) \left( \frac{C}{(1 + r)} + \frac{C}{(1 + r)^2} + \ldots + \frac{C}{(1 + r)^t} \right)
\]

\[
PVA_{\text{due}} = (1 + r)PVA
\]

And the FVA due is:

\[
FVA = C + C(1 + r) + C(1 + r)^2 + \ldots + C(1 + r)^{t-1}
\]

\[
FVA_{\text{due}} = C(1 + r) + C(1 + r)^2 + \ldots + C(1 + r)^t
\]

\[
FVA_{\text{due}} = (1 + r)\left[ C + C(1 + r) + \ldots + C(1 + r)^{t-1} \right]
\]

\[
FVA_{\text{due}} = (1 + r)FVA
\]

72. \(a\). The APR is the interest rate per week times 52 weeks in a year, so:

\[
\text{APR} = 52(7\%) = 364\%
\]

\[
\text{EAR} = (1 + .07)^{52} - 1 = 32.7253, \text{ or } 3,272.53\%
\]

\(b\). In a discount loan, the amount you receive is lowered by the discount, and you repay the full principal. With a 7 percent discount, you would receive \$9.30 for every \$10 in principal, so the weekly interest rate would be:

\[
\$10 = \$9.30(1 + r)
\]

\[
r = (\$10 / \$9.30) - 1 = .0753, \text{ or } 7.53\%
\]
Note the dollar amount we use is irrelevant. In other words, we could use $0.93 and $1, $93 and $100, or any other combination and we would get the same interest rate. Now we can find the APR and the EAR:

\[
\text{APR} = 52(7.53\%) = 391.40\%
\]

\[
\text{EAR} = (1 + .0753)^{52} - 1 = 42.5398, \text{ or } 4,253.98\%
\]

c. Using the cash flows from the loan, we have the PVA and the annuity payments and need to find the interest rate, so:

\[
PVA = $68.92 = \frac{25\left[1 - \left(\frac{1}{1 + r}\right)^4\right]}{r}
\]

Using a spreadsheet, trial and error, or a financial calculator, we find:

\[
r = 16.75\% \text{ per week}
\]

\[
\text{APR} = 52(16.75\%) = 871.002\%
\]

\[
\text{EAR} = 1.1675^{52} - 1 = 3,142.1572, \text{ or } 314,215.72\%
\]

73. To answer this, we can diagram the perpetuity cash flows, which are: (Note, the subscripts are only to differentiate when the cash flows begin. The cash flows are all the same amount.)

\[
\begin{array}{c|c|c|c|c|c}
C_1 & C_2 & C_3 & C_4 & \cdots \\
\hline
C_1 & C_2 & C_1 & C_1 & \cdots \\
\end{array}
\]

Thus, each of the increased cash flows is a perpetuity in itself. So, we can write the cash flows stream as:

\[
\frac{C_1}{r} \quad \frac{C_2}{r} \quad \frac{C_3}{r} \quad \frac{C_4}{r} \quad \cdots
\]

So, we can write the cash flows as the present value of a perpetuity with a perpetuity payment of:

\[
\frac{C_2}{r} \quad \frac{C_3}{r} \quad \frac{C_4}{r} \quad \cdots
\]
The present value of this perpetuity is:

\[ PV = \frac{C}{r} \]

So, the present value equation of a perpetuity that increases by \( C \) each period is:

\[ PV = \frac{C}{r} \]

74. Since it is only an approximation, we know the Rule of 72 is exact for only one interest rate. Using the basic future value equation for an amount that doubles in value and solving for \( t \), we find:

\[ FV = PV(1 + r)^t \]
\[ 2 = 1(1 + r)^t \]
\[ \ln(2) = t \ln(1 + r) \]
\[ t = \frac{\ln(2)}{\ln(1 + r)} \]

We also know the Rule of 72 approximation is:

\[ t = \frac{72}{r} \]

We can set these two equations equal to each other and solve for \( r \). We also need to remember that the exact future value equation uses decimals, so the equation becomes:

\[ \frac{.72}{r} = \frac{\ln(2)}{\ln(1 + r)} \]
\[ 0 = \frac{.72}{r} \cdot \frac{\ln(2)}{\ln(1 + r)} \]

It is not possible to solve this equation directly for \( r \), but using Solver, we find the interest rate for which the Rule of 72 is exact is 7.846894 percent.

75. We are only concerned with the time it takes money to double, so the dollar amounts are irrelevant. So, we can write the future value of a lump sum with continuously compounded interest as:

\[ FV = PV e^{rt} \]
\[ 2 = 1 e^{rt} \]
\[ rt = \ln(2) \]
\[ rt = .693147 \]
\[ t = \frac{.693147}{r} \]

Since we are using percentage interest rates while the equation uses decimal form, to make the equation correct with percentages, we can multiply by 100:

\[ t = \frac{69.3147}{r} \]
Calculator Solutions

1. 
Enter
10 8% $5,000
N I/Y PV PMT FV
Solve for
$10,794.62
$10,794.62 – 9,000 = $1,794.62

2. 
Enter
10 5% $1,000
N I/Y PV PMT FV
Solve for
$1,628.89

Enter
10 10% $1,000
N I/Y PV PMT FV
Solve for
$2,593.74

Enter
20 5% $1,000
N I/Y PV PMT FV
Solve for
$2,653.30

3. 
Enter
6 7% $13,827
N I/Y PV PMT FV
Solve for
$9,213.51

Enter
9 15% $43,852
N I/Y PV PMT FV
Solve for
$12,465.48

Enter
18 11% $725,380
N I/Y PV PMT FV
Solve for
$110,854.15

Enter
23 18% $590,710
N I/Y PV PMT FV
Solve for
$13,124.66

4. 
Enter
4 $242 ±$307
N I/Y PV PMT FV
Solve for
6.13%
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8. Enter 4 N I/Y PV PMT FV +$1,680,000 $1,100,000 Solve for –13.17%

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12. Enter 9 N I/Y PV PMT FV 5% $4,500 Solve for $31,985.20

Enter 5 N I/Y PV PMT FV 5% $7,000 Solve for $30,306.34

Enter 9 N I/Y PV PMT FV 22% $4,500 Solve for $17,038.28

Enter 5 N I/Y PV PMT FV 22% $7,000 Solve for $20,045.48

13. Enter 15 N I/Y PV PMT FV 8% $4,900 Solve for $41,941.45

Enter 40 N I/Y PV PMT FV 8% $4,900 Solve for $58,430.61
### 15.

Enter 75% 8% $4,900  
N I/Y PV PMT FV

Solve for $61,059.31

### 16.

Enter 7% 4  
NOM EFF C/Y

Solve for 7.19%

Enter 16% 12  
NOM EFF C/Y

Solve for 17.23%

Enter 11% 365  
NOM EFF C/Y

Solve for 11.63%

### 17.

Enter 11.2% 12  
NOM EFF C/Y

Solve for 11.79%

Enter 11.4% 2  
NOM EFF C/Y

Solve for 11.72%

### 18.

2nd BGN 2nd SET

Enter 12 $108 ±$10

N I/Y PV PMT FV

Solve for 1.98%

APR = 1.98% × 52 = 102.77%
Enter 102.77% NOM 176.68% EFF
Solve for C/Y

19. Enter 1.3% I/Y $21,500 PV ±$700 PMT FV
Solve for 39.46

20. Enter 1,733.33% NOM 313,916,515.69% EFF C/Y
Solve for

21. Enter 6 N 9% I/Y $1,000 PV PMT FV
Solve for $1,677.10

Enter 6 × 2 N I/Y 9%/2 $1,000 PV PMT FV
Solve for $1,695.88

Enter 6 × 12 N I/Y 9%/12 $1,000 PV PMT FV
Solve for $1,712.55

23. Stock account:

Enter 360 N 11% / 12 $800 PV PMT FV
Solve for $2,243,615.79

Bond account:

Enter 360 N 6% / 12 $350 PV PMT FV
Solve for $351,580.26

Savings at retirement = $2,243,615.79 + 351,580.26 = $2,595,196.05

Enter 300 N 8% / 12 $2,595,196.05 PV PMT FV
Solve for $20,030.14
24. Enter 12 / 3 ±$1 $4
Solve for N 41.42% PV PMT FV

25. Enter 6 ±$65,000 $125,000
Solve for N 11.51% PV PMT FV

Enter 10 ±65,000 $185,000
Solve for N 11.03% PV PMT FV

28. Enter 23 7% PV $6,500
Solve for N $73,269.22 PV PMT FV

Enter 2 7% PV $73,269.22
Solve for N $63,996.17 PV PMT FV

29. Enter 15 13% PV $650
Solve for N $4,200.55 PV PMT FV

Enter 5 11% PV $4,200.55
Solve for N $2,492.82 PV PMT FV

30. Enter 360 6.1%/12 .80($550,000)
Solve for N $2,666.38 PV PMT FV

Enter 22 × 12 6.1%/12 $2,666.38
Solve for N $386,994.11 PV PMT FV

31. Enter 6 2.40% / 12 $7,500
Solve for N $7,590.45 PV PMT FV
Enter 6 18% / 12 $7,590.45
Solve for $8,299.73
$8,299.73 – 7,500 = $799.73

35.
Enter 15 10% $6,800
Solve for $51,721.34

Enter 15 5% $6,800
Solve for $70,581.67

Enter 15 15% $6,800
Solve for $39,762.12

36.
Enter 10% / 12 ±$350 $35,000
Solve for 73.04

37.
Enter 60 5.3% / 12 $950
Solve for 0.672%
0.672% x 12 = 8.07%

38.
Enter 360 5.3% / 12 $78,922.74
Solve for $171,077.26
$250,000 – 171,077.26 = $78,922.74

Enter 360 5.3% / 12 $78,922.74
Solve for $385,664.73

$250,000 – 171,077.26 = $78,922.74
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I = 8%
NPV CPT
$5,663.82

PV of missing CF = $7,300 − $5,663.82 = $1,636.18
Value of missing CF:

```
Enter 2 8% $1,636.18
Solve for     $1,908.44
```

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<td>F08</td>
<td>1</td>
</tr>
<tr>
<td>C09</td>
<td>$3,475,000</td>
</tr>
<tr>
<td>F09</td>
<td>1</td>
</tr>
<tr>
<td>C010</td>
<td>$3,750,000</td>
</tr>
</tbody>
</table>

I = 9%
NPV CPT
$15,885,026.33
41. Enter 360 \( \times \) .80($4,500,000) ±27,500
Solve for 0.702%

\[ \text{APR} = 0.702\% \times 12 = 8.43\% \]

Enter 8.43% 12
Solve for 8.76%

42. Enter 3 13% $115,000
Solve for $79,700.77

Profit = $79,700.77 – 76,000 = $3,700.77

Enter 3 ±76,000 $115,000
Solve for 14.81%

43. Enter 20 7% $5,000
Solve for $52,970.07

Enter 5 7% $52,970.07
Solve for $37,766.93

44. Enter 96 6% / 12 $1,500
Solve for $114,142.83

Enter 84 12% / 12 $1,500 $114,142.83
Solve for $134,455.36
45. Enter \(15 \times 12\) 8.7%/12 $1,500
\[\text{Solve for} \quad FV = 552,490.07\]

\[FV = 522,490.07 = e^{0.08(15)} \cdot PV = 552,490.07 e^{-1.20} = 166,406.81\]

46. \(PV@ t = 14: 2,500 / 0.061 = 40,983.61\)

Enter \(7\) 6.1% \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)
\[\text{Solve for} \quad FV = 27,077.12\]

47. Enter \(12\) \$26,000 \(N\) \(I/Y\) \(PV\) \(+\$2,513.33\) \(PMT\) \(FV\)
\[\text{Solve for} \quad \text{APR} = 2.361\% \times 12 = 28.33\%\]

Enter \(28.33\%\) \(NOM\) \(EFF\) \(C/Y\)
\[\text{Solve for} \quad 32.31\%\]

48. Monthly rate = .12 / 12 = .01; semiannual rate = \((1.01)^6 - 1 = 6.15\%\)

Enter \(10\) 6.15% \(N\) \(I/Y\) \(PV\) \$5,300 \(PMT\) \(FV\)
\[\text{Solve for} \quad FV = 38,729.05\]

Enter \(8\) 6.15% \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)
\[\text{Solve for} \quad FV = 24,022.10\]

Enter \(12\) 6.15% \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)
\[\text{Solve for} \quad FV = 18,918.99\]

Enter \(18\) 6.15% \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)
\[\text{Solve for} \quad FV = 13,222.95\]
49.  
   \(a.\) Enter 5 \(7\%\) $20,000 \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)  
   Solve for $82,003.95  
   2\(^{nd}\) BGN 2\(^{nd}\) SET  
   Enter 5 \(7\%\) $20,000 \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)  
   Solve for $87,744.23  
   \(b.\) Enter 5 \(7\%\) $20,000 \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)  
   Solve for $115,014.78  
   2\(^{nd}\) BGN 2\(^{nd}\) SET  
   Enter 5 \(7\%\) $20,000 \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)  
   Solve for $123,065.81  

50.  
   2\(^{nd}\) BGN 2\(^{nd}\) SET  
   Enter 60 \(6.45\% / 12\) $73,000 \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)  
   Solve for $1,418.99  

51.  
   2\(^{nd}\) BGN 2\(^{nd}\) SET  
   Enter 2 \(10.4\% / 12\) $2,300 \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)  
   Solve for $105.64  

52.  
   PV of college expenses:  
   Enter 4 \(7.5\%\) $45,000 \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)  
   Solve for $150,719.68  
   Cost today of oldest child’s expenses:  
   Enter 14 \(7.5\%\) $150,719.68 \(N\) \(I/Y\) \(PV\) \(PMT\) \(FV\)  
   Solve for $54,758.49
Cost today of youngest child’s expenses:

Enter 16 7.5% $150,719.68
Solve for $47,384.31

Total cost today = $54,758.49 + 47,384.31 = $102,142.80

Enter 15 7.5% $102,142.80
Solve for $11,571.48

54. Option A:
Aftertax cash flows = Pretax cash flows(1 – tax rate)
Aftertax cash flows = $250,000(1 – .28)
Aftertax cash flows = $180,000

2^ND BGN 2^nd SET
Enter 31 7% $180,000
Solve for $2,413,627.41

Option B:
Aftertax cash flows = Pretax cash flows(1 – tax rate)
Aftertax cash flows = $200,000(1 – .28)
Aftertax cash flows = $144,000

2^ND BGN 2^nd SET
Enter 30 7% $144,000
Solve for $1,786,901.93

$1,786,901.93 + 530,000 = $2,316,901.93

56. Enter 5 × 12 7.2% / 12 $29,000
Solve for $576.98

Enter 34 7.2% / 12 $576.98
Solve for $17,697.79

Total payment = Amount due(1 + Prepayment penalty) + Last payment
Total payment = $17,697.79(1 + .01) + $576.98
Total payment = $18,451.74
57. Pre-retirement APR:

Enter  NOM  11%  EFF  12

Solve for  10.48%

Post-retirement APR:

Enter  NOM  8%  EFF  12

Solve for  7.72%

At retirement, he needs:

Enter  240  7.72% / 12  PV  $23,000  $1,000,000

Solve for  N  I/Y  PMT  FV

In 10 years, his savings will be worth:

Enter  120  10.48% / 12  PV  $2,100

Solve for  N  I/Y  PMT  FV

After purchasing the cabin, he will have: $442,239.69 – 320,000 = $122,239.69

Each month between years 10 and 30, he needs to save:

Enter  240  10.48% / 12 $122,239.69± $3,022,336.00

Solve for  N  I/Y  PMT  FV

–$2,519.10

58. PV of purchase:

Enter  36  6% / 12  PV  $20,000

Solve for  N  I/Y  PMT  FV

$31,000 – 16,712.90 = $14,287.10

PV of lease:

Enter  36  6% / 12  PV  $405

Solve for  N  I/Y  PMT  FV

$13,312.76 + 1,500 = $14,812.76

Buy the car.
You would be indifferent when the PV of the two cash flows are equal. The present value of the purchase decision must be $14,812.76. Since the difference in the two cash flows is $31,000 – $14,812.76 = $16,187.24, this must be the present value of the future resale price of the car. The break-even resale price of the car is:

Enter 36 6% / 12 $16,187.24
Solve for PMT FV $19,370.95

59.
Enter 5% 365
Solve for NOM EFF C/Y 5.13%

\[
\begin{align*}
\text{CF}_0 & = 8,500,000 \\
\text{C}_1 & = 3,900,000 \\
\text{F}_1 & = 1 \\
\text{C}_2 & = 4,600,000 \\
\text{F}_2 & = 1 \\
\text{C}_3 & = 5,300,000 \\
\text{F}_3 & = 1 \\
\text{C}_4 & = 5,800,000 \\
\text{F}_4 & = 1 \\
\text{C}_5 & = 6,400,000 \\
\text{F}_5 & = 1 \\
\text{C}_6 & = 7,300,000 \\
\text{F}_6 & = 1
\end{align*}
\]

\[I = 5.13\%\]

\[\text{NPV CPT} = 36,075,085.12\]

\[\text{New contract value} = 36,075,085.12 + 1,500,000 = 37,575,085.12\]

\[\text{PV of payments} = 37,575,085.12 – 10,000,000 = 27,575,085.12\]

\[\text{Effective quarterly rate} = (1 + (0.05/365))^{91.25} – 1 = 1.258\%\]

Enter 24 1.258% $27,575,085.12
Solve for PMT FV $1,338,243.52

60.
Enter 1 17.65% $17,000
Solve for PMT FV ±$20,000

61.
Enter 8.65%
Solve for EFF C/Y
Enter 12 8.65% / 12 $37,000 / 12
Solve for $38,502.50

Enter 1 9% $38,502.50
Solve for $41,967.73

Enter 12 8.65% / 12 $39,000 / 12
Solve for $40,583.72

Enter 60 8.65% / 12 $43,000 / 12
Solve for $174,046.93

Award = $41,967.73 + 40,583.72 + 174,046.93 + 150,000 + 25,000 = $431,598.39

62.
Enter 1 $9,700 ± $10,800
Solve for 11.34%

Enter 1 $9,800 ± $11,100
Solve for 13.27%

63. Refundable fee: With the $2,400 application fee, you will need to borrow $202,400 to have $200,000 after deducting the fee. Solve for the payment under these circumstances.
Enter 30 × 12 5.39% / 12 $202,400
Solve for $1,123.94

Enter 30 × 12 $200,000 ± $1,123.94
Solve for 0.4506%

APR = 0.4506% × 12 = 5.41%
Enter 5.41% NOM
Solve for 5.54% EFF
Without refundable fee: APR = 5.30%

Enter 5.30%  NOM  12
Solve for EFF  C/Y  5.43%

64. Enter 36  N  12  \$1,000  PV  \$45.64  PMT  FV
Solve for EFF  2.98%

APR = 2.98% \times 12 = 35.71%
Enter 35.71%  NOM  12
Solve for EFF  C/Y  42.18%

65. Without fee:
Enter 18.6% / 12  N  12  \$10,000  PMT  \$200  FV
Solve for I/Y  96.98

Enter 8.2% / 12  N  12  \$10,000  PMT  \$200  FV
Solve for I/Y  61.39

With fee:
Enter 8.2% / 12  N  12  \$10,200  PMT  \$200  FV
Solve for I/Y  62.92

66. Value at Year 6:
Enter 5 11%  N  5  \$500  PMT  FV  \$842.53
Solve for I/Y  $500

Enter 4 11%  N  4  \$600  PMT  FV  \$910.84
Solve for I/Y  $600

Enter 3 11%  N  3  \$700  PMT  FV  \$957.34
Solve for I/Y  $700
Enter 2 11% $800
Solve for $985.68

Enter 1 11% $900
Solve for $999.00

So, at Year 6, the value is: $842.53 + 910.84 + 957.34 + 985.68 + 999.00 + 1,000 = $5,695.39

At Year 65, the value is:
Enter 59 7% $5,695.39
Solve for $308,437.08

The policy is not worth buying; the future value of the payments is $308,437.08 but the policy contract will pay off $275,000.

67. Effective six-month rate = (1 + Daily rate)\(^{180} - 1\)
Effective six-month rate = (1 + .09/360)\(^{180} - 1\)
Effective six-month rate = .0460 or 4.60%
Enter 40 4.60% $1,250,000
Solve for $22,670,253.86

Enter 1 4.60% $22,670,253.86
Solve for $21,672,827.50

Value of winnings today = $22,670,253.86 + 2,500,000
Value of winnings today = $24,172,827.50

68.

\[
\begin{array}{l}
\text{CF}_0 \pm11,000 \\
\text{CF}_1 \pm11,000 \\
\text{F01} 5 \\
\text{CF}_2 \pm25,000 \\
\text{F02} 4 \\
\end{array}
\]

IRR CPT 8.54%
72.

a. \( \text{APR} = 7\% \times 52 = 364\% \)

Enter \( 364\% \) \\
Solve for \( \text{NOM} \) \( \text{EFF} \) \( \text{C/Y} \) \( 3,272.53\% \)

b. Enter \( 1 \) \( \text{N} \) \( \text{I/Y} \) \( 9.30 \) \( \text{PV} \) \( \pm 10.00 \) \( \text{PMT} \) \( \text{FV} \) \\
Solve for \( 7.53\% \)

\[ \text{APR} = 7.53\% \times 52 = 391.40\% \]

Enter \( 391.40\% \) \( \text{NOM} \) \( \text{EFF} \) \( \text{C/Y} \) \\
Solve for \( 4,253.98\% \)

c. Enter \( 4 \) \( \text{N} \) \( \text{I/Y} \) \( 68.92 \) \( \text{PV} \) \( \pm 25 \) \( \text{PMT} \) \( \text{FV} \) \\
Solve for \( 16.75\% \)

\[ \text{APR} = 16.75\% \times 52 = 871.00\% \]

Enter \( 871.00\% \) \( \text{NOM} \) \( \text{EFF} \) \( \text{C/Y} \) \\
Solve for \( 314,215.72\% \)
CHAPTER 5
NET PRESENT VALUE AND OTHER INVESTMENT RULES

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

Project A:

Cumulative cash flows Year 1 = $9,500 = $9,500
Cumulative cash flows Year 2 = $9,500 + 6,000 = $15,500

Companies can calculate a more precise value using fractional years. To calculate the fractional payback period, find the fraction of year 2’s cash flows that is needed for the company to have cumulative undiscounted cash flows of $15,000. Divide the difference between the initial investment and the cumulative undiscounted cash flows as of year 1 by the undiscounted cash flow of year 2.

Payback period = 1 + ($15,000 – 9,500) / $6,000
Payback period = 1.917 years

Project B:

Cumulative cash flows Year 1 = $10,500 = $10,500
Cumulative cash flows Year 2 = $10,500 + 7,000 = $17,500
Cumulative cash flows Year 3 = $10,500 + 7,000 + 6,000 = $23,500

To calculate the fractional payback period, find the fraction of year 3’s cash flows that is needed for the company to have cumulative undiscounted cash flows of $18,000. Divide the difference between the initial investment and the cumulative undiscounted cash flows as of year 2 by the undiscounted cash flow of year 3.

