CHAPTER 10
RISK AND RETURN: LESSONS FROM MARKET HISTORY

Solutions to Questions and Problems

1. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. The return of this stock is:

\[ R = \frac{[\$86 - 75] + 1.20}{75} \]
\[ R = .1627, \text{ or } 16.27\% \]

2. The dividend yield is the dividend divided by the price at the beginning of the period, so:

\[ \text{Dividend yield} = \frac{1.20}{75} \]
\[ \text{Dividend yield} = .0160, \text{ or } 1.60\% \]

And the capital gains yield is the increase in price divided by the initial price, so:

\[ \text{Capital gains yield} = \frac{\$86 - 75}{75} \]
\[ \text{Capital gains yield} = .1467, \text{ or } 14.67\% \]

3. Using the equation for total return, we find:

\[ R = \frac{[\$67 - 75] + 1.20}{75} \]
\[ R = -.0907, \text{ or } -9.07\% \]

And the dividend yield and capital gains yield are:

\[ \text{Dividend yield} = \frac{1.20}{75} \]
\[ \text{Dividend yield} = .0160, \text{ or } 1.60\% \]

\[ \text{Capital gains yield} = \frac{\$67 - 75}{75} \]
\[ \text{Capital gains yield} = -.1067, \text{ or } -10.67\% \]

Here’s a question for you: Can the dividend yield ever be negative? No, that would mean you were paying the company for the privilege of owning the stock. It has happened on bonds.

4. The total dollar return is the change in price plus the coupon payment, so:

\[ \text{Total dollar return} = 1,063 - 1,040 + 60 \]
\[ \text{Total dollar return} = 83 \]

The total nominal percentage return of the bond is:
\[ R = \frac{[(1,063 - 1,040) + 60]}{1,040} \]
\[ R = .0798, \text{ or } 7.98\% \]

Notice here that we could have simply used the total dollar return of $83 in the numerator of this equation.

Using the Fisher equation, the real return was:

\[(1 + R) = (1 + r)(1 + h)\]
\[ r = \frac{1.0798}{1.030} - 1 \]
\[ r = .0484, \text{ or } 4.84\% \]

5. The nominal return is the stated return, which is 11.80 percent. Using the Fisher equation, the real return was:

\[(1 + R) = (1 + r)(1 + h)\]
\[ r = \frac{1.1180}{1.031} - 1 \]
\[ r = .0844, \text{ or } 8.44\% \]

6. Using the Fisher equation, the real returns for government and corporate bonds were:

\[ r_G = \frac{1.061}{1.031} - 1 \]
\[ r_G = .0291, \text{ or } 2.91\% \]
\[ r_C = \frac{1.064}{1.031} - 1 \]
\[ r_C = .0320, \text{ or } 3.20\% \]

7. The average return is the sum of the returns, divided by the number of returns. The average return for each stock was:

\[
\bar{X} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{.08 + .21 - .27 + .11 + .18}{5} = .0620, \text{ or } 6.20\% 
\]

\[
\bar{Y} = \frac{\sum_{i=1}^{N} y_i}{N} = \frac{.12 + .27 - .32 + .18 + .24}{5} = .0980, \text{ or } 9.80\% 
\]

We calculate the variance of each stock as:
The standard deviation is the square root of the variance, so the standard deviation of each stock is:

\[
\sigma_X = (.037170)^{1/2} = .1928, \text{ or } 19.28% \\
\sigma_Y = (.057920)^{1/2} = .2407, \text{ or } 24.07% 
\]

8. We will calculate the sum of the returns for each asset and the observed risk premium first. Doing so, we get:

<table>
<thead>
<tr>
<th>Year</th>
<th>Large co. stock return</th>
<th>T-bill return</th>
<th>Risk premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>1973</td>
<td>–14.69%</td>
<td>7.29%</td>
<td>–21.98%</td>
</tr>
<tr>
<td>1975</td>
<td>37.23</td>
<td>5.87</td>
<td>31.36</td>
</tr>
<tr>
<td>1976</td>
<td>23.93</td>
<td>5.07</td>
<td>18.86</td>
</tr>
<tr>
<td>1977</td>
<td>–7.16</td>
<td>5.45</td>
<td>–12.61</td>
</tr>
<tr>
<td>1978</td>
<td>6.57</td>
<td>7.64</td>
<td>–1.07</td>
</tr>
</tbody>
</table>

\[
\text{Large company stock average return} = \frac{19.41\%}{6} = 3.24% \\
\text{T-bills average return} = \frac{39.31\%}{6} = 6.55%
\]

b. Using the equation for variance, we find the variance for large company stocks over this period was:

\[
\text{Variance} = \frac{1}{5}[(-.1469 - .0324)^2 + (-.2647 - .0324)^2 + (.3723 - .0324)^2 + (.2393 - .0324)^2 + (-.0716 - .0324)^2 + (.0657 - .0324)^2] \\
\text{Variance} = 0.058136
\]