Payback period = 2 + ($18,000 – 10,500 – 7,000) / $6,000
Payback period = 2.083 years

Since project A has a shorter payback period than project B has, the company should choose project A.
b. Discount each project’s cash flows at 15 percent. Choose the project with the highest NPV.

Project A:
NPV = –$15,000 + $9,500 / 1.15 + $6,000 / 1.15^2 + $2,400 / 1.15^3
NPV = –$624.23

Project B:
NPV = –$18,000 + $10,500 / 1.15 + $7,000 / 1.15^2 + $6,000 / 1.15^3
NPV = $368.54

The firm should choose Project B since it has a higher NPV than Project A has.

2. To calculate the payback period, we need to find the time that the project has taken to recover its initial investment. The cash flows in this problem are an annuity, so the calculation is simpler. If the initial cost is $3,200, the payback period is:

Payback = 3 + ($680 / $840) = 3.81 years

There is a shortcut to calculate the payback period if the future cash flows are an annuity. Just divide the initial cost by the annual cash flow. For the $3,200 cost, the payback period is:

Payback = $3,200 / $840 = 3.81 years

For an initial cost of $4,800, the payback period is:

Payback = $4,800 / $840 = 5.71 years

The payback period for an initial cost of $7,300 is a little trickier. Notice that the total cash inflows after eight years will be:

Total cash inflows = 8($840) = $6,720

If the initial cost is $7,300, the project never pays back. Notice that if you use the shortcut for annuity cash flows, you get:

Payback = $7,300 / $840 = 8.69 years

This answer does not make sense since the cash flows stop after eight years, so there is no payback period.

3. When we use discounted payback, we need to find the value of all cash flows today. The value today of the project cash flows for the first four years is:

Value today of Year 1 cash flow = $5,000/1.14 = $4,385.96
Value today of Year 2 cash flow = $5,500/1.14^2 = $4,232.07
Value today of Year 3 cash flow = $6,000/1.14^3 = $4,049.83
Value today of Year 4 cash flow = $7,000/1.14^4 = $4,144.56

To find the discounted payback, we use these values to find the payback period. The discounted first year cash flow is $4,385.96, so the discounted payback for an initial cost of $8,000 is:
Discounted payback = 1 + ($8,000 – 4,385.96)/$4,232.07 = 1.85 years

For an initial cost of $12,000, the discounted payback is:

Discounted payback = 2 + ($12,000 – 4,385.96 – 4,232.07)/$4,049.83 = 2.84 years

Notice the calculation of discounted payback. We know the payback period is between two and three years, so we subtract the discounted values of the Year 1 and Year 2 cash flows from the initial cost. This is the numerator, which is the discounted amount we still need to make to recover our initial investment. We divide this amount by the discounted amount we will earn in Year 3 to get the fractional portion of the discounted payback.

If the initial cost is $16,000, the discounted payback is:

Discounted payback = 3 + ($16,000 – 4,385.96 – 4,232.07 – 4,049.83) / $4,144.56 = 3.80 years

4. To calculate the discounted payback, discount all future cash flows back to the present, and use these discounted cash flows to calculate the payback period. To find the fractional year, we divide the amount we need to make in the last year to payback the project by the amount we will make. Doing so, we find:

- \( r = 0\% \):
  \[
  3 + \frac{($3,600)}{($3,800)} = 3.95 \text{ years}
  \]
  Discounted payback = Regular payback = 3.95 years

- \( r = 10\% \):
  \[
  \frac{$3,800}{1.10} + \frac{$3,800}{1.10^2} + \frac{$3,800}{1.10^3} + \frac{$3,800}{1.10^4} + \frac{$3,800}{1.10^5} = $14,040.99
  \]
  \[
  \frac{$3,800}{1.10^6} = $2,145.00
  \]
  Discounted payback = 5 + \frac{($15,000 – 14,040.99)}{2,145.00} = 5.28 years

- \( r = 15\% \):
  \[
  \frac{$3,800}{1.15} + \frac{$3,800}{1.15^2} + \frac{$3,800}{1.15^3} + \frac{$3,800}{1.15^4} + \frac{$3,800}{1.15^5} + \frac{$3,800}{1.15^6} = $14,381.03
  \]
  The project never pays back.

5. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

\[
0 = C_0 + C_1 / (1 + IRR) + C_2 / (1 + IRR)^2 + C_3 / (1 + IRR)^3
\]

\[
0 = -$20,000 + $8,500/(1 + IRR) + $10,200/(1 + IRR)^2 + $6,200/(1 + IRR)^3
\]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 12.41%

Since the IRR is greater than the required return we would accept the project.

6. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this Project A is:

\[
0 = C_0 + C_1 / (1 + IRR) + C_2 / (1 + IRR)^2 + C_3 / (1 + IRR)^3
\]

\[
0 = -$5,300 + $2,000/(1 + IRR) + $2,800/(1 + IRR)^2 + $1,600/(1 + IRR)^3
\]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
IRR = 10.38%

And the IRR for Project B is:

\[ 0 = C_0 + C_1 / (1 + IRR) + C_2 / (1 + IRR)^2 + C_3 / (1 + IRR)^3 \]
\[ 0 = -2,900 + 1,100/(1 + IRR) + 1,800/(1 + IRR)^2 + 1,200/(1 + IRR)^3 \]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 19.16%

7. The profitability index is defined as the PV of the cash inflows divided by the PV of the cash outflows. The cash flows from this project are an annuity, so the equation for the profitability index is:

\[ PI = C(PVIFA_{R%, t}) / C_0 \]
\[ PI = $84,000(PVIFA_{13%, 7}) / $385,000 \]
\[ PI = 0.965 \]

8. a. The profitability index is the present value of the future cash flows divided by the initial cost. So, for Project Alpha, the profitability index is:

\[ PI_{\text{Alpha}} = \left[ \frac{1,200}{1.10} + \frac{1,100}{1.10^2} + \frac{900}{1.10^3} \right] / 2,300 = 1.164 \]

And for Project Beta the profitability index is:

\[ PI_{\text{Beta}} = \left[ \frac{800}{1.10} + \frac{2,300}{1.10^2} + \frac{2,900}{1.10^3} \right] / 3,900 = 1.233 \]

b. According to the profitability index, you would accept Project Beta. However, remember the profitability index rule can lead to an incorrect decision when ranking mutually exclusive projects.

Intermediate

9. a. To have a payback equal to the project’s life, given C is a constant cash flow for N years:

\[ C = I/N \]

b. To have a positive NPV, I < C (PVIFA_{R%, N}). Thus, C > I / (PVIFA_{R%, N}).

c. Benefit = C (PVIFA_{R%, N}) = 2 × costs = 2I

\[ C = 2I / (PVIFA_{R%, N}) \]

10. a. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

\[ 0 = C_0 + C_1 / (1 + IRR) + C_2 / (1 + IRR)^2 + C_3 / (1 + IRR)^3 + C_4 / (1 + IRR)^4 \]
\[ 0 = 7,000 – 3,700 / (1 + IRR) – 2,400 / (1 + IRR)^2 – 1,500 / (1 + IRR)^3 – 1,200 / (1 + IRR)^4 \]
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

\[ \text{IRR} = 12.40\% \]

b. This problem differs from previous ones because the initial cash flow is positive and all future cash flows are negative. In other words, this is a financing-type project, while previous projects were investing-type projects. For financing situations, accept the project when the IRR is less than the discount rate. Reject the project when the IRR is greater than the discount rate.

\[
\begin{align*}
\text{IRR} &= 12.40\% \\
\text{Discount Rate} &= 10\% \\
\text{IRR} &> \text{Discount Rate}
\end{align*}
\]

Reject the offer when the discount rate is less than the IRR.

c. Using the same reason as part b., we would accept the project if the discount rate is 20 percent.

\[
\begin{align*}
\text{IRR} &= 12.40\% \\
\text{Discount Rate} &= 20\% \\
\text{IRR} &< \text{Discount Rate}
\end{align*}
\]

Accept the offer when the discount rate is greater than the IRR.

d. The NPV is the sum of the present value of all cash flows, so the NPV of the project if the discount rate is 10 percent will be:

\[
\begin{align*}
\text{NPV} &= \$7,000 – \$3,700 \div 1.1 – \$2,400 \div 1.1^2 – \$1,500 \div 1.1^3 – \$1,200 \div 1.1^4 \\
\text{NPV} &= –\$293.70
\end{align*}
\]

When the discount rate is 10 percent, the NPV of the offer is –$293.70. Reject the offer.

And the NPV of the project if the discount rate is 20 percent will be:

\[
\begin{align*}
\text{NPV} &= \$7,000 – \$3,700 \div 1.2 – \$2,400 \div 1.2^2 – \$1,500 \div 1.2^3 – \$1,200 \div 1.2^4 \\
\text{NPV} &= $803.24
\end{align*}
\]

When the discount rate is 20 percent, the NPV of the offer is $803.24. Accept the offer.

e. Yes, the decisions under the NPV rule are consistent with the choices made under the IRR rule since the signs of the cash flows change only once.

11. a. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR for each project is:

Deepwater Fishing IRR:

\[
\begin{align*}
0 &= C_0 + C_1 / (1 + \text{IRR}) + C_2 / (1 + \text{IRR})^2 + C_3 / (1 + \text{IRR})^3 \\
0 &= –\$950,000 + \$370,000 / (1 + \text{IRR}) + \$510,000 / (1 + \text{IRR})^2 + \$420,000 / (1 + \text{IRR})^3
\end{align*}
\]
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

**IRR = 17.07%**

**Submarine Ride IRR:**

\[
0 = C_0 + C_1 / (1 + IRR) + C_2 / (1 + IRR)^2 + C_3 / (1 + IRR)^3
\]

\[
0 = -1,850,000 + 900,000 / (1 + IRR) + 800,000 / (1 + IRR)^2 + 750,000 / (1 + IRR)^3
\]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

**IRR = 16.03%**

Based on the IRR rule, the deepwater fishing project should be chosen because it has the higher IRR.

**b.** To calculate the incremental IRR, we subtract the smaller project’s cash flows from the larger project’s cash flows. In this case, we subtract the deepwater fishing cash flows from the submarine ride cash flows. The incremental IRR is the IRR of these incremental cash flows. So, the incremental cash flows of the submarine ride are:

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Submarine Ride</td>
<td>–$1,850,000</td>
<td>$900,000</td>
<td>$800,000</td>
</tr>
<tr>
<td>Deepwater Fishing</td>
<td>–950,000</td>
<td>370,000</td>
<td>510,000</td>
</tr>
<tr>
<td>Submarine – Fishing</td>
<td>–$900,000</td>
<td>$530,000</td>
<td>$290,000</td>
</tr>
</tbody>
</table>

Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

\[
0 = C_0 + C_1 / (1 + IRR) + C_2 / (1 + IRR)^2 + C_3 / (1 + IRR)^3
\]

\[
0 = -900,000 + 530,000 / (1 + IRR) + 290,000 / (1 + IRR)^2 + 330,000 / (1 + IRR)^3
\]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

**Incremental IRR = 14.79%**

For investing-type projects, accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 14.79 percent, is greater than the required rate of return of 14 percent, choose the submarine ride project. Note that this is not the choice when evaluating only the IRR of each project. The IRR decision rule is flawed because there is a scale problem. That is, the submarine ride has a greater initial investment than does the deepwater fishing project. This problem is corrected by calculating the IRR of the incremental cash flows, or by evaluating the NPV of each project.

**c.** The NPV is the sum of the present value of the cash flows from the project, so the NPV of each project will be:
Deepwater fishing:

\[ \text{NPV} = -\$950,000 + \frac{\$370,000}{1.14} + \frac{\$510,000}{1.14^2} + \frac{\$420,000}{1.14^3} \]

\[ \text{NPV} = \$50,477.88 \]

Submarine ride:

\[ \text{NPV} = -\$1,850,000 + \frac{\$900,000}{1.14} + \frac{\$800,000}{1.14^2} + \frac{\$750,000}{1.14^3} \]

\[ \text{NPV} = \$61,276.34 \]

Since the NPV of the submarine ride project is greater than the NPV of the deepwater fishing project, choose the submarine ride project. The incremental IRR rule is always consistent with the NPV rule.

12. a. The profitability index is the PV of the future cash flows divided by the initial investment. The cash flows for both projects are an annuity, so:

\[ \text{PI}_I = \frac{18,000 \times \text{PVIFA}_{10\%,3}}{30,000} = 1.492 \]

\[ \text{PI}_II = \frac{7,500 \times \text{PVIFA}_{10\%,3}}{12,000} = 1.554 \]

The profitability index decision rule implies that we accept project II, since PI$_{II}$ is greater than the PI$_I$.

b. The NPV of each project is:

\[ \text{NPV}_I = -\$30,000 + \frac{18,000 \times \text{PVIFA}_{10\%,3}}{30,000} = \$14,763.34 \]

\[ \text{NPV}_{II} = -\$12,000 + \frac{7,500 \times \text{PVIFA}_{10\%,3}}{12,000} = \$6,651.39 \]

The NPV decision rule implies accepting Project I, since the NPV$_I$ is greater than the NPV$_{II}$.

c. Using the profitability index to compare mutually exclusive projects can be ambiguous when the magnitudes of the cash flows for the two projects are of different scales. In this problem, project I is 2.5 times as large as project II and produces a larger NPV, yet the profitability index criterion implies that project II is more acceptable.

13. a. The equation for the NPV of the project is:

\[ \text{NPV} = -\$85,000,000 + \frac{\$125,000,000}{1.1} - \frac{\$15,000,000}{1.1^2} = \$16,239,669.42 \]

The NPV is greater than 0, so we would accept the project.

b. The equation for the IRR of the project is:

\[ 0 = -\$85,000,000 + \frac{\$125,000,000}{(1+\text{IRR})} - \frac{\$15,000,000}{(1+\text{IRR})^2} \]

From Descartes’ rule of signs, we know there are two IRRs since the cash flows change signs twice. From trial and error, the two IRRs are:

\[ \text{IRR} = 33.88\%, -86.82\% \]
When there are multiple IRRs, the IRR decision rule is ambiguous. Both IRRs are correct; that is, both interest rates make the NPV of the project equal to zero. If we are evaluating whether or not to accept this project, we would not want to use the IRR to make our decision.

14. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

Board game:

Cumulative cash flows Year 1 = $600 = $600
Cumulative cash flows Year 2 = $600 + 450 = $1,050

Payback period = 1 + $150 / $450 = 1.33 years

DVD:

Cumulative cash flows Year 1 = $1,300 = $1,300
Cumulative cash flows Year 2 = $1,300 + 850 = $2,150

Payback period = 1 + ($1,800 – 1,300) / 850
Payback period = 1.59 years

Since the board game has a shorter payback period than the DVD project, the company should choose the board game.

b. The NPV is the sum of the present value of the cash flows from the project, so the NPV of each project will be:

Board game:

NPV = –$750 + $600 / 1.10 + $450 / 1.10^2 + $120 / 1.10^3
NPV = $257.51

DVD:

NPV = –$1,850 + $1,300 / 1.10 + $850 / 1.10^2 + $350 / 1.10^3
NPV = $347.26

Since the NPV of the DVD is greater than the NPV of the board game, choose the DVD.

c. The IRR is the interest rate that makes the NPV of a project equal to zero. So, the IRR of each project is:

Board game:

0 = –$750 + $600 / (1 + IRR) + $450 / (1 + IRR)^2 + $120 / (1 + IRR)^3

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 33.79%
DVD:

\[ 0 = -1,850 + 1,300 / (1 + \text{IRR}) + 850 / (1 + \text{IRR})^2 + 350 / (1 + \text{IRR})^3 \]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

\[ \text{IRR} = 23.31\% \]

Since the IRR of the board game is greater than the IRR of the DVD, IRR implies we choose the board game. Note that this is the choice when evaluating only the IRR of each project. The IRR decision rule is flawed because there is a scale problem. That is, the DVD has a greater initial investment than does the board game. This problem is corrected by calculating the IRR of the incremental cash flows, or by evaluating the NPV of each project.

d. To calculate the incremental IRR, we subtract the smaller project’s cash flows from the larger project’s cash flows. In this case, we subtract the board game cash flows from the DVD cash flows. The incremental IRR is the IRR of these incremental cash flows. So, the incremental cash flows of the DVD are:

<table>
<thead>
<tr>
<th>Year</th>
<th>DVD</th>
<th>Board game</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1,800</td>
<td>-750</td>
</tr>
<tr>
<td>1</td>
<td>1,300</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>850</td>
<td>450</td>
</tr>
<tr>
<td>3</td>
<td>350</td>
<td>120</td>
</tr>
</tbody>
</table>

DVD – Board game

\[ 0 = -1,050 + 700 / (1 + \text{IRR}) + 400 / (1 + \text{IRR})^2 + 230 / (1 + \text{IRR})^3 \]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

\[ \text{Incremental IRR} = 15.86\% \]

For investing-type projects, accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 15.86%, is greater than the required rate of return of 10 percent, choose the DVD project.

15. a. The profitability index is the PV of the future cash flows divided by the initial investment. The profitability index for each project is:

\[
\begin{align*}
\text{PI}_{\text{CDMA}} &= \frac{11,000,000 / 1.10 + 7,500,000 / 1.10^2 + 2,500,000 / 1.10^3}{8,000,000} = 2.26 \\
\text{PI}_{\text{G4}} &= \frac{10,000,000 / 1.10 + 25,000,000 / 1.10^2 + 20,000,000 / 1.10^3}{12,000,000} = 3.73 \\
\text{PI}_{\text{Wi-Fi}} &= \frac{18,000,000 / 1.10 + 32,000,000 / 1.10^2 + 20,000,000 / 1.10^3}{20,000,000} = 2.89
\end{align*}
\]
The profitability index implies we accept the G4 project. Remember this is not necessarily correct because the profitability index does not necessarily rank projects with different initial investments correctly.

b. The NPV of each project is:

\[
\begin{align*}
\text{NPV}_{\text{CDMA}} &= -8,000,000 + 11,000,000 / 1.10 + 7,500,000 / 1.10^2 + 2,500,000 / 1.10^3 \\
&= 10,076,634.11 \\
\text{NPV}_{\text{G4}} &= -12,000,000 + 10,000,000 / 1.10 + 25,000,000 / 1.10^2 + 20,000,000 / 1.10^3 \\
&= 32,778,362.13 \\
\text{NPV}_{\text{Wi-Fi}} &= -20,000,000 + 18,000,000 / 1.10 + 32,000,000 / 1.10^2 + 20,000,000 / 1.10^3 \\
&= 37,836,213.37
\end{align*}
\]

NPV implies we accept the Wi-Fi project since it has the highest NPV. This is the correct decision if the projects are mutually exclusive.

c. We would like to invest in all three projects since each has a positive NPV. If the budget is limited to $20 million, we can only accept the CDMA project and the G4 project, or the Wi-Fi project. NPV is additive across projects and the company. The total NPV of the CDMA project and the G4 project is:

\[
\begin{align*}
\text{NPV}_{\text{CDMA and G4}} &= 10,076,634.11 + 32,778,362.13 \\
&= 42,854,996.24
\end{align*}
\]

This is greater than the Wi-Fi project, so we should accept the CDMA project and the G4 project.

16. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

AZM Mini-SUV:

Cumulative cash flows Year 1 = $320,000 = $320,000
Cumulative cash flows Year 2 = $320,000 + 180,000 = $500,000

Payback period = 1 + $130,000 / $180,000 = 1.72 years

AZF Full-SUV:

Cumulative cash flows Year 1 = $350,000 = $350,000
Cumulative cash flows Year 2 = $350,000 + 420,000 = $770,000
Cumulative cash flows Year 2 = $350,000 + 420,000 + 290,000 = $1,060,000

Payback period = 2 + $30,000 / $290,000 = 2.10 years

Since the AZM has a shorter payback period than the AZF, the company should choose the AZM. Remember the payback period does not necessarily rank projects correctly.

b. The NPV of each project is:

\[
\begin{align*}
\text{NPV}_{\text{AZM}} &= -450,000 + 320,000 / 1.10 + 180,000 / 1.10^2 + 150,000 / 1.10^3
\end{align*}
\]
Solutions Manual

NPV_{AZM} = $102,366.64

NPV_{AZF} = -$800,000 + $350,000 / 1.10 + $420,000 / 1.10^2 + $290,000 / 1.10^3
NPV_{AZF} = $83,170.55

The NPV criteria implies we accept the AZM because it has the highest NPV.

c. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the AZM is:

\[ 0 = -450,000 + 320,000 / (1 + IRR) + 180,000 / (1 + IRR)^2 + 150,000 / (1 + IRR)^3 \]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR_{AZM} = 24.65%

And the IRR of the AZF is:

\[ 0 = -800,000 + 350,000 / (1 + IRR) + 420,000 / (1 + IRR)^2 + 290,000 / (1 + IRR)^3 \]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR_{AZF} = 15.97%

The IRR criteria implies we accept the AZM because it has the highest IRR. Remember the IRR does not necessarily rank projects correctly.

d. Incremental IRR analysis is not necessary. The AZM has the smallest initial investment, and the largest NPV, so it should be accepted.

17. a. The profitability index is the PV of the future cash flows divided by the initial investment. The profitability index for each project is:

\[
\begin{align*}
\text{PI}_A &= \left[\frac{110,000}{1 + 1.12} + \frac{110,000}{(1 + 1.12)^2}\right]/150,000 = 1.24 \\
\text{PI}_B &= \left[\frac{200,000}{1 + 1.12} + \frac{200,000}{(1 + 1.12)^2}\right]/300,000 = 1.13 \\
\text{PI}_C &= \left[\frac{120,000}{1 + 1.12} + \frac{90,000}{(1 + 1.12)^2}\right]/150,000 = 1.19
\end{align*}
\]

b. The NPV of each project is:

\[
\begin{align*}
\text{NPV}_A &= -150,000 + 110,000 / 1.12 + 110,000 / 1.12^2 \\
\text{NPV}_A &= 35,905.61 \\
\text{NPV}_B &= -300,000 + 200,000 / 1.12 + 200,000 / 1.12^2 \\
\text{NPV}_B &= 38,010.20 \\
\text{NPV}_C &= -150,000 + 120,000 / 1.12 + 90,000 / 1.12^2 \\
\text{NPV}_C &= 28,890.31
\end{align*}
\]
c. Accept projects A, B, and C. Since the projects are independent, accept all three projects because the respective profitability index of each is greater than one.

d. Accept Project B. Since the Projects are mutually exclusive, choose the Project with the highest PI, while taking into account the scale of the Project. Because Projects A and C have the same initial investment, the problem of scale does not arise when comparing the profitability indices. Based on the profitability index rule, Project C can be eliminated because its PI is less than the PI of Project A. Because of the problem of scale, we cannot compare the PIs of Projects A and B. However, we can calculate the PI of the incremental cash flows of the two projects, which are:

<table>
<thead>
<tr>
<th>Project</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B – A</td>
<td>-$150,000</td>
<td>$90,000</td>
<td>$90,000</td>
</tr>
</tbody>
</table>

When calculating incremental cash flows, remember to subtract the cash flows of the project with the smaller initial cash outflow from those of the project with the larger initial cash outflow. This procedure insures that the incremental initial cash outflow will be negative. The incremental PI calculation is:

\[
\text{PI}(B – A) = \frac{[$90,000 / 1.12 + $90,000 / 1.12^2]}{-150,000} = 1.014
\]

The company should accept Project B since the PI of the incremental cash flows is greater than one.

e. Remember that the NPV is additive across projects. Since we can spend $450,000, we could take two of the projects. In this case, we should take the two projects with the highest NPVs, which are Project B and Project A.

18. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

Dry Prepeg:

Cumulative cash flows Year 1 = $1,100,000 = $1,100,000
Cumulative cash flows Year 2 = $1,100,000 + 900,000 = $2,000,000

Payback period = 1 + ($600,000/$900,000) = 1.67 years

Solvent Prepeg:

Cumulative cash flows Year 1 = $375,000 = $375,000
Cumulative cash flows Year 2 = $375,000 + 600,000 = $975,000

Payback period = 1 + ($375,000/$600,000) = 1.63 years

Since the solvent prepeg has a shorter payback period than the dry prepeg, the company should choose the solvent prepeg. Remember the payback period does not necessarily rank projects correctly.

b. The NPV of each project is:
NPV_{Dry\ prepeg} = -\$1,700,000 + \$1,100,000 / 1.10 + \$900,000 / 1.10^2 + \$750,000 / 1.10^3
NPV_{Dry\ prepeg} = \$607,287.75

NPV_{Solvent\ prepeg} = -\$750,000 + \$375,000 / 1.10 + \$600,000 / 1.10^2 + \$390,000 / 1.10^3
NPV_{Solvent\ prepeg} = \$379,789.63

The NPV criteria implies accepting the dry prepeg because it has the highest NPV.

c. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the dry prepeg is:

0 = -\$1,700,000 + \$1,100,000 / (1 + IRR) + \$900,000 / (1 + IRR)^2 + \$750,000 / (1 + IRR)^3

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR_{Dry\ prepeg} = 30.90%

And the IRR of the solvent prepeg is:

0 = -\$750,000 + \$375,000 / (1 + IRR) + \$600,000 / (1 + IRR)^2 + \$390,000 / (1 + IRR)^3

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR_{Solvent\ prepeg} = 36.51%

The IRR criteria implies accepting the solvent prepeg because it has the highest IRR. Remember the IRR does not necessarily rank projects correctly.

d. Incremental IRR analysis is necessary. The solvent prepeg has a higher IRR, but is relatively smaller in terms of investment and NPV. In calculating the incremental cash flows, we subtract the cash flows from the project with the smaller initial investment from the cash flows of the project with the large initial investment, so the incremental cash flows are:

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry prepeg</td>
<td>-$1,700,000</td>
<td>$1,100,000</td>
<td>$900,000</td>
</tr>
<tr>
<td>Solvent prepeg</td>
<td>-$750,000</td>
<td>$375,000</td>
<td>$600,000</td>
</tr>
<tr>
<td>Dry prepeg – Solvent prepeg</td>
<td>-$950,000</td>
<td>$725,000</td>
<td>$300,000</td>
</tr>
</tbody>
</table>

Setting the present value of these incremental cash flows equal to zero, we find the incremental IRR is:

0 = -\$950,000 + \$725,000 / (1 + IRR) + \$300,000 / (1 + IRR)^2 + \$360,000 / (1 + IRR)^3

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

Incremental IRR = 25.52%
For investing-type projects, we accept the larger project when the incremental IRR is greater than the discount rate. Since the incremental IRR, 25.52%, is greater than the required rate of return of 10 percent, we choose the dry prepeg.

19. a. The payback period is the time that it takes for the cumulative undiscounted cash inflows to equal the initial investment.

NP-30:

Cumulative cash flows Year 1 = $185,000 = $185,000
Cumulative cash flows Year 2 = $185,000 + 185,000 = $370,000
Cumulative cash flows Year 3 = $185,000 + 185,000 + 185,000 = $555,000

Payback period = 2 + ($180,000/$185,000) = 2.97 years

NX-20:

Cumulative cash flows Year 1 = $100,000 = $100,000
Cumulative cash flows Year 2 = $100,000 + 110,000 = $210,000
Cumulative cash flows Year 3 = $100,000 + 110,000 + 121,000 = $331,000
Cumulative cash flows Year 4 = $100,000 + 110,000 + 121,000 + 133,100 = $464,100

Payback period = 3 + ($19,000/$133,100) = 3.14 years

Since the NP-30 has a shorter payback period than the NX-20, the company should choose the NP-30. Remember the payback period does not necessarily rank projects correctly.

b. The IRR is the interest rate that makes the NPV of the project equal to zero, so the IRR of each project is:

NP-30:

$$0 = -550,000 + 185,000 \left\{1 - \left[\frac{1}{(1 + IRR)^5}\right]\right\} / IRR$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$IRR_{NP-30} = 20.27\%$$

And the IRR of the NX-20 is:

$$0 = -350,000 + 100,000 / (1 + IRR) + 110,000 / (1 + IRR)^2 + 121,000 / (1 + IRR)^3 + 133,100 / (1 + IRR)^4 + 146,410 / (1 + IRR)^5$$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

\[ \text{IRR}_{\text{NX-20}} = 20.34\% \]

The IRR criteria implies accepting the NX-20.

c. The profitability index is the present value of all subsequent cash flows, divided by the initial investment, so the profitability index of each project is:

\[ \text{PI}_{\text{NP-30}} = \frac{\left(185,000\left[1 - \left(\frac{1}{1.15}\right)^5\right] / .15\right)}{550,000} = 1.128 \]

\[ \text{PI}_{\text{NX-20}} = \frac{100,000 / 1.15 + 110,000 / 1.15^2 + 121,000 / 1.15^3 + 133,100 / 1.15^4 + 146,410 / 1.15^5}{350,000} = 1.139 \]

The PI criteria implies accepting the NX-20.

d. The NPV of each project is:

\[ \text{NPV}_{\text{NP-30}} = -550,000 + 185,000\left[1 - \left(\frac{1}{1.15}\right)^5\right] / .15 = 70,148.69 \]

\[ \text{NPV}_{\text{NX-20}} = -350,000 + 100,000 / 1.15 + 110,000 / 1.15^2 + 121,000 / 1.15^3 + 133,100 / 1.15^4 + 146,410 / 1.15^5 = 48,583.79 \]

The NPV criteria implies accepting the NP-30.

**Challenge**

20. The equation for the IRR of the project is:

\[ 0 = -75,000 + 155,000/(1+\text{IRR}) - 65,000/(1+\text{IRR})^2 \]

From Descartes’ Rule of Signs, we know there are either zero IRRs or two IRRs since the cash flows change signs twice. We can rewrite this equation as:

\[ 0 = -75,000 + 155,000X - 65,000X^2 \]

where \( X = 1 / (1 + \text{IRR}) \)

This is a quadratic equation. We can solve for the roots of this equation with the quadratic formula:

\[ X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Remember that the quadratic formula is written as:

\[ 0 = aX^2 + bX + c \]

In this case, the equation is:

\[ 0 = -65,000X^2 + 155,000X - 75,000 \]

\[
X = \frac{-155,000 \pm \sqrt{(155,000)^2 - 4(-75,000)(-65,000)}}{2(-65,000)}
\]

\[
X = \frac{-155,000 \pm \sqrt{4,525,000,000}}{2(-65,000)}
\]

\[
X = \frac{-155,000 \pm 67,268.12}{-130,000}
\]

Solving the quadratic equation, we find two Xs:

\[ X = 0.6749, 1.7098 \]

Since:

\[ X = \frac{1}{1 + IRR} \]

\[ 1.7098 = \frac{1}{1 + IRR} \]

\[ IRR = -.4151, \text{ or } -41.51\% \]

And:

\[ X = \frac{1}{1 + IRR} \]

\[ 0.6749 = \frac{1}{1 + IRR} \]

\[ IRR = 0.4818, \text{ or } 48.18\% \]

To find the maximum (or minimum) of a function, we find the derivative and set it equal to zero. The derivative of this IRR function is:

\[ 0 = -155,000(1 + IRR)^{-2} + 130,000(1 + IRR)^{-3} \]

\[ -155,000(1 + IRR)^{-2} = 130,000(1 + IRR)^{-3} \]

\[ -155,000(1 + IRR)^3 = 130,000(1 + IRR)^2 \]

\[ -155,000(1 + IRR) = 130,000 \]

\[ IRR = \frac{130,000}{155,000} - 1 \]

\[ IRR = -.1613, \text{ or } -16.13\% \]
To determine if this is a maximum or minimum, we can find the second derivative of the IRR function. If the second derivative is positive, we have found a minimum and if the second derivative is negative we have found a maximum. Using the reduced equation above, that is:

\[-155,000(1 + \text{IRR}) = 130,000\]

The second derivative is \(-262,722.18\), therefore we have a maximum.

21. Given the six-year payback, the worst case is that the payback occurs at the end of the sixth year. Thus, the worst case:

\[\text{NPV} = -434,000 + 434,000/1.12^6 = -$214,122.09\]

The best case has infinite cash flows beyond the payback point. Thus, the best-case NPV is infinite.

22. The equation for the IRR of the project is:

\[0 = -1,008 + 5,724/(1 + \text{IRR}) - 12,140/(1 + \text{IRR})^2 + 11,400/(1 + \text{IRR})^3 - 4,000/(1 + \text{IRR})^4\]

Using Descartes’ rule of signs, from looking at the cash flows we know there are four IRRs for this project. Even with most computer spreadsheets, we have to do some trial and error. From trial and error, IRRs of 25%, 33.33%, 42.86%, and 66.67% are found.

We would accept the project when the NPV is greater than zero. See for yourself that the NPV is greater than zero for required returns between 25% and 33.33% or between 42.86% and 66.67%.

23. a. Here the cash inflows of the project go on forever, which is a perpetuity. Unlike ordinary perpetuity cash flows, the cash flows here grow at a constant rate forever, which is a growing perpetuity. The PV of the future cash flows from the project is:

\[\text{PV of cash inflows} = \frac{C_i}{R - g}\]

\[\text{PV of cash inflows} = \frac{290,000}{.11 - .05} = 4,833,333.33\]

NPV is the PV of the outflows minus by the PV of the inflows, so the NPV is:

\[\text{NPV of the project} = -3,900,000 + 4,833,333.33 = 933,333.33\]

The NPV is positive, so we would accept the project.

b. Here we want to know the minimum growth rate in cash flows necessary to accept the project. The minimum growth rate is the growth rate at which we would have a zero NPV. The equation for a zero NPV, using the equation for the PV of a growing perpetuity is:

\[0 = -3,900,000 + 290,000/(.11 - g)\]

Solving for \(g\), we get:

\[g = 3.56\%\]
24. a. The project involves three cash flows: the initial investment, the annual cash inflows, and the abandonment costs. The mine will generate cash inflows over its 11-year economic life. To express the PV of the annual cash inflows, apply the growing annuity formula, discounted at the IRR and growing at eight percent.

\[
PV(\text{Cash Inflows}) = C \left\{ \frac{1}{(r - g)} - \frac{1}{(r - g)} \times \frac{(1 + g)(1 + r)}{1 + IRR} \right\}^{11}
\]

At the end of 11 years, the company will abandon the mine, incurring a $400,000 charge. Discounting the abandonment costs back 11 years at the IRR to express its present value, we get:

\[
PV(\text{Abandonment}) = \frac{C_{11}}{(1 + IRR)^{11}}
\]

So, the IRR equation for this project is:

\[
0 = -\$2,400,000 + \frac{\$345,000}{(1 + IRR)^{11}} - \frac{\$400,000}{(1 + IRR)^{11}}
\]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

\[
IRR = 14.74\%
\]

b. Yes. Since the mine’s IRR exceeds the required return of 10 percent, the mine should be opened. The correct decision rule for an investment-type project is to accept the project if the IRR is greater than the discount rate. Although it appears there is a sign change at the end of the project because of the abandonment costs, the last cash flow is actually positive because of the operating cash flow in the last year.

25. First, we need to find the future value of the cash flows for the one year in which they are blocked by the government. So, reinvesting each cash inflow for one year, we find:

\[
\begin{align*}
\text{Year 2 cash flow} &= \$285,000(1.04) = \$296,400 \\
\text{Year 3 cash flow} &= \$345,000(1.04) = \$358,800 \\
\text{Year 4 cash flow} &= \$415,000(1.04) = \$431,600 \\
\text{Year 5 cash flow} &= \$255,000(1.04) = \$265,200
\end{align*}
\]

So, the NPV of the project is:

\[
\text{NPV} = -\$950,000 + \frac{\$296,400}{1.11} + \frac{\$358,800}{1.11^2} + \frac{\$431,600}{1.11^3} + \frac{\$265,200}{1.11^4} + \frac{\$265,200}{1.11^5}
\]

\[
\text{NPV} = -\$5,392.06
\]
And the IRR of the project is:

\[
0 = -950,000 + 296,400/(1 + IRR)^2 + 358,800/(1 + IRR)^3 + 431,600/(1 + IRR)^4 + 265,200/(1 + IRR)^5
\]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

\[
IRR = 10.81\%
\]

While this may look like a MIRR calculation, it is not a MIRR, rather it is a standard IRR calculation. Since the cash inflows are blocked by the government, they are not available to the company for a period of one year. Thus, all we are doing is calculating the IRR based on when the cash flows actually occur for the company.

26. **a.** We can apply the growing perpetuity formula to find the PV of stream \( A \). The perpetuity formula values the stream as of one year before the first payment. Therefore, the growing perpetuity formula values the stream of cash flows as of year 2. Next, discount the PV as of the end of year 2 back two years to find the PV as of today, year 0. Doing so, we find:

\[
PV(A) = \frac{C_3}{r - g} \div (1 + r)^2
\]

\[
PV(A) = \frac{8,900}{0.12 - 0.04} \div (1.12)^2
\]

\[
PV(A) = 88,687.82
\]

We can apply the perpetuity formula to find the PV of stream \( B \). The perpetuity formula discounts the stream back to year 1, one period prior to the first cash flow. Discount the PV as of the end of year 1 back one year to find the PV as of today, year 0. Doing so, we find:

\[
PV(B) = \frac{C_2}{r} \div (1 + r)
\]

\[
PV(B) = \frac{-10,000}{0.12} \div (1.12)
\]

\[
PV(B) = -74,404.76
\]

**b.** If we combine the cash flow streams to form Project C, we get:

Project A = \( C_3 / (r - g) \div (1 + r)^2 \)

Project B = \( C_2 / r \div (1 + r) \)

Project C = Project A + Project B

Project C = \( C_3 / (r - g) \div (1 + r)^2 + C_2 / r \div (1 + r) \)

\[
0 = \frac{8,900}{(IRR - .04)} \div (1 + IRR)^2 + \frac{-10,000}{IRR} \div (1 + IRR)
\]

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

\[
IRR = 16.80\%
\]

**c.** The correct decision rule for an investing-type project is to accept the project if the discount rate is below the IRR. Since there is one IRR, a decision can be made. At a point in the future, the cash flows from stream \( A \) will be greater than those from stream \( B \). Therefore, although there are many cash flows, there will be only one change in sign. When the sign of the cash flows change more than once over the life of the project, there may be multiple internal rates of return.
In such cases, there is no correct decision rule for accepting and rejecting projects using the internal rate of return.

27. To answer this question, we need to examine the incremental cash flows. To make the projects equally attractive, Project Billion must have a larger initial investment. We know this because the subsequent cash flows from Project Billion are larger than the subsequent cash flows from Project Million. So, subtracting the Project Million cash flows from the Project Billion cash flows, we find the incremental cash flows are:

<table>
<thead>
<tr>
<th>Year</th>
<th>Incremental cash flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1,200</td>
</tr>
<tr>
<td>1</td>
<td>240</td>
</tr>
<tr>
<td>2</td>
<td>240</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
</tr>
</tbody>
</table>

Now we can find the present value of the subsequent incremental cash flows at the discount rate, 12 percent. The present value of the incremental cash flows is:

\[ PV = 1,200 + 240 / 1.12 + 240 / 1.12^2 + 400 / 1.12^3 \]
\[ PV = 1,890.32 \]

So, if \( I_0 \) is greater than $1,890.32, the incremental cash flows will be negative. Since we are subtracting Project Million from Project Billion, this implies that for any value over $1,890.32 the NPV of Project Billion will be less than that of Project Million, so \( I_0 \) must be less than $1,890.32.

28. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the project is:

\[ 0 = 20,000 - 26,000 / (1 + IRR) + 13,000 / (1 + IRR)^2 \]

Even though it appears there are two IRRs, a spreadsheet, financial calculator, or trial and error will not give an answer. The reason is that there is no real IRR for this set of cash flows. If you examine the IRR equation, what we are really doing is solving for the roots of the equation. Going back to high school algebra, in this problem we are solving a quadratic equation. In case you don’t remember, the quadratic equation is:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

In this case, the equation is:

\[ x = \frac{-(-26,000) \pm \sqrt{(-26,000)^2 - 4(20,000)(13,000)}}{2(20,000)} \]

The square root term works out to be:

\[ 676,000,000 - 1,040,000,000 = -364,000,000 \]
The square root of a negative number is a complex number, so there is no real number solution, meaning the project has no real IRR.

Calculator Solutions

1. \(b\).

<table>
<thead>
<tr>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF(_0)</td>
<td>(-$15,000)</td>
</tr>
<tr>
<td>C(01)</td>
<td>$9,500</td>
</tr>
<tr>
<td>F(01)</td>
<td>1</td>
</tr>
<tr>
<td>C(02)</td>
<td>$6,000</td>
</tr>
<tr>
<td>F(02)</td>
<td>1</td>
</tr>
<tr>
<td>C(03)</td>
<td>$2,400</td>
</tr>
<tr>
<td>F(03)</td>
<td>1</td>
</tr>
<tr>
<td>I = 15%</td>
<td></td>
</tr>
<tr>
<td>NPV CPT</td>
<td>$-624.23</td>
</tr>
</tbody>
</table>

5.

<table>
<thead>
<tr>
<th>CF(_0)</th>
<th>(-$20,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(01)</td>
<td>$8,500</td>
</tr>
<tr>
<td>F(01)</td>
<td>1</td>
</tr>
<tr>
<td>C(02)</td>
<td>$10,200</td>
</tr>
<tr>
<td>F(02)</td>
<td>1</td>
</tr>
<tr>
<td>C(03)</td>
<td>$6,200</td>
</tr>
<tr>
<td>F(03)</td>
<td>1</td>
</tr>
<tr>
<td>IRR CPT</td>
<td>12.41%</td>
</tr>
</tbody>
</table>

6. \(\textit{Project A} \quad \textit{Project B}\)

<table>
<thead>
<tr>
<th>CF(_0)</th>
<th>(-$5,300)</th>
<th>CF(_0)</th>
<th>(-$2,900)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(01)</td>
<td>$2,000</td>
<td>C(01)</td>
<td>$1,100</td>
</tr>
<tr>
<td>F(01)</td>
<td>1</td>
<td>F(01)</td>
<td>1</td>
</tr>
<tr>
<td>C(02)</td>
<td>$2,800</td>
<td>C(02)</td>
<td>$1,800</td>
</tr>
<tr>
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<td>F(02)</td>
<td>1</td>
</tr>
<tr>
<td>C(03)</td>
<td>$1,600</td>
<td>C(03)</td>
<td>$1,200</td>
</tr>
<tr>
<td>F(03)</td>
<td>1</td>
<td>F(03)</td>
<td>1</td>
</tr>
<tr>
<td>IRR CPT</td>
<td>10.38%</td>
<td>IRR CPT</td>
<td>19.16%</td>
</tr>
</tbody>
</table>
Solutions Manual

7. $
\begin{array}{l}
\text{CF}_0 & 0 \\
\text{C}_01 & $84,000 \\
\text{F}_01 & 7 \\
\end{array}
$

$I = 13\%$

NPV CPT

$\$371,499.28$

$PI = \$371,499.28 / \$385,000 = 0.965$

10. $
\begin{array}{l}
\text{CF}_0 & $7,000 \\
\text{C}_01 & -$3,700 \\
\text{F}_01 & 1 \\
\text{C}_02 & -$2,400 \\
\text{F}_02 & 1 \\
\text{C}_03 & -$1,500 \\
\text{F}_03 & 1 \\
\text{C}_04 & -$1,200 \\
\text{F}_04 & 1 \\
\end{array}
$

IRR CPT

$12.40\%$

$
\begin{array}{l}
\text{CF}_0 & $7,000 \\
\text{C}_01 & -$3,700 \\
\text{F}_01 & 1 \\
\text{C}_02 & -$2,400 \\
\text{F}_02 & 1 \\
\text{C}_03 & -$1,500 \\
\text{F}_03 & 1 \\
\text{C}_04 & -$1,200 \\
\text{F}_04 & 1 \\
\end{array}
$

$I = 10\%$

NPV CPT

$-$293.70

$I = 20\%$

NPV CPT

$\$803.24$

11. a. Deepwater fishing

$\begin{array}{l}
\text{CF}_0 & -$950,000 \\
\text{C}_01 & $370,000 \\
\text{F}_01 & 1 \\
\text{C}_02 & $510,000 \\
\text{F}_02 & 1 \\
\text{C}_03 & $420,000 \\
\text{F}_03 & 1 \\
\text{IRR CPT} & 17.07\% \\
\end{array}
$

Submarine ride

$\begin{array}{l}
\text{CF}_0 & -$1,850,000 \\
\text{C}_01 & $900,000 \\
\text{F}_01 & 1 \\
\text{C}_02 & $800,000 \\
\text{F}_02 & 1 \\
\text{C}_03 & $750,000 \\
\text{F}_03 & 1 \\
\text{IRR CPT} & 16.03\% \\
\end{array}$
Solutions Manual

b. 

<table>
<thead>
<tr>
<th></th>
<th>CF</th>
<th>C01</th>
<th>F01</th>
<th>C02</th>
<th>F02</th>
<th>C03</th>
<th>F03</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-900,000</td>
<td>$530,000</td>
<td>1</td>
<td>$290,000</td>
<td>1</td>
<td>$330,000</td>
<td>1</td>
</tr>
<tr>
<td>IRR CPT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14.79%</td>
</tr>
</tbody>
</table>

c. 