And the standard deviation for large company stocks over this period was:

\[
\text{Standard deviation} = (0.058136)^{1/2} = 0.2411 \text{ or } 24.11%
\]
Using the equation for variance, we find the variance for T-bills over this period was:

\[
\text{Variance} = \frac{1}{5}[(.0729 - .0655)^2 + (.0799 - .0655)^2 + (.0587 - .0655)^2 + (.0507 - .0655)^2 + (.0545 - .0655)^2 + (.0764 - .0655)^2]
\]

\[
\text{Variance} = 0.000153
\]

And the standard deviation for T-bills over this period was:

\[
\text{Standard deviation} = (0.000153)^{1/2}
\]

\[
\text{Standard deviation} = 0.0124 \text{ or } 1.24\%
\]

c. The average observed risk premium over this period was:

\[
\text{Average observed risk premium} = \frac{-19.90\%}{6}
\]

\[
\text{Average observed risk premium} = -3.32\%
\]

The variance of the observed risk premium was:

\[
\text{Variance} = \frac{1}{5}[(-.2198 - (-.0332))^2 + (-.3446 - (-.0332))^2 + (.3136 - (-.0332))^2 + (.1886 - (-.0332))^2 + (-.1261 - (-.0332))^2 + (-.0107 - (-.0332))^2]
\]

\[
\text{Variance} = 0.062078
\]

And the standard deviation of the observed risk premium was:

\[
\text{Standard deviation} = (0.06278)^{1/2}
\]

\[
\text{Standard deviation} = 0.2492 \text{ or } 24.92\%
\]

9. 

a. To find the average return, we sum all the returns and divide by the number of returns, so:

\[
\text{Arithmetic average return} = \frac{.27 + .13 + .18 - .14 + .09}{5}
\]

\[
\text{Arithmetic average return} = .1060, \text{ or } 10.60\%
\]

b. Using the equation to calculate variance, we find:

\[
\text{Variance} = \frac{1}{4}[(.27 - .106)^2 + (.13 - .106)^2 + (.18 - .106)^2 + (-.14 - .106)^2 + (.09 - .106)^2]
\]

\[
\text{Variance} = 0.023430
\]

So, the standard deviation is:

\[
\text{Standard deviation} = (0.023430)^{1/2}
\]

\[
\text{Standard deviation} = 0.1531, \text{ or } 15.31\%
\]

10. 

a. To calculate the average real return, we can use the average return of the asset and the average inflation rate in the Fisher equation. Doing so, we find:

\[
(1 + R) = (1 + r)(1 + \hat{h})
\]

\[
\mathcal{R} = (1.1060/1.042) - 1
\]

\[
\mathcal{R} = .0614, \text{ or } 6.14\%
\]
b. The average risk premium is simply the average return of the asset, minus the average real risk-free rate, so, the average risk premium for this asset would be:

\[
\overline{RP} = \overline{R} - \overline{R}_f
\]
\[
\overline{RP} = .1060 - .0510
\]
\[
\overline{RP} = .0550, \text{ or } 5.50\%
\]

11. We can find the average real risk-free rate using the Fisher equation. The average real risk-free rate was:

\[
(1 + R) = (1 + r)(1 + h)
\]
\[
\bar{r}_f = (1.051/1.042) - 1
\]
\[
\bar{r}_f = .0086, \text{ or } 0.86\%
\]

And to calculate the average real risk premium, we can subtract the average risk-free rate from the average real return. So, the average real risk premium was:

\[
\overline{rp} = \bar{r} - \bar{r}_f = 6.14\% - 0.86\%
\]
\[
\overline{rp} = 5.28\%
\]

12. Apply the five-year holding-period return formula to calculate the total return of the stock over the five-year period, we find:

\[
\text{5-year holding-period return} = [(1 + R_1)(1 + R_2)(1 + R_3)(1 + R_4)(1 + R_5)] - 1
\]
\[
\text{5-year holding-period return} = [(1 + .1612)(1 + .1211)(1 + .0583)(1 + .2614)(1 - .1319)] - 1
\]
\[
\text{5-year holding-period return} = 0.5086, \text{ or } 50.86\%
\]

13. To find the return on the zero coupon bond, we first need to find the price of the bond today. Since one year has elapsed, the bond now has 24 years to maturity. Using semiannual compounding, the price today is:

\[
P_1 = \frac{1,000}{1.04548}
\]
\[
P_1 = $120.90
\]

There are no intermediate cash flows on a zero coupon bond, so the return is the capital gains, or:

\[
R = (120.90 - 109.83) / 109.83
\]
\[
R = .1008, \text{ or } 10.08\%
\]

14. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. This preferred stock paid a dividend of $4, so the return for the year was:

\[
R = (96.12 - 94.89 + 4.00) / 94.89
\]
\[
R = .0551, \text{ or } 5.51\%
\]
15. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. This stock paid no dividend, so the return was:

\[ R = \frac{46.21 - 43.18}{43.18} \]
\[ R = .0702, \text{ or } 7.02\% \]