<table>
<thead>
<tr>
<th></th>
<th>Deepwater fishing</th>
<th>Submarine ride</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CF0</td>
<td>CF0</td>
</tr>
<tr>
<td></td>
<td>$-950,000</td>
<td>$-1,850,000</td>
</tr>
<tr>
<td></td>
<td>C01</td>
<td>C01</td>
</tr>
<tr>
<td></td>
<td>$370,000</td>
<td>$900,000</td>
</tr>
<tr>
<td></td>
<td>F01</td>
<td>F01</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>C02</td>
<td>C02</td>
</tr>
<tr>
<td></td>
<td>$510,000</td>
<td>$800,000</td>
</tr>
<tr>
<td></td>
<td>F02</td>
<td>F02</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>C03</td>
<td>C03</td>
</tr>
<tr>
<td></td>
<td>$420,000</td>
<td>$750,000</td>
</tr>
<tr>
<td></td>
<td>F03</td>
<td>F03</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I = 14%</td>
<td>$50,477.88</td>
<td>$61,276.34</td>
</tr>
<tr>
<td>NPV CPT</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. 

<table>
<thead>
<tr>
<th></th>
<th>Project I</th>
<th>Project II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CF0</td>
<td>CF0</td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$-30,000</td>
</tr>
<tr>
<td></td>
<td>C01</td>
<td>C01</td>
</tr>
<tr>
<td></td>
<td>$18,000</td>
<td>$18,000</td>
</tr>
<tr>
<td></td>
<td>F01</td>
<td>F01</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>I = 10%</td>
<td>$44,763.34</td>
<td>$14,763.34</td>
</tr>
<tr>
<td>NPV CPT</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{PI} = \frac{44,763.34}{30,000} = 1.492
\]

<table>
<thead>
<tr>
<th></th>
<th>Project II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CF0</td>
<td>CF0</td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$-12,000</td>
</tr>
<tr>
<td></td>
<td>C01</td>
<td>C01</td>
</tr>
<tr>
<td></td>
<td>$7,500</td>
<td>$7,500</td>
</tr>
<tr>
<td></td>
<td>F01</td>
<td>F01</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>I = 10%</td>
<td>$18,651.39</td>
<td>$6,651.39</td>
</tr>
<tr>
<td>NPV CPT</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{PI} = \frac{18,651.39}{12,000} = 1.554
\]
13. | CFo  | $-85,000,000 | CFo  | $-85,000,000 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C01</td>
<td>$125,000,000</td>
<td>C01</td>
<td>$125,000,000</td>
</tr>
<tr>
<td>F01</td>
<td>1</td>
<td>F01</td>
<td>1</td>
</tr>
<tr>
<td>C02</td>
<td>$-15,000,000</td>
<td>C02</td>
<td>$-15,000,000</td>
</tr>
<tr>
<td>F02</td>
<td>1</td>
<td>F02</td>
<td>1</td>
</tr>
</tbody>
</table>

I = 10% IRR CPT
NPV CPT 33.88%
$16,239,669.42

Financial calculators will only give you one IRR, even if there are multiple IRRs. Using trial and error, or a root solving calculator, the other IRR is –86.82%.

14. b. Board game DVD

<table>
<thead>
<tr>
<th>CFo</th>
<th>$-750</th>
<th>CFo</th>
<th>$-1,800</th>
</tr>
</thead>
<tbody>
<tr>
<td>C01</td>
<td>$600</td>
<td>C01</td>
<td>$1,300</td>
</tr>
<tr>
<td>F01</td>
<td>1</td>
<td>F01</td>
<td>1</td>
</tr>
<tr>
<td>C02</td>
<td>$450</td>
<td>C02</td>
<td>$850</td>
</tr>
<tr>
<td>F02</td>
<td>1</td>
<td>F02</td>
<td>1</td>
</tr>
<tr>
<td>C03</td>
<td>$120</td>
<td>C03</td>
<td>$350</td>
</tr>
<tr>
<td>F03</td>
<td>1</td>
<td>F03</td>
<td>1</td>
</tr>
</tbody>
</table>

I = 10% IRR CPT
NPV CPT $257.51 $347.26

14. c. Board game DVD

<table>
<thead>
<tr>
<th>CFo</th>
<th>$-750</th>
<th>CFo</th>
<th>$-1,800</th>
</tr>
</thead>
<tbody>
<tr>
<td>C01</td>
<td>$600</td>
<td>C01</td>
<td>$1,300</td>
</tr>
<tr>
<td>F01</td>
<td>1</td>
<td>F01</td>
<td>1</td>
</tr>
<tr>
<td>C02</td>
<td>$450</td>
<td>C02</td>
<td>$850</td>
</tr>
<tr>
<td>F02</td>
<td>1</td>
<td>F02</td>
<td>1</td>
</tr>
<tr>
<td>C03</td>
<td>$120</td>
<td>C03</td>
<td>$350</td>
</tr>
<tr>
<td>F03</td>
<td>1</td>
<td>F03</td>
<td>1</td>
</tr>
</tbody>
</table>

IRR CPT 33.79% 23.31%

14. d.

<table>
<thead>
<tr>
<th>CFo</th>
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</tr>
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<tbody>
<tr>
<td>C01</td>
<td>$700</td>
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<tr>
<td>F01</td>
<td>1</td>
</tr>
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<td>C02</td>
<td>$400</td>
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<td>F02</td>
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</tr>
<tr>
<td>C03</td>
<td>$230</td>
</tr>
<tr>
<td>F03</td>
<td>1</td>
</tr>
</tbody>
</table>

IRR CPT 15.86%
### 15. a.

<table>
<thead>
<tr>
<th></th>
<th><strong>CDMA</strong></th>
<th><strong>G4</strong></th>
<th><strong>Wi-Fi</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CFo</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>C01</strong></td>
<td>$11,000,000</td>
<td>$10,000,000</td>
<td>$18,000,000</td>
</tr>
<tr>
<td><strong>F01</strong></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>C02</strong></td>
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<td>$25,000,000</td>
<td>$32,000,000</td>
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<tr>
<td><strong>F02</strong></td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>C03</strong></td>
<td>$2,500,000</td>
<td>$20,000,000</td>
<td>$20,000,000</td>
</tr>
<tr>
<td><strong>F03</strong></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

I = 10%  
NPV CPT  
$18,076,634.11  
$44,778,362.13  
$57,836,213.37

\[ PI_{CDMA} = \frac{18,076,634.11}{8,000,000} = 2.26 \]
\[ PI_{G4} = \frac{44,778,362.13}{12,000,000} = 3.73 \]
\[ PI_{Wi-Fi} = \frac{57,836,213.37}{20,000,000} = 2.89 \]

### 15. b.

<table>
<thead>
<tr>
<th></th>
<th><strong>CDMA</strong></th>
<th><strong>G4</strong></th>
<th><strong>Wi-Fi</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CFo</strong></td>
<td>–$8,000,000</td>
<td>–$12,000,000</td>
<td>–$20,000,000</td>
</tr>
<tr>
<td><strong>C01</strong></td>
<td>$11,000,000</td>
<td>$10,000,000</td>
<td>$18,000,000</td>
</tr>
<tr>
<td><strong>F01</strong></td>
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</tr>
<tr>
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<tr>
<td><strong>F02</strong></td>
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</tr>
<tr>
<td><strong>C03</strong></td>
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<tr>
<td><strong>F03</strong></td>
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</tbody>
</table>

I = 10%  
NPV CPT  
$10,076,634.11  
$32,778,362.13  
$37,836,213.37

\[ PI_{AZM} = \frac{10,076,634.11}{8,000,000} = 1.26 \]
\[ PI_{AZF} = \frac{32,778,362.13}{12,000,000} = 2.73 \]
\[ PI_{Wi-Fi} = \frac{37,836,213.37}{20,000,000} = 1.89 \]

### 16. b.

<table>
<thead>
<tr>
<th></th>
<th><strong>AZM</strong></th>
<th><strong>AZF</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CFo</strong></td>
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<td>–$800,000</td>
</tr>
<tr>
<td><strong>C01</strong></td>
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<tr>
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<td><strong>F02</strong></td>
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<tr>
<td><strong>C03</strong></td>
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</tr>
<tr>
<td><strong>F03</strong></td>
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</table>

I = 10%  
NPV CPT  
$102,366.64  
$83,170.55

### 16. c.

<table>
<thead>
<tr>
<th></th>
<th><strong>AZM</strong></th>
<th><strong>AZF</strong></th>
</tr>
</thead>
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<tr>
<td><strong>C03</strong></td>
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<td>$290,000</td>
</tr>
<tr>
<td><strong>F03</strong></td>
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<td>1</td>
</tr>
</tbody>
</table>

IRR CPT  
24.65%  
15.97%

### 17. a.

**Project A**  
**Project B**  
**Project C**
### Solutions Manual

<table>
<thead>
<tr>
<th></th>
<th>CFo</th>
<th>C01</th>
<th>F01</th>
<th>C02</th>
<th>F02</th>
<th>I = 12%</th>
<th>NPV CPT</th>
<th>$185,905.61</th>
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</thead>
<tbody>
<tr>
<td>Project A</td>
<td>0</td>
<td>$110,000</td>
<td>1</td>
<td>$110,000</td>
<td>1</td>
<td></td>
<td>$35,905.61</td>
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</tr>
<tr>
<td>Project B</td>
<td>0</td>
<td>$200,000</td>
<td>1</td>
<td>$200,000</td>
<td>1</td>
<td></td>
<td>$38,010.20</td>
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</tr>
<tr>
<td>Project C</td>
<td>0</td>
<td>$120,000</td>
<td>1</td>
<td>$90,000</td>
<td>1</td>
<td></td>
<td>$178,890.31</td>
<td></td>
</tr>
</tbody>
</table>

PL_A = $185,905.61 / $150,000 = 1.24
PL_B = $338,010.20 / $300,000 = 1.13
PL_C = $178,890.31 / $150,000 = 1.19

#### b. Project A

<table>
<thead>
<tr>
<th></th>
<th>CFo</th>
<th>C01</th>
<th>F01</th>
<th>C02</th>
<th>F02</th>
<th>I = 12%</th>
<th>NPV CPT</th>
<th>$35,905.61</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project A</td>
<td>–$150,000</td>
<td>$110,000</td>
<td>1</td>
<td>$110,000</td>
<td>1</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Project B</td>
<td>–$300,000</td>
<td>$200,000</td>
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<td>1</td>
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</table>

#### 18. b. Dry prepeg

<table>
<thead>
<tr>
<th></th>
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<th>F01</th>
<th>C02</th>
<th>F02</th>
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<th>$607,287.75</th>
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<tbody>
<tr>
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<td>–$1,700,000</td>
<td>$1,100,000</td>
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<td>$900,000</td>
<td>1</td>
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<td></td>
</tr>
<tr>
<td>Solvent prepeg</td>
<td>–$750,000</td>
<td>$375,000</td>
<td>1</td>
<td>$600,000</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

#### c. Dry prepeg

<table>
<thead>
<tr>
<th></th>
<th>CFo</th>
<th>C01</th>
<th>F01</th>
<th>C02</th>
<th>F02</th>
<th>I = 10%</th>
<th>NPV CPT</th>
<th>$379,789.63</th>
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</thead>
<tbody>
<tr>
<td>Dry prepeg</td>
<td>–$1,700,000</td>
<td>$1,100,000</td>
<td>1</td>
<td>$900,000</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solvent prepeg</td>
<td>–$750,000</td>
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<td>1</td>
<td>$600,000</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### IRR CPT

<table>
<thead>
<tr>
<th></th>
<th>IRR CPT</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry prepeg</td>
<td>30.90%</td>
<td></td>
</tr>
<tr>
<td>Solvent prepeg</td>
<td>36.51%</td>
<td></td>
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</tbody>
</table>
### d.

<table>
<thead>
<tr>
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<th>20.34%</th>
<th>19. b.</th>
<th>20.27%</th>
</tr>
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<tbody>
<tr>
<td>CF&lt;sub&gt;0&lt;/sub&gt;</td>
<td>$-950,000</td>
<td>1</td>
<td>CF&lt;sub&gt;0&lt;/sub&gt;</td>
<td>$-350,000</td>
</tr>
<tr>
<td>C&lt;sub&gt;01&lt;/sub&gt;</td>
<td>$725,000</td>
<td>1</td>
<td>C&lt;sub&gt;01&lt;/sub&gt;</td>
<td>$100,000</td>
</tr>
<tr>
<td>F&lt;sub&gt;01&lt;/sub&gt;</td>
<td>1</td>
<td>1</td>
<td>F&lt;sub&gt;01&lt;/sub&gt;</td>
<td>1</td>
</tr>
<tr>
<td>C&lt;sub&gt;02&lt;/sub&gt;</td>
<td>$300,000</td>
<td>1</td>
<td>C&lt;sub&gt;02&lt;/sub&gt;</td>
<td>$110,000</td>
</tr>
<tr>
<td>F&lt;sub&gt;02&lt;/sub&gt;</td>
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<td>1</td>
<td>F&lt;sub&gt;02&lt;/sub&gt;</td>
<td>1</td>
</tr>
<tr>
<td>C&lt;sub&gt;03&lt;/sub&gt;</td>
<td>$360,000</td>
<td>1</td>
<td>C&lt;sub&gt;03&lt;/sub&gt;</td>
<td>$121,000</td>
</tr>
<tr>
<td>F&lt;sub&gt;03&lt;/sub&gt;</td>
<td>1</td>
<td>1</td>
<td>F&lt;sub&gt;03&lt;/sub&gt;</td>
<td>1</td>
</tr>
</tbody>
</table>
| IRR CPT | 25.52% | IRR CPT | 20.27% 

### 19. b.

#### NP-30

- CF<sub>0</sub> = $-550,000
- C<sub>01</sub> = $185,000
- F<sub>01</sub> = 5
- C<sub>02</sub> = $110,000
- F<sub>02</sub> = 1
- C<sub>03</sub> = $121,000
- F<sub>03</sub> = 1
- C<sub>04</sub> = $133,100
- F<sub>04</sub> = 1
- C<sub>05</sub> = $146,410
- F<sub>05</sub> = 1

IRR CPT = 20.27%

#### NX-20

- CF<sub>0</sub> = $-350,000
- C<sub>01</sub> = $100,000
- F<sub>01</sub> = 1
- C<sub>02</sub> = $110,000
- F<sub>02</sub> = 1
- C<sub>03</sub> = $121,000
- F<sub>03</sub> = 1
- C<sub>04</sub> = $133,100
- F<sub>04</sub> = 1
- C<sub>05</sub> = $146,410
- F<sub>05</sub> = 1

IRR CPT = 20.34%

### c.

#### NP-30

- CF<sub>0</sub> = $-550,000
- C<sub>01</sub> = $185,000
- F<sub>01</sub> = 5
- C<sub>02</sub> = $110,000
- F<sub>02</sub> = 1
- C<sub>03</sub> = $121,000
- F<sub>03</sub> = 1
- C<sub>04</sub> = $133,100
- F<sub>04</sub> = 1
- C<sub>05</sub> = $146,410
- F<sub>05</sub> = 1

I = 15%

NPV CPT = $620,148.69

#### NX-20

- CF<sub>0</sub> = $-350,000
- C<sub>01</sub> = $100,000
- F<sub>01</sub> = 1
- C<sub>02</sub> = $110,000
- F<sub>02</sub> = 1
- C<sub>03</sub> = $121,000
- F<sub>03</sub> = 1
- C<sub>04</sub> = $133,100
- F<sub>04</sub> = 1
- C<sub>05</sub> = $146,410
- F<sub>05</sub> = 1

I = 15%

NPV CPT = $398,583.79

\[
P_{NP-30} = \frac{620,148.69}{550,000} = 1.128
\]

\[
P_{NX-20} = \frac{398,583.79}{350,000} = 1.139
\]
### d. NP-30 vs. NY-20

<table>
<thead>
<tr>
<th></th>
<th>NP-30</th>
<th></th>
<th></th>
<th>NY-20</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>CF&lt;sub&gt;0&lt;/sub&gt;</td>
<td>$-350,000</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>C01</td>
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<td>C01</td>
<td>$100,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F01</td>
<td>5</td>
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<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C02</td>
<td></td>
<td>C02</td>
<td>$110,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F02</td>
<td></td>
<td>F02</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C03</td>
<td></td>
<td>C03</td>
<td>$121,000</td>
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<td></td>
</tr>
<tr>
<td>F03</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>F04</td>
<td></td>
<td>F04</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>C05</td>
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<td>C05</td>
<td>$146,410</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F05</td>
<td></td>
<td>F05</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I = 15%  
NPV CPT  
$70,148.66  

I = 15%  
NPV CPT  
$48,583.79  

---

### 28.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CF&lt;sub&gt;0&lt;/sub&gt;</td>
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<td></td>
</tr>
<tr>
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<td>$-26,000</td>
<td>F01</td>
<td>1</td>
</tr>
<tr>
<td>C02</td>
<td>$13,000</td>
<td>F02</td>
<td>1</td>
</tr>
</tbody>
</table>

IRR CPT  
ERROR 7
CHAPTER 6
MAKING CAPITAL INVESTMENT DECISIONS

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. Using the tax shield approach to calculating OCF, we get:

   \[ OCF = (Sales - Costs)(1 - tc) + tc \cdot Depreciation \]
   \[ OCF = \left[ ($4.75 \times 1,500) - ($2.30 \times 1,500) \right] (1 - 0.34) + 0.34 \left( \frac{$9,000}{5} \right) \]
   \[ OCF = $3,037.50 \]

   So, the NPV of the project is:

   \[ NPV = -$9,000 + $3,037.50 \times PVIFA_{14\%,5} \]
   \[ NPV = $1,427.98 \]

2. We will use the bottom-up approach to calculate the operating cash flow for each year. We also must be sure to include the net working capital cash flows each year. So, the net income and total cash flow each year will be:

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$12,500</td>
<td>$13,000</td>
<td>$13,500</td>
<td>$10,500</td>
</tr>
<tr>
<td>Costs</td>
<td>2,700</td>
<td>2,800</td>
<td>2,900</td>
<td>2,100</td>
</tr>
<tr>
<td>Depreciation</td>
<td>6,000</td>
<td>6,000</td>
<td>6,000</td>
<td>6,000</td>
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<tr>
<td>EBT</td>
<td>$3,800</td>
<td>$4,200</td>
<td>$4,600</td>
<td>$2,400</td>
</tr>
<tr>
<td>Tax</td>
<td>1,292</td>
<td>1,428</td>
<td>1,564</td>
<td>816</td>
</tr>
<tr>
<td></td>
<td>2022</td>
<td>2023</td>
<td>2024</td>
<td>2025</td>
</tr>
<tr>
<td>------------------------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Net income</td>
<td>$2,508</td>
<td>$2,772</td>
<td>$3,036</td>
<td>$1,584</td>
</tr>
<tr>
<td>OCF</td>
<td>0</td>
<td>$8,508</td>
<td>$8,772</td>
<td>$9,036</td>
</tr>
<tr>
<td>Capital spending</td>
<td>−$24,000</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NWC</td>
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<td>−350</td>
<td>−400</td>
<td>−300</td>
</tr>
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<td>−$24,300</td>
<td>$8,158</td>
<td>$8,372</td>
<td>$8,736</td>
</tr>
</tbody>
</table>
The NPV for the project is:

\[
NPV = -24,300 + \frac{8,158}{1.12} + \frac{8,372}{1.12^2} + \frac{8,736}{1.12^3} + \frac{8,934}{1.12^4}
\]

\[
NPV = 1,553.87
\]

3. Using the tax shield approach to calculating OCF, we get:

\[
OCF = (Sales - Costs)(1 - t_c) + t_c \times Depreciation
\]

\[
OCF = (1,120,000 - 480,000)(1 - 0.35) + 0.35(\frac{1,400,000}{3})
\]

\[
OCF = 579,333.33
\]

So, the NPV of the project is:

\[
NPV = -1,400,000 + 579,333.33(PVIFA_{12\%},3)
\]

\[
NPV = -8,539.09
\]

4. The cash outflow at the beginning of the project will increase because of the spending on NWC. At the end of the project, the company will recover the NWC, so it will be a cash inflow. The sale of the equipment will result in a cash inflow, but we also must account for the taxes which will be paid on this sale. So, the cash flows for each year of the project will be:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1,685,000 = -1,400,000 - 285,000</td>
</tr>
<tr>
<td>1</td>
<td>579,333.33</td>
</tr>
<tr>
<td>2</td>
<td>579,333.33</td>
</tr>
<tr>
<td>3</td>
<td>1,010,583.33 = 579,333.33 + 285,000 + 225,000 + (0 - 25,000)(.35)</td>
</tr>
</tbody>
</table>

And the NPV of the project is:

\[
NPV = -1,685,000 + 579,333.33(PVIFA_{12\%},2) + (\frac{1,010,583.33}{1.12^3})
\]

\[
NPV = 13,416.15
\]

5. First we will calculate the annual depreciation for the equipment necessary for the project. The depreciation amount each year will be:

Year 1 depreciation = $1,400,000(0.3333) = 466,620
Year 2 depreciation = $1,400,000(0.4445) = $622,300  
Year 3 depreciation = $1,400,000(0.1481) = $207,340

So, the book value of the equipment at the end of three years, which will be the initial investment minus the accumulated depreciation, is:

Book value in 3 years = $1,400,000 – ($466,620 + 622,300 + 207,340)
Book value in 3 years = $103,740

The asset is sold at a gain to book value, so this gain is taxable.

Aftertax salvage value = $225,000 + ($103,740 – 225,000)(0.35)
Aftertax salvage value = $182,559
To calculate the OCF, we will use the tax shield approach, so the cash flow each year is:

\[
OCF = (Sales - Costs)(1 - t_c) + t_c Depreciation
\]

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-1,685,000 = -$1,400,000 - 285,000</td>
</tr>
<tr>
<td>1</td>
<td>$579,317 = ($640,000)(.65) + 0.35($466,620)</td>
</tr>
<tr>
<td>2</td>
<td>$633,805 = ($640,000)(.65) + 0.35($622,300)</td>
</tr>
<tr>
<td>3</td>
<td>$956,128 = ($640,000)(.65) + 0.35($207,340) + $182,559 + $85,000</td>
</tr>
</tbody>
</table>

Remember to include the NWC cost in Year 0, and the recovery of the NWC at the end of the project. The NPV of the project with these assumptions is:

\[
NPV = -1,685,000 + \frac{579,317}{1.12} + \frac{633,805}{1.12^2} + \frac{956,128}{1.12^3}
\]

\[
NPV = $18,065.81
\]

6. First, we will calculate the annual depreciation of the new equipment. It will be:

Annual depreciation charge = $670,000/5
Annual depreciation charge = $134,000

The aftertax salvage value of the equipment is:

Aftertax salvage value = $50,000(1 - 0.35)
Aftertax salvage value = $32,500

Using the tax shield approach, the OCF is:

\[
OCF = 240,000(1 - 0.35) + 0.35($134,000)
OCF = $202,900
\]

Now we can find the project IRR. There is an unusual feature that is a part of this project. Accepting this project means that we will reduce NWC. This reduction in NWC is a cash inflow at Year 0. This reduction in NWC implies that when the project ends, we will have to increase NWC. So, at the end of the project, we will have a cash outflow to restore the NWC to its level before the project. We also must include the aftertax salvage value at the end of the project. The IRR of the project is:
NPV = 0 = –$670,000 + 85,000 + $202,900(\text{PVIFA}_{\text{IRR}%,4})
+ \left[\frac{($202,900 + 32,500 - 85,000)}{(1+\text{IRR})^5}\right]

\text{IRR} = 20.06\%

7. First, we will calculate the annual depreciation of the new equipment. It will be:

Annual depreciation = $375,000/5
Annual depreciation = $75,000

Now, we calculate the aftertax salvage value. The aftertax salvage value is the market price minus (or plus) the taxes on the sale of the equipment, so:

Aftertax salvage value = MV + (BV – MV)\text{t}_c

Very often, the book value of the equipment is zero as it is in this case. If the book value is zero, the equation for the aftertax salvage value becomes:

Aftertax salvage value = MV + (0 – MV)\text{t}_c
Aftertax salvage value = MV(1 – \text{t}_c)

We will use this equation to find the aftertax salvage value since we know the book value is zero. So, the aftertax salvage value is:

Aftertax salvage value = $40,000(1 – 0.34)
Aftertax salvage value = $26,400

Using the tax shield approach, we find the OCF for the project is:

OCF = $105,000(1 – 0.34) + 0.34($75,000)
OCF = $94,800

Now we can find the project NPV. Notice that we include the NWC in the initial cash outlay. The recovery of the NWC occurs in Year 5, along with the aftertax salvage value.