This is the return for three months, so the APR is:
\[ \text{APR} = 4(7.02\%) \]
\[ \text{APR} = 28.07\% \]

And the EAR is:
\[ \text{EAR} = (1 + .0702)^4 - 1 \]
\[ \text{EAR} = .3116, \text{ or } 31.16\% \]

16. To find the real return each year, we will use the Fisher equation, which is:

\[ 1 + R = (1 + r)(1 + h) \]

Using this relationship for each year, we find:

<table>
<thead>
<tr>
<th>Year</th>
<th>T-bills</th>
<th>Inflation</th>
<th>Real Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1926</td>
<td>.0330</td>
<td>(.0112)</td>
<td>.0447</td>
</tr>
<tr>
<td>1927</td>
<td>.0315</td>
<td>(.0226)</td>
<td>.0554</td>
</tr>
<tr>
<td>1928</td>
<td>.0405</td>
<td>(.0116)</td>
<td>.0527</td>
</tr>
<tr>
<td>1929</td>
<td>.0447</td>
<td>.0058</td>
<td>.0387</td>
</tr>
<tr>
<td>1930</td>
<td>.0227</td>
<td>(.0640)</td>
<td>.0926</td>
</tr>
<tr>
<td>1931</td>
<td>.0115</td>
<td>(.0932)</td>
<td>.1155</td>
</tr>
<tr>
<td>1932</td>
<td>.0088</td>
<td>(.0127)</td>
<td>.1243</td>
</tr>
</tbody>
</table>

So, the average real return was:
\[ \text{Average} = (.0447 + .0554 + .0527 + .0387 + .0926 + .1155 + .1243) / 7 \]
\[ \text{Average} = .0748 \text{ or } 7.48\% \]

Notice the real return was higher than the nominal return during this period because of deflation, or negative inflation.

17. Looking at the long-term corporate bond return history in Table 10.2, we see that the mean return was 6.4 percent, with a standard deviation of 8.4 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:
\[ R \in \mu \pm 1\sigma = 6.4\% \pm 8.4\% = -2.00\% \text{ to } 14.80\% \]
The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

\[ R \in \mu \pm 2\sigma = 6.4\% \pm 2(8.4\%) = -10.40\% \text{ to } 23.20\% \]

18. Looking at the large-company stock return history in Table 10.2, we see that the mean return was 11.8 percent, with a standard deviation of 20.3 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:

\[ R \in \mu \pm 1\sigma = 11.8\% \pm 20.3\% = -8.50\% \text{ to } 32.10\% \]

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

\[ R \in \mu \pm 2\sigma = 11.8\% \pm 2(20.3\%) = -28.80\% \text{ to } 52.40\% \]

Intermediate

19. Here we know the average stock return, and four of the five returns used to compute the average return. We can work the average return equation backward to find the missing return. The average return is calculated as:

\[ 5(.11) = .12 -.21 + .09 + .32 + R \]

\[ R = .23 \text{ or } 23\% \]

The missing return has to be 23 percent. Now we can use the equation for the variance to find:

\[ \text{Variance} = \frac{1}{4}[(.12 - .11)^2 + (-.21 -.11)^2 + (.09 -.11)^2 + (.32 -.11)^2 + (.23 -.11)^2]\]

\[ \text{Variance} = 0.04035 \]

And the standard deviation is:

\[ \text{Standard deviation} = (0.04035)^{1/2} \]

\[ \text{Standard deviation} = 0.2009, \text{ or } 20.09\% \]

20. The arithmetic average return is the sum of the known returns divided by the number of returns, so:

\[ \text{Arithmetic average return} = (0.27 + .12 + .32 -.12 + .19 -.31) / 6 \]

\[ \text{Arithmetic average return} = .0783, \text{ or } 7.83\% \]

Using the equation for the geometric return, we find:

\[ \text{Geometric average return} = [(1 + R_1) \times (1 + R_2) \times \ldots \times (1 + R_T)]^{1/T} - 1 \]

\[ \text{Geometric average return} = [(1 + .27)(1 + .12)(1 + .32)(1 -.12)(1 + .19)(1 -.31)]^{(1/6)} - 1 \]

\[ \text{Geometric average return} = .0522, \text{ or } 5.22\% \]

Remember, the geometric average return will always be less than the arithmetic average return if the returns have any variation.
21. To calculate the arithmetic and geometric average returns, we must first calculate the return for each year. The return for each year is:

\[
R_1 = \frac{($64.83 - 61.18 + 0.72)}{61.18} = .0714, \text{ or } 7.14\%
\]

\[
R_2 = \frac{($72.18 - 64.83 + 0.78)}{64.83} = .1254, \text{ or } 12.54\%
\]

\[
R_3 = \frac{($63.12 - 72.18 + 0.86)}{72.18} = -0.1136, \text{ or } -11.36\%
\]

\[
R_4 = \frac{($69.27 - 63.12 + 0.95)}{63.12} = .1125, \text{ or } 11.25\%
\]

\[
R_5 = \frac{($76.93 - 69.27 + 1.08)}{69.27} = .1262, \text{ or } 12.62\%
\]

The arithmetic average return was:

\[
R_A = \frac{0.0714 + 0.1254 - 0.1136 + 0.1125 + 0.1262}{5}
\]

\[
R_A = 0.0644, \text{ or } 6.44\%
\]

And the geometric average return was:

\[
R_G = [(1 + .0714)(1 + .1254)(1 - .1136)(1 + .1125)(1 + .1262)]^{1/5} - 1
\]

\[
R_G = 0.0601, \text{ or } 6.01\%
\]

22. To find the real return we need to use the Fisher equation. Re-writing the Fisher equation to solve for the real return, we get:

\[
r = \left[\frac{1 + R}{1 + h}\right] - 1
\]

So, the real return each year was:

<table>
<thead>
<tr>
<th>Year</th>
<th>T-bill return</th>
<th>Inflation</th>
<th>Real return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1973</td>
<td>0.0729</td>
<td>0.0871</td>
<td>-0.013</td>
</tr>
<tr>
<td>1974</td>
<td>0.0799</td>
<td>0.1234</td>
<td>-0.038</td>
</tr>
<tr>
<td>1975</td>
<td>0.0587</td>
<td>0.0694</td>
<td>-0.010</td>
</tr>
<tr>
<td>1976</td>
<td>0.0507</td>
<td>0.0486</td>
<td>0.0020</td>
</tr>
<tr>
<td>1977</td>
<td>0.0545</td>
<td>0.0670</td>
<td>-0.011</td>
</tr>
<tr>
<td>1978</td>
<td>0.0764</td>
<td>0.0902</td>
<td>-0.012</td>
</tr>
<tr>
<td>1979</td>
<td>0.1056</td>
<td>0.1329</td>
<td>-0.024</td>
</tr>
<tr>
<td>1980</td>
<td>0.1210</td>
<td>0.1252</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

\[
a. \text{ The average return for T-bills over this period was:}
\]

\[
\text{Average return} = 0.6197 / 8 = .0775, \text{ or } 7.75\% 
\]
And the average inflation rate was:

Average inflation = 0.7438 / 8
Average inflation = .0930, or 9.30%

b. Using the equation for variance, we find the variance for T-bills over this period was:

\[
\text{Variance} = \frac{1}{7}[(.0729 - .0775)^2 + (.0799 - .0775)^2 + (.0587 - .0775)^2 + (.0507 - .0775)^2 + (.0545 - .0775)^2 + (.0764 - .0775)^2 + (.1056 - .0775)^2 + (.1210 - .0775)^2]
\]

Variance = 0.000616

And the standard deviation for T-bills was:

Standard deviation = (0.000616)\(^{1/2}\)
Standard deviation = 0.0248, or 2.48%

The variance of inflation over this period was:

\[
\text{Variance} = \frac{1}{7}[(.0871 - .0930)^2 + (.1234 - .0930)^2 + (.0694 - .0930)^2 + (.0486 - .0930)^2 + (.0670 - .0930)^2 + (.0902 - .0930)^2 + (.1329 - .0930)^2 + (.1252 - .0930)^2]
\]

Variance = 0.000971

And the standard deviation of inflation was:

Standard deviation = (0.000971)\(^{1/2}\)
Standard deviation = 0.0312, or 3.12%

c. The average observed real return over this period was:

Average observed real return = –.1122 / 8
Average observed real return = –.0140, or –1.40%

d. The statement that T-bills have no risk refers to the fact that there is only an extremely small chance of the government defaulting, so there is little default risk. Since T-bills are short term, there is also very limited interest rate risk. However, as this example shows, there is inflation risk, i.e. the purchasing power of the investment can actually decline over time even if the investor is earning a positive return.

23. To find the return on the coupon bond, we first need to find the price of the bond today. Since one year has elapsed, the bond now has six years to maturity, so the price today is:

\[
P_1 = 70(PVIFA_{5.9\%, 6}) + \frac{1,000}{1.055^6}
\]

\[
P_1 = 1,074.93
\]

You received the coupon payments on the bond, so the nominal return was:

\[
R = (1,074.93 - 1,080.50 + 70) / 1,080.50
R = .0596, or 5.96%
\]

And using the Fisher equation to find the real return, we get:
24. Looking at the long-term government bond return history in Table 10.2, we see that the mean return was 6.1 percent, with a standard deviation of 9.8 percent. In the normal probability distribution, approximately 2/3 of the observations are within one standard deviation of the mean. This means that 1/3 of the observations are outside one standard deviation away from the mean. Or:

\[ \Pr(R < -3.7 \text{ or } R > 15.9) \approx \frac{1}{3} \]