\text{NPV} = –$375,000 – 28,000 + $94,800(\text{PVIFA}_{10\%,5}) + \left[\frac{($26,400 + 28,000)}{1.15}\right]
\text{NPV} = –$9,855.29
8. To find the BV at the end of four years, we need to find the accumulated depreciation for the first four years. We could calculate a table with the depreciation each year, but an easier way is to add the MACRS depreciation amounts for each of the first four years and multiply this percentage times the cost of the asset. We can then subtract this from the asset cost. Doing so, we get:

\[ BV_4 = 7,100,000 - 7,100,000(0.2000 + 0.3200 + 0.1920 + 0.1152) \]
\[ BV_4 = 1,226,880 \]

The asset is sold at a gain to book value, so this gain is taxable.

Aftertax salvage value = \(1,400,000 + (1,226,880 - 1,400,000)(.35)\)
\[ \text{Aftertax salvage value} = 1,339,408 \]

9. We will begin by calculating the initial cash outlay, that is, the cash flow at Time 0. To undertake the project, we will have to purchase the equipment and increase net working capital. So, the cash outlay today for the project will be:

| Equipment | –3,800,000 |
| NWC      | –150,000   |
| Total    | –3,950,000 |

Using the bottom-up approach to calculating the operating cash flow, we find the operating cash flow each year will be:

| Sales      | $2,500,000 |
| Costs      | 625,000    |
| Depreciation | 950,000   |
| EBT        | $925,000   |
| Tax        | 323,750    |
| Net income | $601,250   |

The operating cash flow is:

\[ \text{OCF} = \text{Net income} + \text{Depreciation} \]
\[ \text{OCF} = 601,250 + 950,000 \]
\[ \text{OCF} = 1,551,250 \]
To find the NPV of the project, we add the present value of the project cash flows. We must be sure to add back the net working capital at the end of the project life, since we are assuming the net working capital will be recovered. So, the project NPV is:

\[
NPV = -3,950,000 + 1,551,250(PVIFA_{16\%\,4}) + 150,000 / 1.16^4
\]

\[
NPV = 473,521.38
\]

10. We will need the aftertax salvage value of the equipment to compute the EAC. Even though the equipment for each product has a different initial cost, both have the same salvage value. The aftertax salvage value for both is:

Both cases: aftertax salvage value = $20,000(1 – 0.35) = $13,000

To calculate the EAC, we first need the OCF and NPV of each option. The OCF and NPV for Techron I is:

\[
OCF = -35,000(1 – 0.35) + 0.35(215,000/3) = 2,333.33
\]

\[
NPV = -215,000 + 2,333.33(PVIFA_{12\%,3}) + (13,000/1.12^3) = -200,142.58
\]

\[
EAC = -200,142.58 / (PVIFA_{12\%,3}) = -83,329.16
\]

And the OCF and NPV for Techron II is:

\[
OCF = -44,000(1 – 0.35) + 0.35(270,000/5) = -9,700
\]

\[
NPV = -270,000 – 9,700(PVIFA_{12\%,5}) + (13,000/1.12^5) = -297,589.78
\]

\[
EAC = -297,589.78 / (PVIFA_{12\%,5}) = -82,554.30
\]

The two milling machines have unequal lives, so they can only be compared by expressing both on an equivalent annual basis, which is what the EAC method does. Thus, you prefer the Techron II because it has the lower (less negative) annual cost.

Intermediate

11. First, we will calculate the depreciation each year, which will be:
\[ D_1 = 640,000 \times 0.2000 = 128,000 \]
\[ D_2 = 640,000 \times 0.3200 = 204,800 \]
\[ D_3 = 640,000 \times 0.1920 = 122,880 \]
\[ D_4 = 640,000 \times 0.1150 = 73,728 \]

The book value of the equipment at the end of the project is:
\[ BV_4 = 640,000 - (128,000 + 204,800 + 122,880 + 73,728) = 110,592 \]

The asset is sold at a loss to book value, so this creates a tax refund. After-tax salvage value = \[ 70,000 + (110,592 - 70,000)(0.35) = 84,207.20 \]

So, the OCF for each year will be:
\[ OCF_1 = 270,000(1 - 0.35) + 0.35(128,000) = 220,300.00 \]
\[ OCF_2 = 270,000(1 - 0.35) + 0.35(204,800) = 247,180.00 \]
\[ OCF_3 = 270,000(1 - 0.35) + 0.35(122,880) = 218,508.00 \]
\[ OCF_4 = 270,000(1 - 0.35) + 0.35(73,728) = 201,304.80 \]

Now we have all the necessary information to calculate the project NPV. We need to be careful with the NWC in this project. Notice the project requires $20,000 of NWC at the beginning, and $3,500 more in NWC each successive year. We will subtract the $20,000 from the initial cash flow and subtract $3,500 each year from the OCF to account for this spending. In Year 4, we will add back the total spent on NWC, which is $30,500. The $3,500 spent on NWC capital during Year 4 is irrelevant. Why? Well, during this year the project required an additional $3,500, but we would get the money back immediately. So, the net cash flow for additional NWC would be zero. With all this, the equation for the NPV of the project is:
\[
NPV = -640,000 - 20,000 + \frac{(220,300 - 3,500)}{1.14} + \frac{(247,180 - 3,500)}{1.14^2} + \frac{(218,508 - 3,500)}{1.14^3} + \frac{(201,304.80 + 30,500 + 84,207.20)}{1.14^4}
\]

\[ NPV = 49,908.03 \]

12. If we are trying to decide between two projects that will not be replaced when they wear out, the proper capital budgeting method to use is NPV. Both projects only have costs
associated with them, not sales, so we will use these to calculate the NPV of each project. Using the tax shield approach to calculate the OCF, the NPV of System A is:

\[
OCF_A = -85,000(1 - 0.34) + 0.34(290,000/4)
\]
\[
OCF_A = -31,450
\]

\[
NPV_A = -290,000 - 31,450(PVIFA_{11\%},4)
\]
\[
NPV_A = -387,571.92
\]

And the NPV of System B is:

\[
OCF_B = -75,000(1 - 0.34) + 0.34(405,000/6)
\]
\[
OCF_B = -26,550
\]

\[
NPV_B = -405,000 - 26,550(PVIFA_{11\%},6)
\]
\[
NPV_B = -517,320.78
\]

If the system will not be replaced when it wears out, then System A should be chosen, because it has the less negative NPV.

13. If the equipment will be replaced at the end of its useful life, the correct capital budgeting technique is EAC. Using the NPVs we calculated in the previous problem, the EAC for each system is:

\[
EAC_A = -387,571.92 / (PVIFA_{11\%},4)
\]
\[
EAC_A = -124,924.64
\]

\[
EAC_B = -517,320.78 / (PVIFA_{11\%},6)
\]
\[
EAC_B = -122,282.51
\]

If the conveyor belt system will be continually replaced, we should choose System B since it has the less negative EAC.

14. Since we need to calculate the EAC for each machine, sales are irrelevant. EAC only uses the costs of operating the equipment, not the sales. Using the bottom up approach, or net income plus depreciation, method to calculate OCF, we get:

<table>
<thead>
<tr>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Solutions Manual

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable costs</td>
<td>-$4,200,000</td>
<td>-$3,600,000</td>
</tr>
<tr>
<td>Fixed costs</td>
<td>-195,000</td>
<td>-165,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>-483,333</td>
<td>-633,333</td>
</tr>
<tr>
<td>EBT</td>
<td>-$4,878,333</td>
<td>-$4,398,333</td>
</tr>
<tr>
<td>Tax</td>
<td>1,707,417</td>
<td>1,539,417</td>
</tr>
<tr>
<td>Net income</td>
<td>-$3,170,917</td>
<td>-$2,858,917</td>
</tr>
<tr>
<td>+ Depreciation</td>
<td>483,333</td>
<td>633,333</td>
</tr>
<tr>
<td>OCF</td>
<td>-$2,687,583</td>
<td>-$2,225,583</td>
</tr>
</tbody>
</table>

The NPV and EAC for Machine A is:

\[
\text{NPV}_A = -$2,900,000 - 2,687,583 \times (\text{PVIFA}_{10\%,6})
\]

\[
\text{NPV}_A = -$14,605,126.07
\]

\[
\text{EAC}_A = - \frac{-14,605,126.07}{(\text{PVIFA}_{10\%,6})}
\]

\[
\text{EAC}_A = -$3,353,444.74
\]

And the NPV and EAC for Machine B is:

\[
\text{NPV}_B = -$5,700,000 - 2,225,583 \times (\text{PVIFA}_{10\%,9})
\]

\[
\text{NPV}_B = -$18,517,187.42
\]

\[
\text{EAC}_B = - \frac{-18,517,187.42}{(\text{PVIFA}_{10\%,9})}
\]

\[
\text{EAC}_B = -$3,215,334.41
\]

You should choose Machine B since it has a less negative EAC.

15. When we are dealing with nominal cash flows, we must be careful to discount cash flows at the nominal interest rate, and we must discount real cash flows using the real interest rate. Project A’s cash flows are in real terms, so we need to find the real interest rate. Using the Fisher equation, the real interest rate is:

\[
1 + R = (1 + r)(1 + h)
\]

\[
1.13 = (1 + r)(1 + .04)
\]

\[
r = .0865, \text{ or } 8.65\%
\]

So, the NPV of Project A’s real cash flows, discounting at the real interest rate, is:
NPV = –$50,000 + $30,000 / 1.0865 + $25,000 / 1.0865² + $20,000 / 1.0865³
NPV = $14,378.65

Project B’s cash flow are in nominal terms, so the NPV discounted at the nominal interest rate is:

NPV = –$65,000 + $29,000 / 1.13 + $38,000 / 1.13² + $41,000 / 1.13³
NPV = $18,838.35

We should accept Project B if the projects are mutually exclusive since it has the highest NPV.

16. To determine the value of a firm, we can simply find the present value of the firm’s future cash flows. No depreciation is given, so we can assume depreciation is zero. Using the tax shield approach, we can find the present value of the aftertax revenues, and the present value of the aftertax costs. The required return, growth rates, price, and costs are all given in real terms. Subtracting the costs from the revenues will give us the value of the firm’s cash flows. We must calculate the present value of each separately since each is growing at a different rate. First, we will find the present value of the revenues. The revenues in year 1 will be the number of bottles sold, times the price per bottle, or:

Aftertax revenue in year 1 in real terms = (2,800,000 × $1.25)(1 – 0.34)
Aftertax revenue in year 1 in real terms = $2,310,000

Revenues will grow at six percent per year in real terms forever. Apply the growing perpetuity formula, we find the present value of the revenues is:

\[
PV \text{ of revenues} = \frac{C_1}{(R - g)}
\]

\[
PV \text{ of revenues} = \frac{2,310,000}{0.10 - 0.06}
\]

PV of revenues = $57,750,000
The real aftertax costs in year 1 will be:

Aftertax costs in year 1 in real terms = \((2,800,000 \times 0.90)(1 – 0.34)\)
Aftertax costs in year 1 in real terms = $1,663,200

Costs will grow at five percent per year in real terms forever. Applying the growing perpetuity formula, we find the present value of the costs is:

\[
PV \text{ of costs} = \frac{C_1}{(R – g)}
\]
\[
PV \text{ of costs} = \frac{1,663,200}{0.10 – 0.05}
\]
\[
PV \text{ of costs} = $33,264,000
\]

Now we can find the value of the firm, which is:

\[
\text{Value of the firm} = PV \text{ of revenues} – PV \text{ of costs}
\]
\[
\text{Value of the firm} = $57,750,000 – 33,264,000
\]
\[
\text{Value of the firm} = $24,486,000
\]

17. To calculate the nominal cash flows, we increase each item in the income statement by the inflation rate, except for depreciation. Depreciation is a nominal cash flow, so it does not need to be adjusted for inflation in nominal cash flow analysis. Since the resale value is given in nominal terms as of the end of year 5, it does not need to be adjusted for inflation. Also, no inflation adjustment is needed for net working capital since it is already expressed in nominal terms. Note that an increase in required net working capital is a negative cash flow whereas a decrease in required net working capital is a positive cash flow. We first need to calculate the taxes on the salvage value. Remember, to calculate the taxes paid (or tax credit) on the salvage value, we take the book value minus the market value, times the tax rate, which, in this case, would be:

\[
\text{Taxes on salvage value} = (BV – MV)t_c
\]
\[
\text{Taxes on salvage value} = (0 – 45,000)(.34)
\]
\[
\text{Taxes on salvage value} = –$15,300
\]

So, the nominal aftertax salvage value is:

\[
\begin{align*}
\text{Market price} & \quad $45,000 \\
\text{Tax on sale} & \quad –15,300 \\
\text{Aftertax salvage value} & \quad $29,700
\end{align*}
\]
Now we can find the nominal cash flows each year using the income statement. Doing so, we find:

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales</th>
<th>Expenses</th>
<th>Depreciation</th>
<th>EBT</th>
<th>Tax</th>
<th>Net income</th>
<th>OCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$245,000</td>
<td>70,000</td>
<td>73,000</td>
<td>$102,000</td>
<td>34,680</td>
<td>$67,320</td>
<td>$140,320</td>
</tr>
<tr>
<td>1</td>
<td>$252,350</td>
<td>72,100</td>
<td>73,000</td>
<td>$107,250</td>
<td>36,465</td>
<td>$70,785</td>
<td>$143,785</td>
</tr>
<tr>
<td>2</td>
<td>$259,921</td>
<td>74,263</td>
<td>73,000</td>
<td>$112,658</td>
<td>38,304</td>
<td>$74,354</td>
<td>$147,354</td>
</tr>
<tr>
<td>3</td>
<td>$267,718</td>
<td>76,491</td>
<td>73,000</td>
<td>$118,227</td>
<td>40,197</td>
<td>$78,030</td>
<td>$151,030</td>
</tr>
<tr>
<td>4</td>
<td>$275,750</td>
<td>78,786</td>
<td>73,000</td>
<td>$123,964</td>
<td>42,148</td>
<td>$81,816</td>
<td>$154,816</td>
</tr>
</tbody>
</table>

18. The present value of the company is the present value of the future cash flows generated by the company. Here we have real cash flows, a real interest rate, and a real growth rate. The cash flows are a growing perpetuity, with a negative growth rate. Using the growing perpetuity equation, the present value of the cash flows is:

\[
PV = \frac{C_1}{(R - g)}
\]

\[
PV = \frac{$190,000}{.11 - (-.04)}
\]

\[
PV = $1,266,666.67
\]

19. To find the EAC, we first need to calculate the NPV of the incremental cash flows. We will begin with the aftertax salvage value, which is:

\[
\text{Taxes on salvage value} = (BV - MV)t_c
\]

\[
\text{Taxes on salvage value} = ($0 - 18,000)(.34)
\]

\[
\text{Taxes on salvage value} = -$6,120
\]

\[
\text{Market price} = $18,000
\]

\[
\text{Tax on sale} = -$6,120
\]

\[
\text{Aftertax salvage value} = $11,880
\]
Now we can find the operating cash flows. Using the tax shield approach, the operating cash flow each year will be:

\[
OCF = -\$8,600(1 - 0.34) + 0.34(\frac{\$94,000}{3})
OCF = \$4,977.33
\]

So, the NPV of the cost of the decision to buy is:

\[
NPV = -\$94,000 + \$4,977.33(PVIFA_{12\%,3}) + \frac{\$11,880}{1.12^3}
NPV = -\$73,589.34
\]

In order to calculate the equivalent annual cost, set the NPV of the equipment equal to an annuity with the same economic life. Since the project has an economic life of three years and is discounted at 12 percent, set the NPV equal to a three-year annuity, discounted at 12 percent.

\[
EAC = -\frac{\$73,589.34}{(PVIFA_{12\%,3})}
EAC = -\$30,638.84
\]

**20.** We will calculate the aftertax salvage value first. The aftertax salvage value of the equipment will be:

\[
\text{Taxes on salvage value} = (BV - MV)\tau_c
\]
\[
\text{Taxes on salvage value} = (\$0 - 60,000)(0.34)
\]
\[
\text{Taxes on salvage value} = -\$20,400
\]

<table>
<thead>
<tr>
<th>Market price</th>
<th>$60,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax on sale</td>
<td>-20,400</td>
</tr>
<tr>
<td>Aftertax salvage value</td>
<td>$39,600</td>
</tr>
</tbody>
</table>

Next, we will calculate the initial cash outlay, that is, the cash flow at Time 0. To undertake the project, we will have to purchase the equipment. The new project will decrease the net working capital, so this is a cash inflow at the beginning of the project. So, the cash outlay today for the project will be:

<table>
<thead>
<tr>
<th>Equipment</th>
<th>-$360,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>NWC</td>
<td>80,000</td>
</tr>
<tr>
<td>Total</td>
<td>-$280,000</td>
</tr>
</tbody>
</table>
Now we can calculate the operating cash flow each year for the project. Using the bottom up approach, the operating cash flow will be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saved salaries</td>
<td>$105,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>72,000</td>
</tr>
<tr>
<td>EBT</td>
<td>$33,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>11,220</td>
</tr>
<tr>
<td>Net income</td>
<td>$21,780</td>
</tr>
</tbody>
</table>

And the OCF will be:

$$\text{OCF} = 21,780 + 72,000$$
$$\text{OCF} = 93,780$$

Now we can find the NPV of the project. In Year 5, we must replace the saved NWC, so:

$$\text{NPV} = -280,000 + 93,780(PVIFA_{12\%,5}) + (39,600 - 80,000) / 1.125$$
$$\text{NPV} = 35,131.87$$

21. Replacement decision analysis is the same as the analysis of two competing projects, in this case, keep the current equipment, or purchase the new equipment. We will consider the purchase of the new machine first.

Purchase new machine:

The initial cash outlay for the new machine is the cost of the new machine, plus the increased net working capital. So, the initial cash outlay will be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase new machine</td>
<td>$18,000,000</td>
</tr>
<tr>
<td>Net working capital</td>
<td>$250,000</td>
</tr>
<tr>
<td>Total</td>
<td>$18,250,000</td>
</tr>
</tbody>
</table>

Next, we can calculate the operating cash flow created if the company purchases the new machine. The saved operating expense is an incremental cash flow. Additionally, the reduced operating expense is a cash inflow, so it should be treated as such in the income statement. The pro forma income statement, and adding depreciation to net income, the annual operating cash flow created by purchasing the new machine will be:
Operating expense $6,700,000  
Depreciation 4,500,000  
EBT $2,200,000  
Taxes 858,000  
Net income $1,342,000  
OCF $5,842,000  

So, the NPV of purchasing the new machine, including the recovery of the net working capital, is:

\[ NPV = -18,250,000 + 5,842,000(PVIFA_{10\%,4}) + 250,000 / 1.10^4 \]
\[ NPV = 439,107.30 \]

And the IRR is:

\[ 0 = -18,250,000 + 5,842,000(PVIFA_{IRR,4}) + 250,000 / (1 + IRR)^4 \]

Using a spreadsheet or financial calculator, we find the IRR is:

IRR = 11.10%

Now we can calculate the decision to keep the old machine:

Keep old machine:

The initial cash outlay for the old machine is the market value of the old machine, including any potential tax consequence. The decision to keep the old machine has an opportunity cost, namely, the company could sell the old machine. Also, if the company sells the old machine at its current value, it will receive a tax benefit. Both of these cash flows need to be included in the analysis. So, the initial cash flow of keeping the old machine will be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keep machine</td>
<td>-$4,500,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>-585,000</td>
</tr>
<tr>
<td>Total</td>
<td>-$5,085,000</td>
</tr>
</tbody>
</table>
Next, we can calculate the operating cash flow created if the company keeps the old machine. There are no incremental cash flows from keeping the old machine, but we need to account for the cash flow effects of depreciation. The income statement, adding depreciation to net income to calculate the operating cash flow will be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation</td>
<td>$1,500,000</td>
</tr>
<tr>
<td>EBT</td>
<td>–$1,500,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>–585,000</td>
</tr>
<tr>
<td>Net income</td>
<td>–$915,000</td>
</tr>
<tr>
<td>OCF</td>
<td>$585,000</td>
</tr>
</tbody>
</table>

So, the NPV of the decision to keep the old machine will be:

\[ NPV = -5,085,000 + 585,000(PVIFA_{10\%,4}) \]
\[ NPV = -3,230,628.71 \]

And the IRR is:

\[ 0 = -5,085,000 + 585,000(PVIFA_{IRR,4}) \]

Using a spreadsheet or financial calculator, we find the IRR is:

\[ IRR = -25.15\% \]

There is another way to analyze a replacement decision that is often used. It is an incremental cash flow analysis of the change in cash flows from the existing machine to the new machine, assuming the new machine is purchased. In this type of analysis, the initial cash outlay would be the cost of the new machine, the increased NWC, and the cash inflow (including any applicable taxes) of selling the old machine. In this case, the initial cash flow under this method would be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase new machine</td>
<td>–$18,000,000</td>
</tr>
<tr>
<td>Net working capital</td>
<td>–250,000</td>
</tr>
<tr>
<td>Sell old machine</td>
<td>4,500,000</td>
</tr>
<tr>
<td>Taxes on old machine</td>
<td>585,000</td>
</tr>
<tr>
<td>Total</td>
<td>–$13,165,000</td>
</tr>
</tbody>
</table>
The cash flows from purchasing the new machine would be the saved operating expenses. We would also need to include the change in depreciation. The old machine has a depreciation of $1.5 million per year, and the new machine has a depreciation of $4.5 million per year, so the increased depreciation will be $3 million per year. The pro forma income statement and operating cash flow under this approach will be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating expense savings</td>
<td>$6,700,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$3,000,000</td>
</tr>
<tr>
<td>EBT</td>
<td>$3,700,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>$1,443,000</td>
</tr>
<tr>
<td>Net income</td>
<td>$2,257,000</td>
</tr>
<tr>
<td>OCF</td>
<td>$5,257,000</td>
</tr>
</tbody>
</table>

The NPV under this method is:

\[
\text{NPV} = -13,165,000 + 5,257,000 \times (\text{PVIFA}_{10\%,4}) + \frac{250,000}{1.10^4} \\
\text{NPV} = 3,669,736.02
\]

And the IRR is:

\[
0 = -13,165,000 + 5,257,000 \times (\text{PVIFA}_{\text{IRR,4}}) + \frac{250,000}{(1 + \text{IRR})^4}
\]

Using a spreadsheet or financial calculator, we find the IRR is:

\[\text{IRR} = 22.23\%\]

So, this analysis still tells us the company should purchase the new machine. This is really the same type of analysis we originally did. Consider this: Subtract the NPV of the decision to keep the old machine from the NPV of the decision to purchase the new machine. You will get:

\[
\text{Differential NPV} = 439,107.30 - (-3,230,628.71) = 3,669,736.02
\]

This is the exact same NPV we calculated when using the second analysis method.

22. We can find the NPV of a project using nominal cash flows or real cash flows. Either method will result in the same NPV. For this problem, we will calculate the NPV using
both nominal and real cash flows. The initial investment in either case is $270,000 since it
will be spent today. We will begin with the nominal cash flows. The revenues and
production costs increase at different rates, so we must be careful to increase each at the
appropriate growth rate. The nominal cash flows for each year will be:

<table>
<thead>
<tr>
<th>Year</th>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$105,000.00</td>
<td>$110,250.00</td>
<td>$115,762.50</td>
<td></td>
</tr>
<tr>
<td>Costs</td>
<td>$30,000.00</td>
<td>31,800.00</td>
<td>33,708.00</td>
<td></td>
</tr>
<tr>
<td>Depreciation</td>
<td>38,571.43</td>
<td>38,571.43</td>
<td>38,571.43</td>
<td></td>
</tr>
<tr>
<td>EBT</td>
<td>$36,428.57</td>
<td>$39,878.57</td>
<td>$43,483.07</td>
<td></td>
</tr>
<tr>
<td>Taxes</td>
<td>12,385.71</td>
<td>13,558.71</td>
<td>14,784.24</td>
<td></td>
</tr>
<tr>
<td>Net income</td>
<td>$24,042.86</td>
<td>$26,319.86</td>
<td>$28,698.83</td>
<td></td>
</tr>
<tr>
<td>OCF</td>
<td>$62,614.29</td>
<td>$64,891.29</td>
<td>$67,270.26</td>
<td></td>
</tr>
</tbody>
</table>

Capital spending $–270,000

Total cash flow $–270,000 $62,614.29 $64,891.29 $67,270.26

Now that we have the nominal cash flows, we can find the NPV. We must use the nominal
required return with nominal cash flows. Using the Fisher equation to find the nominal
required return, we get:

\[(1 + R) = (1 + r)(1 + h)\]
\[(1 + R) = (1 + 0.08)(1 + 0.05)\]
\[R = 0.1340, or 13.40\%\]
So, the NPV of the project using nominal cash flows is:

\[
\text{NPV} = -270,000 + \frac{62,614.29}{1.1340} + \frac{64,891.29}{1.1340^2} + \frac{67,270.26}{1.1340^3} + \frac{69,755.58}{1.1340^4} + \frac{72,351.83}{1.1340^5} + \frac{75,063.73}{1.1340^6} + \frac{77,896.24}{1.1340^7} \\
\text{NPV} = 30,170.71
\]

We can also find the NPV using real cash flows and the nominal required return. This will allow us to find the operating cash flow using the tax shield approach. Both the revenues and expenses are growing annuities, but growing at different rates. This means we must find the present value of each separately. We also need to account for the effect of taxes, so we will multiply by one minus the tax rate. So, the present value of the aftertax revenues using the growing annuity equation is:

\[
\text{PV of aftertax revenues} = C \left[ \frac{1}{(r - g)} - \frac{1}{(r - g)} \times \frac{(1 + g)}{(1 + r)} \right] t (1 - t_c) \\
\text{PV of aftertax revenues} = 105,000 \left[ \frac{1}{(.134 - .05)} - \frac{1}{(.134 - .05)} \times \frac{(1 + .05)}{(1 + .134)} \right] 7 (1 - .34) \\
\text{PV of aftertax revenues} = 343,620.42
\]

And the present value of the aftertax costs will be:

\[
\text{PV of aftertax costs} = C \left[ \frac{1}{(r - g)} - \frac{1}{(r - g)} \times \frac{(1 + g)}{(1 + r)} \right] t (1 - t_c) \\
\text{PV of aftertax costs} = 30,000 \left[ \frac{1}{(.134 - .06)} - \frac{1}{(.134 - .06)} \times \frac{(1 + .06)}{(1 + .134)} \right] 7 (1 - .34) \\
\text{PV of aftertax costs} = 100,734.11
\]

Now we need to find the present value of the depreciation tax shield. The depreciation amount in the first year is a real value, so we can find the present value of the depreciation tax shield as an ordinary annuity using the real required return. So, the present value of the depreciation tax shield will be:

\[
\text{PV of depreciation tax shield} = \left( \frac{270,000}{7} \right)(.34)(\text{PVIFA}_{13.40\%,7}) \\
\text{PV of depreciation tax shield} = 57,284.40
\]

Using the present value of the real cash flows to find the NPV, we get:

\[
\text{NPV} = \text{Initial cost} + \text{PV of revenues} - \text{PV of costs} + \text{PV of depreciation tax shield} \\
\text{NPV} = -270,000 + 343,620.42 - 100,734.11 + 57,284.40
\]
Solutions Manual

NPV = $30,170.71

Notice, the NPV using nominal cash flows or real cash flows is identical, which is what we would expect.

23. Here we have a project in which the quantity sold each year increases. First, we need to calculate the quantity sold each year by increasing the current year’s quantity by the growth rate. So, the quantity sold each year will be:

Year 1 quantity = 7,000
Year 2 quantity = 7,000(1 + .08) = 7,560
Year 3 quantity = 7,560(1 + .08) = 8,165
Year 4 quantity = 8,165(1 + .08) = 8,818
Year 5 quantity = 8,818(1 + .08) = 9,523

Now we can calculate the sales revenue and variable costs each year. The pro forma income statements and operating cash flow each year will be:

<table>
<thead>
<tr>
<th>Year</th>
<th>Revenues</th>
<th>Fixed costs</th>
<th>Variable costs</th>
<th>Depreciation</th>
<th>EBT</th>
<th>Taxes</th>
<th>Net income</th>
<th>OCF</th>
<th>Capital spending</th>
<th>NWC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$336,000</td>
<td>95,000</td>
<td>140,000</td>
<td>35,000</td>
<td>$66,000</td>
<td>22,440</td>
<td>$43,560</td>
<td>$78,560</td>
<td>–$175,000</td>
<td>–35,000</td>
</tr>
<tr>
<td>1</td>
<td>$362,880</td>
<td>95,000</td>
<td>151,200</td>
<td>35,000</td>
<td>$81,680</td>
<td>27,771</td>
<td>$53,908.80</td>
<td>$88,908.80</td>
<td>35,000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$391,910.40</td>
<td>95,000</td>
<td>163,296.00</td>
<td>35,000</td>
<td>$98,614.40</td>
<td>33,528.90</td>
<td>$65,085.50</td>
<td>$100,085.50</td>
<td>35,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$423,263.23</td>
<td>95,000</td>
<td>176,359.68</td>
<td>35,000</td>
<td>$116,903.55</td>
<td>39,747.21</td>
<td>$77,156.34</td>
<td>$112,156.34</td>
<td>35,000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$457,124.29</td>
<td>95,000</td>
<td>190,468.45</td>
<td>35,000</td>
<td>$136,655.84</td>
<td>46,462.98</td>
<td>$90,192.85</td>
<td>$125,192.85</td>
<td>35,000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

So, the NPV of the project is:

\[
NPV = -210,000 + 75,860 / 1.25 + 88,908.80 / 1.25^2 + 100,085.50 / 1.25^3 + 112,156.34 / 1.25^4 + 160,192.85 / 1.25^5
\]
NPV = $59,424.64

We could also have calculated the cash flows using the tax shield approach, with growing annuities and ordinary annuities. The sales and variable costs increase at the same rate as sales, so both are growing annuities. The fixed costs and depreciation are both ordinary annuities. Using the growing annuity equation, the present value of the revenues is:

\[
PV \text{ of revenues} = C \left\{ \frac{1}{(r - g)} - \frac{1}{r} \times \left[ \frac{(1 + g)(1 + r)}{1 + (r - g)} \right] \right\}^t
\]

\[
PV \text{ of revenues} = $336,000 \left\{ \frac{1}{.25 - .08} - \frac{1}{.25} \times \left[ \frac{(1 + .08)(1 + .25)}{1 + (1.25 - .08)} \right] \right\}^5
\]

\[
PV \text{ of revenues} = $1,024,860.43
\]

And the present value of the variable costs will be:

\[
PV \text{ of variable costs} = C \left\{ \frac{1}{(r - g)} - \frac{1}{r} \times \left[ \frac{(1 + g)(1 + r)}{1 + (r - g)} \right] \right\}^t
\]

\[
PV \text{ of variable costs} = $140,000 \left\{ \frac{1}{.25 - .08} - \frac{1}{.25} \times \left[ \frac{(1 + .08)(1 + .25)}{1 + (1.25 - .08)} \right] \right\}^5
\]

\[
PV \text{ of variable costs} = $427,025.18
\]

The fixed costs and depreciation are both ordinary annuities. The present value of each is:

\[
PV \text{ of fixed costs} = C \left\{ \frac{1}{(1 + r)} - \frac{1}{1 + (1 + r)} \right\}^t
\]

\[
PV \text{ of fixed costs} = $95,000 \times (PVIFA_{25\%,5})
\]

\[
PV \text{ of depreciation} = C \left\{ \frac{1}{(1 + r)} \right\}^t
\]

\[
PV \text{ of depreciation} = $35,000 \times (PVIFA_{25\%,5})
\]

Now, we can use the depreciation tax shield approach to find the NPV of the project, which is:

\[
NPV = -$210,000 + ($1,024,860.43 - 427,025.18 - 255,481.60)(1 - .34) + ($94,124.80)(.34)
\]

\[
+ \frac{$35,000}{1.25^5}
\]

\[
NPV = $59,424.64
\]

24. We will begin by calculating the aftertax salvage value of the equipment at the end of the project’s life. The aftertax salvage value is the market value of the equipment minus any taxes paid (or refunded), so the aftertax salvage value in four years will be:
Taxes on salvage value = (BV – MV)\( t_c \)

Taxes on salvage value = (\$0 – 300,000)\( t_c \)

Taxes on salvage value = –\$114,000

Market price $300,000
Tax on sale –\$114,000
After tax salvage value $186,000

Now we need to calculate the operating cash flow each year. Note, we assume that the net working capital cash flow occurs immediately. Using the bottom up approach to calculating operating cash flow, we find:

<table>
<thead>
<tr>
<th>Year</th>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$1,842,500</td>
<td>$2,062,500</td>
<td>$2,502,500</td>
<td>$1,705,000</td>
<td></td>
</tr>
<tr>
<td>Fixed costs</td>
<td>350,000</td>
<td>350,000</td>
<td>350,000</td>
<td>350,000</td>
<td></td>
</tr>
<tr>
<td>Variable costs</td>
<td>276,375</td>
<td>309,375</td>
<td>375,375</td>
<td>255,750</td>
<td></td>
</tr>
<tr>
<td>Depreciation</td>
<td>1,033,230</td>
<td>1,377,950</td>
<td>459,110</td>
<td>229,710</td>
<td></td>
</tr>
<tr>
<td>EBT</td>
<td>$182,895</td>
<td>$25,175</td>
<td>$1,318,015</td>
<td>$869,540</td>
<td></td>
</tr>
<tr>
<td>Taxes</td>
<td>69,500</td>
<td>9,567</td>
<td>500,846</td>
<td>330,425</td>
<td></td>
</tr>
<tr>
<td>Net income</td>
<td>$113,395</td>
<td>$15,609</td>
<td>$817,169</td>
<td>$539,115</td>
<td></td>
</tr>
<tr>
<td>OCF</td>
<td>$1,146,625</td>
<td>$1,393,559</td>
<td>$1,276,279</td>
<td>$768,825</td>
<td></td>
</tr>
</tbody>
</table>

Capital spending –\$3,100,00
Land –\$900,000
NWC –\$120,000

–\$4,120,000

Total cash flow 0 $1,146,625 $1,393,559 $1,276,279 $2,274,825
Notice the calculation of the cash flow at time 0. The capital spending on equipment and investment in net working capital are cash outflows. The aftertax selling price of the land is also a cash outflow. Even though no cash is actually spent on the land because the company already owns it, the aftertax cash flow from selling the land is an opportunity cost, so we need to include it in the analysis. With all the project cash flows, we can calculate the NPV, which is:

\[
NPV = -\$4,120,000 + \frac{\$1,146,625}{1.13} + \frac{\$1,393,559}{1.13^2} + \frac{\$1,246,279}{1.13^3} + \frac{\$2,274,825}{1.13^4}
\]

\[
NPV = \$265,791.25
\]

The company should accept the new product line.

25. Replacement decision analysis is the same as the analysis of two competing projects, in this case, keep the current equipment, or purchase the new equipment. We will consider the purchase of the new machine first.

Purchase new machine:

The initial cash outlay for the new machine is the cost of the new machine. We can calculate the operating cash flow created if the company purchases the new machine. The maintenance cost is an incremental cash flow, so using the pro forma income statement, and adding depreciation to net income, the operating cash flow created each year by purchasing the new machine will be:

<table>
<thead>
<tr>
<th>Maintenance cost</th>
<th>$330,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation</td>
<td>$860,000</td>
</tr>
<tr>
<td>EBT</td>
<td>$1,190,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>$476,000</td>
</tr>
<tr>
<td>Net income</td>
<td>$714,000</td>
</tr>
<tr>
<td>OCF</td>
<td>$146,000</td>
</tr>
</tbody>
</table>

Notice the taxes are negative, implying a tax credit. The new machine also has a salvage value at the end of five years, so we need to include this in the cash flows analysis. The aftertax salvage value will be:

<table>
<thead>
<tr>
<th>Sell machine</th>
<th>$800,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxes</td>
<td>$320,000</td>
</tr>
</tbody>
</table>
The NPV of purchasing the new machine is:

\[
\text{NPV} = -\$4,300,000 + \$146,000(PVIFA_{8\%, 5}) + \$480,000 / 1.08^5
\]

\[
\text{NPV} = -\$3,390,384.40
\]

Notice the NPV is negative. This does not necessarily mean we should not purchase the new machine. In this analysis, we are only dealing with costs, so we would expect a negative NPV. The revenue is not included in the analysis since it is not incremental to the machine. Similar to an EAC analysis, we will use the machine with the least negative NPV. Now we can calculate the decision to keep the old machine:

Keep old machine:

The initial cash outlay for the keeping the old machine is the market value of the old machine, including any potential tax. The decision to keep the old machine has an opportunity cost, namely, the company could sell the old machine. Also, if the company sells the old machine at its current value, it will incur taxes. Both of these cash flows need to be included in the analysis. So, the initial cash flow of keeping the old machine will be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keep machine</td>
<td>–$2,200,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>320,000</td>
</tr>
<tr>
<td>Total</td>
<td>–$1,880,000</td>
</tr>
</tbody>
</table>

Next, we can calculate the operating cash flow created if the company keeps the old machine. We need to account for the cost of maintenance, as well as the cash flow effects of depreciation. The pro forma income statement, adding depreciation to net income to calculate the operating cash flow will be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maintenance cost</td>
<td>$845,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>280,000</td>
</tr>
<tr>
<td>EBT</td>
<td>–$1,125,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>–450,000</td>
</tr>
<tr>
<td>Net income</td>
<td>–$675,000</td>
</tr>
<tr>
<td>OCF</td>
<td>–$395,000</td>
</tr>
</tbody>
</table>
The old machine also has a salvage value at the end of five years, so we need to include this in the cash flows analysis. The aftertax salvage value will be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell machine</td>
<td>$120,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>–48,000</td>
</tr>
<tr>
<td>Total</td>
<td>$72,000</td>
</tr>
</tbody>
</table>

So, the NPV of the decision to keep the old machine will be:

\[
\text{NPV} = -\$1,880,000 - 395,000 \text{PVIFA}_{0.05,5} + \frac{72,000}{1.08^5} \\
\text{NPV} = -\$3,408,118.47
\]

The company should buy the new machine since it has a greater NPV.

There is another way to analyze a replacement decision that is often used. It is an incremental cash flow analysis of the change in cash flows from the existing machine to the new machine, assuming the new machine is purchased. In this type of analysis, the initial cash outlay would be the cost of the new machine, and the cash inflow (including any applicable taxes) of selling the old machine. In this case, the initial cash flow under this method would be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase new machine</td>
<td>–$4,300,000</td>
</tr>
<tr>
<td>Sell old machine</td>
<td>2,200,000</td>
</tr>
<tr>
<td>Taxes on old machine</td>
<td>–320,000</td>
</tr>
<tr>
<td>Total</td>
<td>–$2,420,000</td>
</tr>
</tbody>
</table>

The cash flows from purchasing the new machine would be the difference in the operating expenses. We would also need to include the change in depreciation. The old machine has a depreciation of $280,000 per year, and the new machine has a depreciation of $860,000 per year, so the increased depreciation will be $580,000 per year. The pro forma income statement and operating cash flow under this approach will be:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maintenance cost</td>
<td>–$515,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>580,000</td>
</tr>
<tr>
<td>EBT</td>
<td>–$65,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>–$26,000</td>
</tr>
<tr>
<td>Net income</td>
<td>–$39,000</td>
</tr>
<tr>
<td>OCF</td>
<td>$541,000</td>
</tr>
</tbody>
</table>
The salvage value of the differential cash flow approach is more complicated. The company will sell the new machine, and incur taxes on the sale in five years. However, we must also include the lost sale of the old machine. Since we assumed we sold the old machine in the initial cash outlay, we lose the ability to sell the machine in five years. This is an opportunity loss that must be accounted for. So, the salvage value is:

Sell machine $800,000
Taxes –320,000
Lost sale of old –120,000
Taxes on lost sale of old  48,000
Total $408,000

The NPV under this method is:

\[ \text{NPV} = -2,420,000 + 541,000 \times \text{PVIFA}_{8\%, 5} + \frac{408,000}{1.08} \]

\[ \text{NPV} = 17,734.07 \]

So, this analysis still tells us the company should purchase the new machine. This is really the same type of analysis we originally did. Consider this: Subtract the NPV of the decision to keep the old machine from the NPV of the decision to purchase the new machine. You will get:

Differential NPV = –3,390,384.40 – (–3,408,118.47) = 17,734.07

This is the exact same NPV we calculated when using the second analysis method.

26. Here we are comparing two mutually exclusive assets, with inflation. Since each will be replaced when it wears out, we need to calculate the EAC for each. We have real cash flows. Similar to other capital budgeting projects, when calculating the EAC, we can use real cash flows with the real interest rate, or nominal cash flows and the nominal interest rate. Using the Fisher equation to find the real required return, we get:

\[ (1 + R) = (1 + r)(1 + h) \]
\[ (1 + .14) = (1 + r)(1 + .05) \]
\[ r = .0857, \text{ or } 8.57\% \]

This is the interest rate we need to use with real cash flows. We are given the real aftertax cash flows for each asset, so the NPV for the XX40 is:
NPV = –$900 – $120(PVIFA_{8.57\%,3})
NPV = –$1,206.09

So, the EAC for the XX40 is:

–$1,206.09 = EAC(PVIFA_{8.57\%,3})
EAC = –$472.84

And the EAC for the RH45 is:

NPV = –$1,400 – $95(PVIFA_{8.57\%,5})
NPV = –$1,773.66

–$1,773.66 = EAC(PVIFA_{8.57\%,5})
EAC = –$450.94

The company should choose the RH45 because it has the greater EAC.

27. The project has a sales price that increases at 5 percent per year, and a variable cost per unit that increases at 6 percent per year. First, we need to find the sales price and variable cost for each year. The table below shows the price per unit and the variable cost per unit each year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales price</th>
<th>Cost per unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$40.00</td>
<td>$15.00</td>
</tr>
<tr>
<td>2</td>
<td>$42.00</td>
<td>$15.90</td>
</tr>
<tr>
<td>3</td>
<td>$44.10</td>
<td>$16.85</td>
</tr>
<tr>
<td>4</td>
<td>$46.31</td>
<td>$17.87</td>
</tr>
<tr>
<td>5</td>
<td>$48.62</td>
<td>$18.94</td>
</tr>
</tbody>
</table>

Using the sales price and variable cost, we can now construct the pro forma income statement for each year. We can use this income statement to calculate the cash flow each year. We must also make sure to include the net working capital outlay at the beginning of the project, and the recovery of the net working capital at the end of the project. The pro forma income statement and cash flows for each year will be:
<table>
<thead>
<tr>
<th></th>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td></td>
<td>$800,000.</td>
<td>$840,000.</td>
<td>$882,000.</td>
<td>$926,100.</td>
<td>$972,405.</td>
</tr>
<tr>
<td></td>
<td>195,000.0</td>
<td>195,000.0</td>
<td>195,000.0</td>
<td>195,000.0</td>
<td>195,000.0</td>
<td></td>
</tr>
<tr>
<td>Fixed costs</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>300,000.0</td>
<td>318,000.0</td>
<td>337,080.0</td>
<td>357,304.8</td>
<td>378,743.0</td>
<td></td>
</tr>
<tr>
<td>Variable costs</td>
<td>195,000.0</td>
<td>195,000.0</td>
<td>195,000.0</td>
<td>195,000.0</td>
<td>195,000.0</td>
<td>9</td>
</tr>
<tr>
<td>Depreciation</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>EBT</td>
<td>$110,000.0</td>
<td>$132,000.0</td>
<td>$154,920.0</td>
<td>$178,795.0</td>
<td>$203,661.0</td>
<td></td>
</tr>
<tr>
<td>Taxes</td>
<td>37,400.00</td>
<td>44,880.00</td>
<td>52,672.80</td>
<td>60,790.37</td>
<td>69,245.05</td>
<td></td>
</tr>
<tr>
<td>Net income</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>83</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>OCF</td>
<td>$267,600.0</td>
<td>$282,120.0</td>
<td>$297,247.0</td>
<td>$313,004.0</td>
<td>$329,416.0</td>
<td></td>
</tr>
</tbody>
</table>

Capital spending: $975,000
NWC: –25,000

Total cash flow: $354,416.