But we are only interested in one tail here, that is, returns less than −3.7 percent, so:

\[ \Pr(R < -3.7) \approx \frac{1}{6} \]

You can use the z-statistic and the cumulative normal distribution table to find the answer as well. Doing so, we find:

\[ z = \frac{(X - \mu)}{\sigma} \]

\[ z = \frac{(-3.7\% - 6.1)}{9.8\%} = -1.00 \]

Looking at the z-table, this gives a probability of 15.87%, or:

\[ \Pr(R < -3.3) \approx 0.1587, \text{ or } 15.87\% \]

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

95% level: \[ R \in \mu \pm 2\sigma = 6.1\% \pm 2(9.8\%) = -13.50\% \text{ to } 25.70\% \]

The range of returns you would expect to see 99 percent of the time is the mean plus or minus 3 standard deviations, or:

99% level: \[ R \in \mu \pm 3\sigma = 6.1\% \pm 3(9.8\%) = -23.30\% \text{ to } 35.50\% \]

25. The mean return for small company stocks was 16.4 percent, with a standard deviation of 33.0 percent. Doubling your money is a 100% return, so if the return distribution is normal, we can use the z-statistic. So:

\[ z = \frac{(X - \mu)}{\sigma} \]

\[ z = \frac{(100\% - 16.5\%)}{32.5\%} = 2.569 \text{ standard deviations above the mean} \]

This corresponds to a probability of \( \approx 0.510\% \), or about once every 200 years. Tripling your money would be:

\[ z = \frac{(200\% - 16.5\%)}{32.5\%} = 5.646 \text{ standard deviations above the mean} \]

This corresponds to a probability of (much) less than 0.5%. The actual answer is \( \approx 0.00000082039\% \), or about once every 1 million years.
26. It is impossible to lose more than 100 percent of your investment. Therefore, return distributions are truncated on the lower tail at –100 percent.

Challenge

27. Using the z-statistic, we find:

\[ z = \frac{(X - \mu)}{\sigma} \]

\[ z = \frac{(0\% - 11.8\%)}{20.3\%} = -0.581 \]

\[ \Pr(R \leq 0) \approx 28.05\% \]

28. For each of the questions asked here, we need to use the z-statistic, which is:

\[ z = \frac{(X - \mu)}{\sigma} \]

\( a. \) \[ z_1 = \frac{(10\% - 6.4\%)}{8.4\%} = 0.4286 \]

This z-statistic gives us the probability that the return is less than 10 percent, but we are looking for the probability the return is greater than 10 percent. Given that the total probability is 100 percent (or 1), the probability of a return greater than 10 percent is 1 minus the probability of a return less than 10 percent. Using the cumulative normal distribution table, we get:

\[ \Pr(R \geq 10\%) = 1 - \Pr(R \leq 10\%) = 33.41\% \]

For a return less than 0 percent:

\[ z_2 = \frac{(0\% - 6.4\%)}{8.4\%} = -0.7619 \]

\[ \Pr(R < 0\%) = 1 - \Pr(R > 0\%) = 22.31\% \]

\( b. \) The probability that T-bill returns will be greater than 10 percent is:

\[ z_3 = \frac{(10\% - 3.6\%)}{3.1\%} = 2.0645 \]

\[ \Pr(R \geq 10\%) = 1 - \Pr(R \leq 10\%) = 1 - .9805 \approx 1.95\% \]

And the probability that T-bill returns will be less than 0 percent is:

\[ z_4 = \frac{(0\% - 3.6\%)}{3.1\%} = -1.1613 \]

\[ \Pr(R \leq 0) \approx 12.28\% \]

\( c. \) The probability that the return on long-term corporate bonds will be less than –4.18 percent is:

\[ z_5 = \frac{(-4.18\% - 6.4\%)}{8.4\%} = -1.2595 \]

\[ \Pr(R \leq -4.18\%) \approx 10.39\% \]
And the probability that T-bill returns will be greater than 10.56 percent is:

\[ z_0 = \frac{(10.56\% - 3.6\%)}{3.1\%} = 2.2452 \]

\[ \Pr(R \geq 10.56\%) = 1 - \Pr(R \leq 10.56\%) = 1 - .9876 \approx 1.24\% \]
CHAPTER 11
RETURN AND RISK: THE CAPITAL ASSET PRICING MODEL (CAPM)

1. The portfolio weight of an asset is total investment in that asset divided by the total portfolio value. First, we will find the portfolio value, which is:

\[
\text{Total value} = 135(\$47) + 105(\$41) = \$10,650
\]

The portfolio weight for each stock is:

\[
X_A = \frac{135(\$47)}{\$10,650} = .5958
\]
\[
X_B = \frac{105(\$41)}{\$10,650} = .4042
\]

2. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. The total value of the portfolio is:

\[
\text{Total value} = \$2,100 + 3,200 = \$5,300
\]