With these cash flows, the NPV of the project is:

\[
\text{NPV} = -1,000,000 + \frac{267,600}{1.11} + \frac{282,120}{1.11^2} + \frac{297,247.20}{1.11^3} \\
+ \frac{313,004.83}{1.11^4} + \frac{354,416.86}{1.11^5}
\]

\[
\text{NPV} = 103,915.73
\]

We could also answer this problem using the depreciation tax shield approach. The revenues and variable costs are growing annuities, growing at different rates. The fixed costs and depreciation are ordinary annuities. Using the growing annuity equation, the present value of the revenues is:

\[
P\text{V of revenues} = C \left\{ \left[ \frac{1}{(r - g)} - \frac{1}{(r - g)} \times \left[ \frac{(1 + g)(1 + r)}{r} \right]^t \right] \right\}
\]
PV of revenues = $800,000 \left\{ \frac{1}{(1.11 - .05)} - \frac{1}{(1.11 - .05)} \times \frac{(1 + .05)/(1 + .11)^5}{(1 + .05)/(1 + .11)^5} \right\}

PV of revenues = $3,234,520.16

And the present value of the variable costs will be:

PV of variable costs = C \left\{ \frac{1}{(r - g)} - \frac{1}{(r - g)} \times \frac{(1 + g)/(1 + r)^5}{(1 + g)/(1 + r)^5} \right\}

PV of variable costs = $300,000 \left\{ \frac{1}{(1.11 - .06)} - \frac{1}{(1.11 - .06)} \times \frac{(1 + .06)/(1 + .11)^5}{(1 + .06)/(1 + .11)^5} \right\}

PV of variable costs = $1,234,969.52

The fixed costs and depreciation are both ordinary annuities. The present value of each is:

PV of fixed costs = C \left\{ 1 - \frac{1}{(1 + r)^5} \right\} / r

PV of fixed costs = $195,000 \left\{ 1 - \frac{1}{(1 + .11)^5} \right\} / .11

PV of fixed costs = $720,699.92

PV of depreciation = C \left\{ 1 - \frac{1}{(1 + r)^5} \right\} / r

PV of depreciation = $195,000 \left\{ 1 - \frac{1}{(1 + .11)^5} \right\} / .11

PV of depreciation = $720,699.92

Now, we can use the depreciation tax shield approach to find the NPV of the project, which is:

NPV = –$1,000,000 + ($3,234,520.16 – 1,234,969.52 – 720,699.92)(1 – .34) +

($720,699.92)(.34)

+ $25,000 / 1.11^5

NPV = $103,915.73

Challenge

28. Probably the easiest OCF calculation for this problem is the bottom up approach, so we will construct an income statement for each year. Beginning with the initial cash flow at time zero, the project will require an investment in equipment. The project will also require an investment in NWC of $1,500,000. So, the cash flow required for the project today will be:

Capital spending $23,000,000
Change in NWC $1,500,000
Total cash flow $24,500,000
Now we can begin the remaining calculations. Sales figures are given for each year, along with the price per unit. The variable costs per unit are used to calculate total variable costs, and fixed costs are given at $2,400,000 per year. To calculate depreciation each year, we use the initial equipment cost of $23 million, times the appropriate MACRS depreciation each year. The remainder of each income statement is calculated below. Notice at the bottom of the income statement we added back depreciation to get the OCF for each year. The section labeled “Net cash flows” will be discussed below:

<table>
<thead>
<tr>
<th>Year</th>
<th>Ending book value</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$19,713,300</td>
<td>$14,080,600</td>
<td>$10,057,900</td>
<td>$7,185,200</td>
<td>$5,131,300</td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>$28,635,000</td>
<td>$31,740,000</td>
<td>$35,880,000</td>
<td>$33,810,000</td>
<td>$28,980,000</td>
<td></td>
</tr>
<tr>
<td>Variable costs</td>
<td>15,770,000</td>
<td>17,480,000</td>
<td>19,760,000</td>
<td>18,620,000</td>
<td>15,960,000</td>
<td></td>
</tr>
<tr>
<td>Fixed costs</td>
<td>2,400,000</td>
<td>2,400,000</td>
<td>2,400,000</td>
<td>2,400,000</td>
<td>2,400,000</td>
<td></td>
</tr>
<tr>
<td>Depreciation</td>
<td>3,286,700</td>
<td>5,632,700</td>
<td>4,022,700</td>
<td>2,872,700</td>
<td>2,053,900</td>
<td></td>
</tr>
<tr>
<td>EBIT</td>
<td>7,178,300</td>
<td>6,227,300</td>
<td>9,697,300</td>
<td>9,917,300</td>
<td>8,566,100</td>
<td></td>
</tr>
<tr>
<td>Taxes</td>
<td>2,512,405</td>
<td>2,179,555</td>
<td>3,394,055</td>
<td>3,471,055</td>
<td>2,998,135</td>
<td></td>
</tr>
<tr>
<td>Net income</td>
<td>4,665,895</td>
<td>4,047,745</td>
<td>6,303,245</td>
<td>6,446,245</td>
<td>5,567,965</td>
<td></td>
</tr>
<tr>
<td>Depreciation</td>
<td>3,286,700</td>
<td>5,632,700</td>
<td>4,022,700</td>
<td>2,872,700</td>
<td>2,053,900</td>
<td></td>
</tr>
<tr>
<td>Operating cash flow</td>
<td>$7,952,595</td>
<td>$9,680,445</td>
<td>$10,325,945</td>
<td>$9,318,945</td>
<td>$7,621,865</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Net cash flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating cash flow</td>
</tr>
<tr>
<td>Change in NWC</td>
</tr>
<tr>
<td>Capital spending</td>
</tr>
<tr>
<td>Total cash flow</td>
</tr>
</tbody>
</table>

After we calculate the OCF for each year, we need to account for any other cash flows. The other cash flows in this case are NWC cash flows and capital spending, which is the aftertax salvage of the equipment. The required NWC is 15 percent of the sales increase in the next year. We will work through the NWC cash flow for Year 1. The total NWC in Year 1 will be 15 percent of sales increase from Year 1 to Year 2, or:

Increase in NWC for Year 1 = \(0.15(31,740,000 - 28,635,000)\)
Increase in NWC for Year 1 = $465,750
Notice that the NWC cash flow is negative. Since the sales are increasing, we will have to spend more money to increase NWC. In Year 4, the NWC cash flow becomes positive when sales are declining. And, in Year 5, the NWC cash flow is the recovery of all NWC the company still has in the project.

To calculate the aftertax salvage value, we first need the book value of the equipment. The book value at the end of the five years will be the purchase price, minus the total depreciation. So, the ending book value is:

\[
\text{Ending book value} = \$23,000,000 - (\$3,286,700 + 5,632,700 + 4,022,700 + 2,872,700 + 2,053,900) \\
\text{Ending book value} = \$5,131,300
\]

The market value of the used equipment is 20 percent of the purchase price, or $4.6 million, so the aftertax salvage value will be:

\[
\text{Aftertax salvage value} = \$4,600,000 + (\$5,131,300 - 4,600,000)(.35) \\
\text{Aftertax salvage value} = \$4,785,955
\]

The aftertax salvage value is included in the total cash flows as capital spending. Now we have all of the cash flows for the project. The NPV of the project is:

\[
\text{NPV} = -\$24,500,000 + \frac{\$7,486,845}{1.18} + \frac{\$9,059,445}{1.18^2} + \frac{\$10,636,445}{1.18^3} + \frac{\$10,043,445}{1.18^4} + \frac{\$13,959,570}{1.18^5} \\
\text{NPV} = \$6,106,958.94
\]

And the IRR is:

\[
\text{IRR} = 0 = -\$24,500,000 + \frac{\$7,486,845}{(1 + \text{IRR})} + \frac{\$9,059,445}{(1 + \text{IRR})^2} + \frac{\$10,636,445}{(1 + \text{IRR})^3} + \frac{\$10,043,445}{(1 + \text{IRR})^4} + \frac{\$13,959,570}{(1 + \text{IRR})^5} \\
\text{IRR} = 27.54\%
\]

We should accept the project.

29. To find the initial pretax cost savings necessary to buy the new machine, we should use the tax shield approach to find the OCF. We begin by calculating the depreciation each year using the MACRS depreciation schedule. The depreciation each year is:
\[ D_1 = \$640,000(0.3333) = \$213,312 \]
\[ D_2 = \$640,000(0.4445) = \$284,480 \]
\[ D_3 = \$640,000(0.1481) = \$94,784 \]
\[ D_4 = \$640,000(0.0741) = \$47,424 \]

Using the tax shield approach, the OCF each year is:

\[ OCF_1 = (S - C)(1 - 0.35) + 0.35(\$213,312) \]
\[ OCF_2 = (S - C)(1 - 0.35) + 0.35(\$284,480) \]
\[ OCF_3 = (S - C)(1 - 0.35) + 0.35(\$94,784) \]
\[ OCF_4 = (S - C)(1 - 0.35) + 0.35(\$47,424) \]
\[ OCF_5 = (S - C)(1 - 0.35) \]

Now we need the aftertax salvage value of the equipment. The aftertax salvage value is:

\[ \text{After-tax salvage value} = \$60,000(1 - 0.35) = \$39,000 \]

To find the necessary cost reduction, we must realize that we can split the cash flows each year. The OCF in any given year is the cost reduction \((S - C)\) times one minus the tax rate, which is an annuity for the project life, and the depreciation tax shield. To calculate the necessary cost reduction, we would require a zero NPV. The equation for the NPV of the project is:

\[ \text{NPV} = 0 = -\$640,000 - 55,000 + (S - C)(0.65)(PVIFA_{12\%,5}) + 0.35(\$213,312/1.12^1 + \$284,480/1.12^2 + \$94,784/1.12^3 + \$47,424/1.12^4) + (\$55,000 + 39,000)/1.12^5 \]
Solving this equation for the sales minus costs, we get:

\[(S - C)(0.65)(PVIFA_{12\%,5}) = $461,465.41\]
\[(S - C) = $196,946.15\]

30. To find the bid price, we need to calculate all other cash flows for the project, and then solve for the bid price. The aftertax salvage value of the equipment is:

Aftertax salvage value = $150,000(1 – 0.35) = $97,500

Now we can solve for the necessary OCF that will give the project a zero NPV. The equation for the NPV of the project is:

\[NPV = 0 = – $1,800,000 – 130,000 + OCF(PVIFA_{14\%,5}) + [(130,000 + 97,500) / 1.145]\]

Solving for the OCF, we find the OCF that makes the project NPV equal to zero is:

\[OCF = $1,811,843.63 / PVIFA_{14\%,5} = $527,760.24\]

The easiest way to calculate the bid price is the tax shield approach, so:

\[OCF = $527,760.24 = [(P – v)Q – FC \times (1 – t_c) + t_cD\]
\[P = $14.81\]

31. a. This problem is basically the same as the previous problem, except that we are given a sales price. The cash flow at Time 0 for all three parts of this question will be:

| Capital spending | –$1,800,000 |
| Change in NWC    | –130,000    |
| Total cash flow  | –$1,930,000 |

We will use the initial cash flow and the salvage value we already found in that problem. Using the bottom up approach to calculating the OCF, we get:

Assume price per unit = $16 and units/year = 140,000

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$2,240,000</td>
<td>$2,240,000</td>
<td>$2,240,000</td>
<td>$2,240,000</td>
<td>$2,240,000</td>
</tr>
</tbody>
</table>
Solutions Manual

Variable costs | 1,190,000 | 1,190,000 | 1,190,000 | 1,190,000 | 1,190,000 |
Fixed costs | 265,000 | 265,000 | 265,000 | 265,000 | 265,000 |
Depreciation | 360,000 | 360,000 | 360,000 | 360,000 | 360,000 |
EBIT | $425,000 | $425,000 | $425,000 | $425,000 | $425,000 |
Taxes (35%) | 148,750 | 148,750 | 148,750 | 148,750 | 148,750 |
Net Income | $276,250 | $276,250 | $276,250 | $276,250 | $276,250 |
Depreciation | 360,000 | 360,000 | 360,000 | 360,000 | 360,000 |
Operating CF | $636,250 | $636,250 | $636,250 | $636,250 | $636,250 |

Year | 1 | 2 | 3 | 4 | 5
Operating CF | $636,250 | $636,250 | $636,250 | $636,250 | $636,250 |
Change in NWC | | | | | 130,000 |
Capital spending | | | | | 97,500 |
Total CF | $636,250 | $636,250 | $636,250 | $636,250 | $863,750 |

With these cash flows, the NPV of the project is:

\[
\text{NPV} = -1,800,000 - 130,000 + \text{OCF}(\text{PVIFA}_{14\%,5}) + \frac{[(130,000 + 97,500)}{1.14^5}\]
\[
\text{NPV} = 372,454.14
\]

If the actual price is above the bid price that results in a zero NPV, the project will have a positive NPV. As for the cartons sold, if the number of cartons sold increases, the NPV will increase, and if the costs increase, the NPV will decrease.

b. To find the minimum number of cartons sold to still breakeven, we need to use the tax shield approach to calculating OCF, and solve the problem similar to finding a bid price. Using the initial cash flow and salvage value we already calculated, the equation for a zero NPV of the project is:

\[
\text{NPV} = 0 = -1,800,000 - 130,000 + \text{OCF}(\text{PVIFA}_{14\%,5}) + \frac{[(130,000 + 97,500)}{1.14^5}\]
\[
\text{OCF} = \frac{1,811,843.63}{\text{PVIFA}_{14\%,5}} = 527,760.24
\]
Now we can use the tax shield approach to solve for the minimum quantity as follows:

\[
\text{OCF} = \$527,760.24 = [(P - v)Q - FC \{1 - t_c\} + t_cD]
\]

\[
\text{\$527,760.24} = [($16.00 - 8.50)Q - 265,000 \{1 - 0.35\} + 0.35($1,800,000/5)]
\]

\[Q = 117,746\]

As a check, we can calculate the NPV of the project with this quantity. The calculations are:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$1,883,931</td>
<td>$1,883,931</td>
<td>$1,883,931</td>
<td>$1,883,931</td>
<td>$1,883,931</td>
</tr>
<tr>
<td>Variable costs</td>
<td>1,000,838</td>
<td>1,000,838</td>
<td>1,000,838</td>
<td>1,000,838</td>
<td>1,000,838</td>
</tr>
<tr>
<td>Fixed costs</td>
<td>265,000</td>
<td>265,000</td>
<td>265,000</td>
<td>265,000</td>
<td>265,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
</tr>
<tr>
<td>EBIT</td>
<td>$258,093</td>
<td>$258,093</td>
<td>$258,093</td>
<td>$258,093</td>
<td>$258,093</td>
</tr>
<tr>
<td>Taxes (35%)</td>
<td>90,332</td>
<td>90,332</td>
<td>90,332</td>
<td>90,332</td>
<td>90,332</td>
</tr>
<tr>
<td>Net Income</td>
<td>$167,760</td>
<td>$167,760</td>
<td>$167,760</td>
<td>$167,760</td>
<td>$167,760</td>
</tr>
<tr>
<td>Depreciation</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
</tr>
<tr>
<td>Operating CF</td>
<td>$527,760</td>
<td>$527,760</td>
<td>$527,760</td>
<td>$527,760</td>
<td>$527,760</td>
</tr>
</tbody>
</table>

\[
\text{NPV} = -\text{\$1,800,000} - 130,000 + \text{\$527,760 (PVIFA}_{14\%}\text{,5)} + [(130,000 + 97,500) / 1.145^5] \approx \text{\$0}
\]

Note that the NPV is not exactly equal to zero because we had to round the number of cartons sold; you cannot sell one-half of a carton.

c. To find the highest level of fixed costs and still breakeven, we need to use the tax shield approach to calculating OCF, and solve the problem similar to finding a bid price. Using the initial cash flow and salvage value we already calculated, the equation for a zero NPV of the project is:
\[ \text{NPV} = 0 = -\$1,800,000 - 130,000 + \text{OCF}(\text{PVIFA}_{14\%,5}) + \left[ (\$130,000 + 97,500) / 1.14^5 \right] \]
\[ \text{OCF} = \$1,811,843.63 / \text{PVIFA}_{14\%,5} = \$527,760.24 \]

Notice this is the same OCF we calculated in part \( b \). Now we can use the tax shield approach to solve for the maximum level of fixed costs as follows:

\[ \text{OCF} = \$527,760.24 = \left[ (P-v)Q - FC \right] (1-t_C) + t_CD \]
\[ \$527,760.24 = \left[ ($16.00 - $8.50)(140,000) - FC \right] (1-0.35) + 0.35($1,800,000/5) \]
\[ \text{FC} = \$431,907.33 \]

As a check, we can calculate the NPV of the project with this quantity. The calculations are:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$2,240,000</td>
<td>$2,240,000</td>
<td>$2,240,000</td>
<td>$2,240,000</td>
<td>$2,240,000</td>
</tr>
<tr>
<td>Variable costs</td>
<td>1,190,000</td>
<td>1,190,000</td>
<td>1,190,000</td>
<td>1,190,000</td>
<td>1,190,000</td>
</tr>
<tr>
<td>Fixed costs</td>
<td>431,907</td>
<td>431,907</td>
<td>431,907</td>
<td>431,907</td>
<td>431,907</td>
</tr>
<tr>
<td>Depreciation</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
</tr>
<tr>
<td>EBIT</td>
<td>$258,093</td>
<td>$258,093</td>
<td>$258,093</td>
<td>$258,093</td>
<td>$258,093</td>
</tr>
<tr>
<td>Taxes (35%)</td>
<td>90,332</td>
<td>90,332</td>
<td>90,332</td>
<td>90,332</td>
<td>90,332</td>
</tr>
<tr>
<td>Net Income</td>
<td>$167,760</td>
<td>$167,760</td>
<td>$167,760</td>
<td>$167,760</td>
<td>$167,760</td>
</tr>
<tr>
<td>Depreciation</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
<td>360,000</td>
</tr>
<tr>
<td>Operating CF</td>
<td>$527,760</td>
<td>$527,760</td>
<td>$527,760</td>
<td>$527,760</td>
<td>$527,760</td>
</tr>
<tr>
<td>Change in NWC</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>130,000</td>
</tr>
<tr>
<td>Capital spending</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>97,500</td>
</tr>
<tr>
<td>Total CF</td>
<td>$527,760</td>
<td>$527,760</td>
<td>$527,760</td>
<td>$527,760</td>
<td>$755,260</td>
</tr>
</tbody>
</table>

\[ \text{NPV} = -\$1,800,000 - 130,000 + \$527,760(\text{PVIFA}_{14\%,5}) + \left[ (\$130,000 + 97,500) / 1.14^5 \right] \approx \$0 \]
32. We need to find the bid price for a project, but the project has extra cash flows. Since we don’t already produce the keyboard, the sales of the keyboard outside the contract are relevant cash flows. Since we know the extra sales number and price, we can calculate the cash flows generated by these sales. The cash flow generated from the sale of the keyboard outside the contract is:

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$820,000</td>
<td>$2,460,000</td>
<td>$2,870,000</td>
</tr>
<tr>
<td>Variable costs</td>
<td>420,000</td>
<td>1,260,000</td>
<td>1,470,000</td>
</tr>
<tr>
<td>EBT</td>
<td>$400,000</td>
<td>$1,200,000</td>
<td>$1,400,000</td>
</tr>
<tr>
<td>Tax</td>
<td>160,000</td>
<td>480,000</td>
<td>560,000</td>
</tr>
<tr>
<td>Net income (and OCF)</td>
<td>$240,000</td>
<td>$720,000</td>
<td>$840,000</td>
</tr>
</tbody>
</table>

So, the addition to NPV of these market sales is:

NPV of market sales = $240,000/1.13 + $720,000/1.13^2 + $840,000/1.13^3 + $420,000/1.13^4

NPV of market sales = $1,616,010.99

You may have noticed that we did not include the initial cash outlay, depreciation, or fixed costs in the calculation of cash flows from the market sales. The reason is that it is irrelevant whether or not we include these here. Remember that we are not only trying to determine the bid price, but we are also determining whether or not the project is feasible. In other words, we are trying to calculate the NPV of the project, not just the NPV of the bid price. We will include these cash flows in the bid price calculation. Whether we include these costs in this initial calculation is irrelevant since you will come up with the same bid price if you include these costs in this calculation, or if you include them in the bid price calculation.

Next, we need to calculate the aftertax salvage value, which is:

Aftertax salvage value = $200,000(1 – .40) = $120,000

Instead of solving for a zero NPV as is usual in setting a bid price, the company president requires an NPV of $100,000, so we will solve for a NPV of that amount. The NPV equation for this project is (remember to include the NWC cash flow at the beginning of the project, and the NWC recovery at the end):
NPV = $100,000 = –$3,400,000 – 75,000 + 1,616,010.99 + OCF (PVIFA_{13\%,4}) + \left(\frac{$120,000 + (75,000)}{1.13^4}\right)

Solving for the OCF, we get:

OCF = $1,839,391.85 / PVIFA_{13\%,4} = $618,392.87

Now we can solve for the bid price as follows:

OCF = $618,392.87 = \left(\frac{P – v}{1 – t_C}\right) + t_C D - FC

$618,392.87 = \left(\frac{P – $105}{15,000} – $700,000\right)(1 – 0.40) + 0.40(\frac{$3,400,000}{4})

P = $182.60

33. a. Since the two computers have unequal lives, the correct method to analyze the decision is the EAC. We will begin with the EAC of the new computer. Using the depreciation tax shield approach, the OCF for the new computer system is:

OCF = ($85,000)(1 – .38) + ($580,000 / 5)(.38) = $96,780

Notice that the costs are positive, which represents a cash inflow. The costs are positive in this case since the new computer will generate a cost savings. The only initial cash flow for the new computer is the cost of $780,000. We next need to calculate the aftertax salvage value, which is:

Aftertax salvage value = $130,000(1 – .38) = $80,600

Now we can calculate the NPV of the new computer as:

NPV = –$580,000 + $96,780(PVIFA_{14\%,5}) + $80,600 / 1.14^5

NPV = –$205,885.31

And the EAC of the new computer is:

EAC = – $205,885.31 / (PVIFA_{14\%,5}) = –$59,971.00

Analyzing the old computer, the only OCF is the depreciation tax shield, so:
OCF = $90,000(.38) = $34,200

The initial cost of the old computer is a little trickier. You might assume that since we already own the old computer there is no initial cost, but we can sell the old computer, so there is an opportunity cost. We need to account for this opportunity cost. To do so, we will calculate the aftertax salvage value of the old computer today. We need the book value of the old computer to do so. The book value is not given directly, but we are told that the old computer has depreciation of $90,000 per year for the next three years, so we can assume the book value is the total amount of depreciation over the remaining life of the system, or $270,000. So, the aftertax salvage value of the old computer is:

Aftertax salvage value = $230,000 + ($270,000 – 230,000)(.38) = $245,200

This is the initial cost of the old computer system today because we are forgoing the opportunity to sell it today. We next need to calculate the aftertax salvage value of the computer system in two years since we are “buying” it today. The aftertax salvage value in two years is:

Aftertax salvage value = $60,000 + ($90,000 – 60,000)(.38) = $71,400

Now we can calculate the NPV of the old computer as:

\[ \text{NPV} = -245,200 + 34,200 \times (PVIFA_{14\%,2}) + \frac{71,400}{1.14^2} \]

\[ \text{NPV} = -133,944.23 \]

And the EAC of the old computer is:

\[ \text{EAC} = - \frac{133,944.23}{(PVIFA_{14\%,2})} = -81,342.95 \]

If we are going to replace the system in two years no matter what our decision today, we should instead replace it today since the EAC is lower.

b. If we are only concerned with whether or not to replace the machine now, and are not worrying about what will happen in two years, the correct analysis is NPV. To calculate the NPV of the decision on the computer system now, we need the difference in the total cash flows of the old computer system and the new computer
system. From our previous calculations, we can say the cash flows for each computer system are:

<table>
<thead>
<tr>
<th></th>
<th>New computer</th>
<th>Old computer</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$580,000</td>
<td>-$245,200</td>
<td>-$334,800</td>
</tr>
<tr>
<td>1</td>
<td>96,780</td>
<td>34,200</td>
<td>62,580</td>
</tr>
<tr>
<td>2</td>
<td>96,780</td>
<td>105,600</td>
<td>-8,820</td>
</tr>
<tr>
<td>3</td>
<td>96,780</td>
<td>0</td>
<td>96,780</td>
</tr>
<tr>
<td>4</td>
<td>96,780</td>
<td>0</td>
<td>96,780</td>
</tr>
<tr>
<td>5</td>
<td>177,380</td>
<td>0</td>
<td>177,380</td>
</tr>
</tbody>
</table>

Since we are only concerned with marginal cash flows, the cash flows of the decision to replace the old computer system with the new computer system are the differential cash flows. The NPV of the decision to replace, ignoring what will happen in two years is:

\[
NPV = -334,800 + \frac{62,580}{1.14} - \frac{8,820}{1.14^2} + \frac{96,780}{1.14^3} + \frac{96,780}{1.14^4} + \frac{177,380}{1.14^5}
\]

\[
NPV = -71,941.08
\]

If we are not concerned with what will happen in two years, we should not replace the old computer system.