So, the expected return of this portfolio is:

\[
E(R_p) = \left(\frac{\$2,100}{\$5,300}\right)(.11) + \left(\frac{\$3,200}{\$5,300}\right)(.14) = .1281, \text{ or } 12.81\%
\]

3. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:

\[
E(R_p) = .25(.11) + .40(.17) + .35(.14) = .1445, \text{ or } 14.45\%
\]

4. Here we are given the expected return of the portfolio and the expected return of each asset in the portfolio and are asked to find the weight of each asset. We can use the equation for the expected return of a portfolio to solve this problem. Since the total weight of a portfolio must equal 1 (100%), the weight of Stock Y must be one minus the weight of Stock X. Mathematically speaking, this means:
E(R_p) = .129 = .14X_X + .09(1 - X_X)

We can now solve this equation for the weight of Stock X as:

.129 = .14X_X + .09 - .10X_X
.039 = .04X_X
X_X = .7800

So, the dollar amount invested in Stock X is the weight of Stock X times the total portfolio value, or:

Investment in X = .7800($10,000) = $7,800

And the dollar amount invested in Stock Y is:

Investment in Y = (1 -.7800)($10,000) = $2,200

5. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock asset is:

E(R_A) = .20(.06) + .55(.07) + .25(.11) = .0780, or 7.80%
E(R_B) = .20(-.20) + .55(.13) + .25(.33) = .1140, or 11.40%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation of each stock are:

\[ \sigma_A^2 = .20(0.06 - 0.0780)^2 + .55(0.07 - 0.0780)^2 + .25(0.11 - 0.0780)^2 = 0.00036 \]
\[ \sigma_A = (0.00036)^{1/2} = 0.189, \text{ or 1.89%} \]
\[ \sigma_B^2 = .20(-0.20 - 0.1140)^2 + .55(0.13 - 0.1140)^2 + .25(0.33 - 0.1140)^2 = 0.03152 \]
\[ \sigma_B = (0.03152)^{1/2} = 0.1775, \text{ or 17.75%} \]
6. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the stock is:

\[
E(R_A) = 0.10(-0.105) + 0.25(0.059) + 0.45(0.130) + 0.20(0.211) = 0.1050, \text{ or } 10.50\%
\]

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation are:

\[
\sigma^2 = 0.10(-0.105 - 0.1050)^2 + 0.25(0.059 - 0.1050)^2 + 0.45(0.130 - 0.1050)^2 + 0.20(0.211 - 0.1050)^2 = 0.00747
\]

\[
\sigma = (0.00747)^{1/2} = 0.0864, \text{ or } 8.64\%
\]

7. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:

\[
E(R_p) = 0.10(0.09) + 0.65(0.11) + 0.25(0.14) = 0.1155, \text{ or } 11.55\%
\]

If we own this portfolio, we would expect to get a return of 11.55 percent.

8. a. To find the expected return of the portfolio, we need to find the return of the portfolio in each state of the economy. This portfolio is a special case since all three assets have the same weight. To find the expected return in an equally weighted portfolio, we can sum the returns of each asset and divide by the number of assets, so the expected return of the portfolio in each state of the economy is:

Boom: \[ R_p = (0.07 + 0.15 + 0.33)/3 = 0.1833 \text{ or } 18.33\% \]

Bust: \[ R_p = (0.13 + 0.03 - 0.06)/3 = 0.0333 \text{ or } 3.33\% \]

To find the expected return of the portfolio, we multiply the return in each state of the economy by the probability of that state occurring, and then sum. Doing this, we find:

\[
E(R_p) = 0.65(0.1833) + 0.35(0.0333) = 0.1308, \text{ or } 13.08\%
\]
b. This portfolio does not have an equal weight in each asset. We still need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

**Boom:**
\[ R_p = 0.20(0.07) + 0.20(0.15) + 0.60(0.33) = 0.2420 \text{ or } 24.20\% \]

**Bust:**
\[ R_p = 0.20(0.13) + 0.20(0.03) + 0.60(-0.06) = -0.0040 \text{ or } -0.40\% \]

And the expected return of the portfolio is:

\[
E(R_p) = 0.65(0.2420) + 0.35(-0.004) = 0.1559, \text{ or } 15.59\%
\]

To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance of the portfolio is:

\[
\sigma_p^2 = 0.65(0.2420 - 0.1559)^2 + 0.35(-0.0040 - 0.1559)^2 = 0.013767
\]

9. a. This portfolio does not have an equal weight in each asset. We first need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

**Boom:**
\[ R_p = 0.30(0.24) + 0.40(0.45) + 0.30(0.33) = 0.3510, \text{ or } 35.10\% \]

**Good:**
\[ R_p = 0.30(0.09) + 0.40(0.10) + 0.30(0.15) = 0.1120, \text{ or } 11.20\% \]

**Poor:**
\[ R_p = 0.30(0.03) + 0.40(-0.10) + 0.30(-0.05) = -0.0460, \text{ or } -4.60\% \]

**Bust:**
\[ R_p = 0.30(-0.05) + 0.40(-0.25) + 0.30(-0.09) = -0.1420, \text{ or } -14.20\% \]

And the expected return of the portfolio is:

\[
E(R_p) = 0.20(0.3510) + 0.35(0.1120) + 0.30(-0.0460) + 0.15(-0.1420) = 0.0743, \text{ or } 7.43\%
\]

b. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation the portfolio is:
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\[
\sigma_p^2 = .20(.3510 - .0743)^2 + .35(.1120 - .0743)^2 + .30(-.0460 - .0743)^2 + .15(-.1420 - .0743)^2
\]

\[
\sigma_p^2 = .02717
\]

\[
\sigma_p = (.02717)^{1/2} = .1648, \text{ or } 16.48\%
\]

10. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. So, the beta of the portfolio is:

\[
\beta_p = .10(.75) + .35(1.90) + .20(1.38) + .35(1.16) = 1.42
\]

11. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. If the portfolio is as risky as the market it must have the same beta as the market. Since the beta of the market is one, we know the beta of our portfolio is one. We also need to remember that the beta of the risk-free asset is zero. It has to be zero since the asset has no risk. Setting up the equation for the beta of our portfolio, we get:

\[
\beta_p = 1.0 = 1/3(0) + 1/3(1.65) + 1/3(\beta_X)
\]

Solving for the beta of Stock X, we get:

\[
\beta_X = 1.35
\]

12. CAPM states the relationship between the risk of an asset and its expected return. CAPM is:

\[
E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i
\]

Substituting the values we are given, we find:

\[
E(R_i) = .05 + (.11 - .05)(1.15) = .1190, \text{ or } 11.90\%
\]

13. We are given the values for the CAPM except for the \(\beta\) of the stock. We need to substitute these values into the CAPM, and solve for the \(\beta\) of the stock. One important thing we need to realize is that we are given the market risk premium. The market risk premium is the expected return of the market minus the risk-free rate. We must be careful not to use this value as the expected return of the market. Using the CAPM, we find:
\[ E(R_i) = .102 = .04 + .07\beta_i \]
\[ \beta_i = .89 \]

14. Here we need to find the expected return of the market using the CAPM. Substituting the values given, and solving for the expected return of the market, we find:

\[ E(R_i) = .134 = .055 + [E(R_M) - .055](1.60) \]

\[ E(R_M) = .1044, \text{ or } 10.44\% \]

15. Here we need to find the risk-free rate using the CAPM. Substituting the values given, and solving for the risk-free rate, we find:

\[ E(R_i) = .131 = R_f + (.11 - R_f)(1.28) \]

\[ .131 = R_f + .1408 - 1.28R_f \]

\[ R_f = .0350, \text{ or } 3.50\% \]

16. a. Again, we have a special case where the portfolio is equally weighted, so we can sum the returns of each asset and divide by the number of assets. The expected return of the portfolio is:

\[ E(R_p) = (.121 + .05)/2 = .0855, \text{ or } 8.55\% \]

b. We need to find the portfolio weights that result in a portfolio with a \( \beta \) of 0.50. We know the \( \beta \) of the risk-free asset is zero. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

\[ \beta_p = 0.50 = X_S(1.13) + (1 - X_S)(0) \]
\[ 0.50 = 1.13X_S + 0 - 0X_S \]
\[ X_S = 0.50/1.13 \]
\[ X_S = .4425 \]

And, the weight of the risk-free asset is:
\[ X_{RF} = 1 - .4425 = .5575 \]

c. We need to find the portfolio weights that result in a portfolio with an expected return of 10 percent. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

\[ E(R_p) = .10 = .121X_S + .05(1 - X_S) \]
\[ .10 = .121X_S + .05 - .05X_S \]
\[ X_S = .7042 \]

So, the \( \beta \) of the portfolio will be:

\[ \beta_p = .7042(1.13) + (1 - .7042)(0) = 0.796 \]

d. Solving for the \( \beta \) of the portfolio as we did in part b, we find:

\[ \beta_p = 2.26 = X_S(1.13) + (1 - X_S)(0) \]
\[ X_S = 2.26/1.13 = 2 \]
\[ X_{RF} = 1 - 2 = -1 \]

The portfolio is invested 200% in the stock and –100% in the risk-free asset. This represents borrowing at the risk-free rate to buy more of the stock.

17. First, we need to find the \( \beta \) of the portfolio. The \( \beta \) of the risk-free asset is zero, and the weight of the risk-free asset is one minus the weight of the stock, so the \( \beta \) of the portfolio is:

\[ \beta_p = X_W(1.3) + (1 - X_W)(0) = 1.3X_W \]

So, to find the \( \beta \) of the portfolio for any weight of the stock, we simply multiply the weight of the stock times its \( \beta \).