34. To answer this question, we need to compute the NPV of all three alternatives, specifically, continue to rent the building, Project A, or Project B. We would choose the project with the highest NPV. If all three of the projects have a positive NPV, the project that is more favorable is the one with the highest NPV.

There are several important cash flows we should not consider in the incremental cash flow analysis. The remaining fraction of the value of the building and depreciation are not incremental and should not be included in the analysis of the two alternatives. The $1,450,000 purchase price of the building is the same for all three options and should be ignored. In effect, what we are doing is finding the NPV of the future cash flows of each option, so the only cash flow today would be the building modifications needed for Project A and Project B. If we did include these costs, the effect would be to lower the NPV of all three options by the same amount, thereby leading to the same conclusion. The cash flows from renting the building after year 15 are also irrelevant. No matter what the company
chooses today, it will rent the building after year 15, so these cash flows are not incremental to any project.

We will begin by calculating the NPV of the decision of continuing to rent the building first.

Continue to rent:

Rent $61,000  
Taxes 20,740  
Net income $40,260

Since there is no incremental depreciation, the operating cash flow is simply the net income. So, the NPV of the decision to continue to rent is:

\[
\text{NPV} = \$40,260(\text{PVIFA}_{12\%,15}) \\
\text{NPV} = \$274,205.40
\]

Product A:

Next, we will calculate the NPV of the decision to modify the building to produce Product A. The income statement for this modification is the same for the first 14 years, and in Year 15, the company will have an additional expense to convert the building back to its original form. This will be an expense in Year 15, so the income statement for that year will be slightly different. The cash flow at time zero will be the cost of the equipment, and the cost of the initial building modifications, both of which are depreciable on a straight-line basis. So, the pro forma cash flows for Product A are:

Initial cash outlay:

| Building modifications | –$95,000  
| Equipment | –$195,000  
| Total cash flow | –$290,000

<table>
<thead>
<tr>
<th></th>
<th>Years 1-14</th>
<th>Year 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>$180,000</td>
<td>$180,000</td>
</tr>
<tr>
<td>Expenditures</td>
<td>70,000</td>
<td>70,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>19,333</td>
<td>19,333</td>
</tr>
<tr>
<td>Restoration cost</td>
<td>0</td>
<td>55,000</td>
</tr>
</tbody>
</table>
The OCF each year is net income plus depreciation. So, the NPV for modifying the building to manufacture Product A is:

\[
NPV = -290,000 + 79,173(PVIFA_{12\%,14}) + 42,873 / 1.12^{15}
\]

\[
NPV = 242,606.97
\]

Product B:

Now we will calculate the NPV of the decision to modify the building to produce Product B. The income statement for this modification is the same for the first 14 years, and in year 15, the company will have an additional expense to convert the building back to its original form. This will be an expense in year 15, so the income statement for that year will be slightly different. The cash flow at time zero will be the cost of the equipment, and the cost of the initial building modifications, both of which are depreciable on a straight-line basis. So, the pro forma cash flows for Product B are:

<table>
<thead>
<tr>
<th>Initial cash outlay:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Building modifications</td>
<td>–$125,000</td>
</tr>
<tr>
<td>Equipment</td>
<td>–230,000</td>
</tr>
<tr>
<td>Total cash flow</td>
<td>–$355,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Years</th>
<th>Revenue</th>
<th>Expenditures</th>
<th>Depreciation</th>
<th>Restoration cost</th>
<th>EBT</th>
<th>Tax</th>
<th>NI</th>
<th>OCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-14</td>
<td>$215,000</td>
<td>90,000</td>
<td>23,667</td>
<td>0</td>
<td>$101,333</td>
<td>34,453</td>
<td>$66,880</td>
<td>$90,547</td>
</tr>
<tr>
<td>15</td>
<td>$215,000</td>
<td>90,000</td>
<td>23,667</td>
<td>80,000</td>
<td>$21,333</td>
<td>7,253</td>
<td>$14,080</td>
<td>$37,747</td>
</tr>
</tbody>
</table>

The OCF each year is net income plus depreciation. So, the NPV for modifying the building to manufacture Product B is:
NPV = –$355,000 + $90,547(PVIFA_{12\%,14}) + $37,747 / 1.1215
NPV = $252,054.71

Since renting has the highest NPV, the company should continue to rent the building.

We could have also done the analysis as the incremental cash flows between Product A and continuing to rent the building, and the incremental cash flows between Product B and continuing to rent the building. The results of this type of analysis would be:

NPV of differential cash flows between Product A and continuing to rent:

\[
NPV = NPV_{Product A} - NPV_{Rent}
\]
\[
NPV = $242,606.97 - 274,205.40
\]
\[
NPV = –$31,598.43
\]

NPV of differential cash flows between Product B and continuing to rent:

\[
NPV = NPV_{Product B} - NPV_{Rent}
\]
\[
NPV = $252,054.71 - 274,205.40
\]
\[
NPV = –$22,150.69
\]

Since the differential NPV of both products and renting is negative, the company should continue to rent, which is the same as our original result.

35. The discount rate is expressed in real terms, and the cash flows are expressed in nominal terms. We can answer this question by converting all of the cash flows to real dollars. We can then use the real interest rate. The real value of each cash flow is the present value of the year 1 nominal cash flows, discounted back to the present at the inflation rate. So, the real value of the revenue and costs will be:

Revenue in real terms = $265,000 / 1.06 = $250,000.00
Labor costs in real terms = $185,000 / 1.06 = $174,528.30
Other costs in real terms = $55,000 / 1.06 = $51,886.79
Lease payment in real terms = $90,000 / 1.06 = $84,905.66
Revenues, labor costs, and other costs are all growing perpetuities. Each has a different growth rate, so we must calculate the present value of each separately. Using the real required return, the present value of each of these is:

\[
PV_{\text{Revenue}} = \frac{250,000.00}{0.10 - 0.04} = 4,166,666.67
\]
\[
PV_{\text{Labor costs}} = \frac{174,528.30}{0.10 - 0.03} = 2,493,261.46
\]
\[
PV_{\text{Other costs}} = \frac{51,886.79}{0.10 - 0.01} = 576,519.92
\]

The lease payments are constant in nominal terms, so they are declining in real terms by the inflation rate. Therefore, the lease payments form a growing perpetuity with a negative growth rate. The real present value of the lease payments is:

\[
PV_{\text{Lease payments}} = \frac{84,905.66}{0.10 + 0.06} = 530,660.38
\]

Now we can use the tax shield approach to calculate the net present value. Since there is no investment in equipment, there is no depreciation; therefore, no depreciation tax shield, so we will ignore this in our calculation. This means the cash flows each year are equal to net income. There is also no initial cash outlay, so the NPV is the present value of the future aftertax cash flows. The NPV of the project is:

\[
NPV = (PV_{\text{Revenue}} - PV_{\text{Labor costs}} - PV_{\text{Other costs}} - PV_{\text{Lease payments}})(1 - t_c)
\]
\[
NPV = (4,166,666.67 - 2,493,261.46 - 576,519.92 - 530,660.38)(1 - .34)
\]
\[
NPV = $373,708.45
\]

Alternatively, we could have solved this problem by expressing everything in nominal terms. This approach yields the same answer as given above. However, in this case, the computation would have been impossible. The reason is that we are dealing with growing perpetuities. In other problems, when calculating the NPV of nominal cash flows, we could simply calculate the nominal cash flow each year since the cash flows were finite. Because of the perpetual nature of the cash flows in this problem, we cannot calculate the nominal cash flows each year until the end of the project. When faced with two alternative approaches, where both are equally correct, always choose the simplest one.
36. We are given the real revenue and costs, and the real growth rates, so the simplest way to solve this problem is to calculate the NPV with real values. While we could calculate the NPV using nominal values, we would need to find the nominal growth rates, and convert all values to nominal terms. The real labor costs will increase at a real rate of two percent per year, and the real energy costs will increase at a real rate of three percent per year, so the real costs each year will be:

<table>
<thead>
<tr>
<th>Year</th>
<th>Real labor cost each year</th>
<th>Real energy cost each year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>$15.75</td>
<td>$3.80</td>
</tr>
<tr>
<td>Year 2</td>
<td>$16.07</td>
<td>$3.91</td>
</tr>
<tr>
<td>Year 3</td>
<td>$16.39</td>
<td>$4.03</td>
</tr>
<tr>
<td>Year 4</td>
<td>$16.71</td>
<td>$4.15</td>
</tr>
</tbody>
</table>

Remember that the depreciation tax shield also affects a firm’s aftertax cash flows. The present value of the depreciation tax shield must be added to the present value of a firm’s revenues and expenses to find the present value of the cash flows related to the project. The depreciation the firm will recognize each year is:

\[
\text{Annual depreciation} = \frac{\text{Investment}}{\text{Economic Life}}
\]

\[
\text{Annual depreciation} = \frac{165,000,000}{4} \quad \text{Annual depreciation} = 41,250,000
\]

Depreciation is a nominal cash flow, so to find the real value of depreciation each year, we discount the real depreciation amount by the inflation rate. Doing so, we find the real depreciation each year is:

- Year 1 real depreciation = \( \frac{41,250,000}{1.05} = 39,285,714.29 \)
- Year 2 real depreciation = \( \frac{41,250,000}{1.05^2} = 37,414,965.99 \)
- Year 3 real depreciation = \( \frac{41,250,000}{1.05^3} = 35,633,300.94 \)
- Year 4 real depreciation = \( \frac{41,250,000}{1.05^4} = 33,936,477.09 \)

Now we can calculate the pro forma income statement each year in real terms. We can then add back depreciation to net income to find the operating cash flow each year. Doing so, we find the cash flow of the project each year is:

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$69,300,000</td>
<td>$74,250,000</td>
<td>$84,150,000</td>
<td>$79,200,000</td>
</tr>
<tr>
<td>Labor cost</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>17,640,000.0</td>
<td>19,278,000.0</td>
<td>22,285,368.0</td>
<td>21,393,953.2</td>
</tr>
</tbody>
</table>
We can use the total cash flows each year to calculate the NPV, which is:

\[
NPV = -165,000,000 + 46,926,062.86 / 1.04 + 48,421,379.44 / 1.04^2 + \\
52,267,491.45 / 1.04^3 + 49,032,658.81 / 1.04^4 \\
NPV = 13,268,433.31
\]

We can find the real revenue and production costs by multiplying each by the units sold. We must be sure to discount the depreciation, which is in nominal terms. We can then find the pro forma net income, and add back depreciation to find the operating cash flow. Discounting the depreciation each year by the inflation rate, we find the following cash flows each year:
<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$25,050,000</td>
<td>$25,050,000</td>
<td>$25,050,000</td>
</tr>
<tr>
<td>Production costs</td>
<td>12,300,000</td>
<td>12,300,000</td>
<td>12,300,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>7,443,366</td>
<td>7,226,569</td>
<td>7,016,086</td>
</tr>
<tr>
<td>EBT</td>
<td>$5,306,634</td>
<td>$5,523,431</td>
<td>$5,733,914</td>
</tr>
<tr>
<td>Tax</td>
<td>1,804,256</td>
<td>1,877,967</td>
<td>1,949,531</td>
</tr>
<tr>
<td>OCF</td>
<td>$10,945,744</td>
<td>$10,872,033</td>
<td>$10,800,469</td>
</tr>
</tbody>
</table>

And the NPV of the headache only pill is:

\[
\text{NPV} = -23,000,000 + \frac{10,945,744}{1.07} + \frac{10,872,033}{1.07^2} + \frac{10,800,469}{1.07^3}
\]

\[
\text{NPV} = 5,542,122.70
\]

Headache and arthritis:

For the headache and arthritis pill project, the equipment has a salvage value. We will find the aftertax salvage value of the equipment first, which will be:

- Market value: $1,000,000
- Taxes: $340,000
- Total: $660,000
Remember, to calculate the taxes on the equipment salvage value, we take the book value minus the market value, times the tax rate. Using the same method as the headache only pill, the cash flows each year for the headache and arthritis pill will be:

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$37,575,000</td>
<td>$37,575,000</td>
<td>$37,575,000</td>
</tr>
<tr>
<td>Production costs</td>
<td>20,925,000</td>
<td>20,925,000</td>
<td>20,925,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>10,355,987</td>
<td>10,054,356</td>
<td>9,761,511</td>
</tr>
<tr>
<td>EBT</td>
<td>$6,294,013</td>
<td>$6,595,644</td>
<td>$6,888,489</td>
</tr>
<tr>
<td>Tax</td>
<td>2,139,964</td>
<td>2,242,519</td>
<td>2,342,086</td>
</tr>
<tr>
<td>Net income</td>
<td>$4,154,049</td>
<td>$4,353,125</td>
<td>$4,546,403</td>
</tr>
<tr>
<td>OCF</td>
<td>$14,510,036</td>
<td>$14,407,481</td>
<td>$14,307,914</td>
</tr>
</tbody>
</table>

So, the NPV of the headache and arthritis pill is:

\[
\text{NPV} = \frac{-32,000,000}{1.07} + \frac{14,510,036}{1.07^2} + \frac{14,407,481}{1.07^3} + (\frac{14,307,914 + 660,000}{1.07^4}) + \frac{14,307,914}{1.07^5}
\]

\[
\text{NPV} = 6,363,109.18
\]

The company should manufacture the headache and arthritis remedy since the project has a higher NPV.

38. Since the project requires an initial investment in inventory as a percentage of sales, we will calculate the sales figures for each year first. The incremental sales will include the sales of the new table, but we also need to include the lost sales of the existing model. This is an erosion cost of the new table. The lost sales of the existing table are constant for every year, but the sales of the new table change every year. So, the total incremental sales figure for the five years of the project will be:

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>$10,980,000</td>
<td>$11,895,000</td>
<td>$15,250,000</td>
<td>$14,335,000</td>
<td>$12,810,000</td>
</tr>
<tr>
<td>Lost sales</td>
<td>$-1,125,000</td>
<td>$-1,125,000</td>
<td>$-1,125,000</td>
<td>$-1,125,000</td>
<td>$-1,125,000</td>
</tr>
<tr>
<td>Total</td>
<td>$9,855,000</td>
<td>$10,770,000</td>
<td>$14,125,000</td>
<td>$13,210,000</td>
<td>$11,685,000</td>
</tr>
</tbody>
</table>

Now we will calculate the initial cash outlay that will occur today. The company has the necessary production capacity to manufacture the new table without adding equipment today. So, the equipment will not be purchased today, but rather in two years. The reason is that the existing capacity is not being used. If the existing capacity were being used, the
new equipment would be required, so it would be a cash flow today. The old equipment would have an opportunity cost if it could be sold. As there is no discussion that the existing equipment could be sold, we must assume it cannot be sold. The only initial cash flow is the cost of the inventory. The company will have to spend money for inventory in the new table, but will be able to reduce inventory of the existing table. So, the initial cash flow today is:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>New table</td>
<td>–$1,098,000</td>
</tr>
<tr>
<td>Old table</td>
<td>112,500</td>
</tr>
<tr>
<td>Total</td>
<td>–$985,500</td>
</tr>
</tbody>
</table>

In year 2, the company will have a cash outflow to pay for the cost of the new equipment. Since the equipment will be purchased in two years rather than now, the equipment will have a higher salvage value. The book value of the equipment in five years will be the initial cost, minus the accumulated depreciation, or:

Book value = $18,000,000 – 2,572,200 – 4,408,200 – 3,148,200
Book value = $7,871,400

The taxes on the salvage value will be:

Taxes on salvage = ($7,871,400 – 7,400,000)(0.40)
Taxes on salvage = $188,560

So, the aftertax salvage value of the equipment in five years will be:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell equipment</td>
<td>$7,400,000</td>
</tr>
<tr>
<td>Taxes</td>
<td>188,560</td>
</tr>
<tr>
<td>Salvage value</td>
<td>$7,588,560</td>
</tr>
</tbody>
</table>

Next, we need to calculate the variable costs each year. The variable costs of the lost sales are included as a variable cost savings, so the variable costs will be:

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>$4,941,000</td>
<td>$5,352,750</td>
<td>$6,862,500</td>
<td>$6,450,750</td>
<td>$5,764,500</td>
</tr>
<tr>
<td>Lost sales</td>
<td>–450,000</td>
<td>–450,000</td>
<td>–450,000</td>
<td>–450,000</td>
<td>–450,000</td>
</tr>
<tr>
<td>Variable costs</td>
<td>$4,491,000</td>
<td>$4,902,750</td>
<td>$6,412,500</td>
<td>$6,000,750</td>
<td>$5,314,500</td>
</tr>
</tbody>
</table>
Now we can prepare the rest of the pro forma income statements for each year. The project will have no incremental depreciation for the first two years as the equipment is not purchased for two years. Adding back depreciation to net income to calculate the operating cash flow, we get:

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales</th>
<th>VC</th>
<th>Fixed costs</th>
<th>Dep.</th>
<th>EBT</th>
<th>Tax</th>
<th>NI</th>
<th>+Dep.</th>
<th>OCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$9,855,000</td>
<td>$4,491,000</td>
<td>1,900,000</td>
<td>0</td>
<td>$3,464,000</td>
<td>$1,385,600</td>
<td>$2,078,400</td>
<td>0</td>
<td>$2,078,400</td>
</tr>
<tr>
<td>2</td>
<td>$10,770,000</td>
<td>4,902,750</td>
<td>1,900,000</td>
<td>0</td>
<td>$3,967,250</td>
<td>1,586,900</td>
<td>$2,380,350</td>
<td>0</td>
<td>$2,380,350</td>
</tr>
<tr>
<td>3</td>
<td>$14,125,000</td>
<td>6,412,500</td>
<td>1,900,000</td>
<td>2,572,200</td>
<td>$3,240,300</td>
<td>1,296,120</td>
<td>$1,944,180</td>
<td>4,408,200</td>
<td>$4,516,380</td>
</tr>
<tr>
<td>4</td>
<td>$13,210,000</td>
<td>6,000,750</td>
<td>1,900,000</td>
<td>4,408,200</td>
<td>$901,050</td>
<td>360,420</td>
<td>$540,630</td>
<td>4,408,200</td>
<td>$4,948,830</td>
</tr>
<tr>
<td>5</td>
<td>$11,685,000</td>
<td>5,314,500</td>
<td>1,900,000</td>
<td>3,148,200</td>
<td>$1,322,300</td>
<td>528,920</td>
<td>$793,380</td>
<td>3,148,200</td>
<td>$3,941,580</td>
</tr>
</tbody>
</table>

Next, we need to account for the changes in inventory each year. The inventory is a percentage of sales. The way we will calculate the change in inventory is the beginning of period inventory minus the end of period inventory. The sign of this calculation will tell us whether the inventory change is a cash inflow, or a cash outflow. The inventory each year, and the inventory change, will be:

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1,098,000</td>
<td>$1,189,500</td>
<td>$1,525,000</td>
<td>$1,433,500</td>
<td>$1,281,000</td>
</tr>
<tr>
<td>2</td>
<td>$1,189,500</td>
<td>$1,525,000</td>
<td>$1,433,500</td>
<td>$1,281,000</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$1,525,000</td>
<td>$1,433,500</td>
<td>$1,281,000</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$1,433,500</td>
<td>$1,281,000</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$1,281,000</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notice that we recover the remaining inventory at the end of the project. The total cash flows for the project will be the sum of the operating cash flow, the capital spending, and the inventory cash flows, so:

<table>
<thead>
<tr>
<th>Year</th>
<th>OCF</th>
<th>Equipment</th>
<th>Inventory</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2,078,400</td>
<td>0</td>
<td>$91,500</td>
<td>$1,986,900</td>
</tr>
<tr>
<td>2</td>
<td>$2,380,350</td>
<td>$18,000,000</td>
<td>$335,500</td>
<td>$15,955,150</td>
</tr>
<tr>
<td>3</td>
<td>$4,516,380</td>
<td>0</td>
<td>91,500</td>
<td>$4,607,880</td>
</tr>
<tr>
<td>4</td>
<td>$4,948,830</td>
<td>0</td>
<td>152,500</td>
<td>$5,101,330</td>
</tr>
<tr>
<td>5</td>
<td>$3,941,580</td>
<td>7,588,560</td>
<td>1,281,000</td>
<td>$12,811,140</td>
</tr>
</tbody>
</table>
The NPV of the project, including the inventory cash flow at the beginning of the project, will be:

\[
\text{NPV} = -985,500 + \frac{1,986,900}{1.11} - \frac{15,955,150}{1.11^2} + \frac{4,607,880}{1.11^3} + \frac{5,101,330}{1.11^4} + \frac{12,811,140}{1.11^5} \\
\]

\[
\text{NPV} = 2,187,376.60
\]

The company should go ahead with the new table.

b. You can perform an IRR analysis, and would expect to find three IRRs since the cash flows change signs three times.

c. The profitability index is intended as a “bang for the buck” measure; that is, it shows how much shareholder wealth is created for every dollar of initial investment. This is usually a good measure of the investment since most projects have conventional cash flows. In this case, the largest investment is not at the beginning of the project, but later in its life, so while the interpretation is the same, it really does not measure the bang for the dollar invested.