Even though we are solving for the \( \beta \) and expected return of a portfolio of one stock and the risk-free asset for different portfolio weights, we are really solving for the SML. Any combination of this stock and the risk-free asset will fall on the SML. For that matter, a portfolio of any stock and the risk-free asset, or any portfolio of stocks, will fall on the
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SML. We know the slope of the SML line is the market risk premium, so using the CAPM and the information concerning this stock, the market risk premium is:

\[ E(R_W) = .123 = .04 + \text{MRP}(1.30) \]
\[ \text{MRP} = .083/1.3 = .0638, \text{ or } 6.38\% \]

So, now we know the CAPM equation for any stock is:

\[ E(R_p) = .04 + .0638\beta_p \]

The slope of the SML is equal to the market risk premium, which is 0.0638. Using these equations to fill in the table, we get the following results:

<p>| | | | |</p>
<table>
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</thead>
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<td>0</td>
<td></td>
</tr>
<tr>
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<td>0.325</td>
<td></td>
</tr>
<tr>
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<td>.0815</td>
<td>0.650</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>.1023</td>
<td>0.975</td>
<td></td>
</tr>
<tr>
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<td>.1230</td>
<td>1.300</td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>.1438</td>
<td>1.625</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>.1645</td>
<td>1.950</td>
<td></td>
</tr>
</tbody>
</table>

18. There are two ways to correctly answer this question. We will work through both. First, we can use the CAPM. Substituting in the value we are given for each stock, we find:

\[ E(R_Y) = .045 + .073(1.35) = .1436, \text{ or } 14.36\% \]

It is given in the problem that the expected return of Stock Y is 14 percent, but according to the CAPM, the return of the stock based on its level of risk should be 14.36 percent. This means the stock return is too low, given its level of risk. Stock Y plots below the SML and is overvalued. In other words, its price must decrease to increase the expected return to 14.36 percent.

For Stock Z, we find:

\[ E(R_Z) = .045 + .073(0.80) = .1034, \text{ or } 10.34\% \]
The return given for Stock Z is 11.5 percent, but according to the CAPM the expected return of the stock should be 10.34 percent based on its level of risk. Stock Z plots above the SML and is undervalued. In other words, its price must increase to decrease the expected return to 10.34 percent.

We can also answer this question using the reward-to-risk ratio. All assets must have the same reward-to-risk ratio, that is, every asset must have the same ratio of the asset risk premium to its beta. This follows from the linearity of the SML in Figure 11.11. The reward-to-risk ratio is the risk premium of the asset divided by its $\beta$. This is also known as the Treynor ratio or Treynor index. We are given the market risk premium, and we know the $\beta$ of the market is one, so the reward-to-risk ratio for the market is 0.073, or 7.3 percent. Calculating the reward-to-risk ratio for Stock Y, we find:

\[
\text{Reward-to-risk ratio } Y = \frac{(.14 - .045)}{1.35} = .0704
\]

The reward-to-risk ratio for Stock Y is too low, which means the stock plots below the SML, and the stock is overvalued. Its price must decrease until its reward-to-risk ratio is equal to the market reward-to-risk ratio. For Stock Z, we find:

\[
\text{Reward-to-risk ratio } Z = \frac{(.115 - .045)}{.80} = .0875
\]

The reward-to-risk ratio for Stock Z is too high, which means the stock plots above the SML, and the stock is undervalued. Its price must increase until its reward-to-risk ratio is equal to the market reward-to-risk ratio.

19. We need to set the reward-to-risk ratios of the two assets equal to each other (see the previous problem), which is:

\[
(\frac{.14 - R_f}{1.35}) = (\frac{.115 - R_f}{0.80})
\]

We can cross multiply to get:

\[
0.80(.14 - R_f) = 1.35(.115 - R_f)
\]

Solving for the risk-free rate, we find:

\[
0.112 - 0.80R_f = 0.15525 - 1.35R_f
\]
Intermediate

20. For a portfolio that is equally invested in large-company stocks and long-term bonds:

\[
\text{Return} = \frac{(11.8\% + 6.1\%)}{2} = 8.95\%
\]

For a portfolio that is equally invested in small stocks and Treasury bills:

\[
\text{Return} = \frac{(16.5\% + 3.6\%)}{2} = 10.05\%
\]

21. We know that the reward-to-risk ratios for all assets must be equal (See Question 19). This can be expressed as:

\[
\left[ E(R_A) - R_f \right]/\beta_A = \left[ E(R_B) - R_f \right]/\beta_B
\]

The numerator of each equation is the risk premium of the asset, so:

\[
RP_A/\beta_A = RPB/\beta_B
\]

We can rearrange this equation to get:

\[
\beta_B/\beta_A = RPB/RPA
\]

If the reward-to-risk ratios are the same, the ratio of the betas of the assets is equal to the ratio of the risk premiums of the assets.

22. a. We need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

\[
\begin{align*}
\text{Boom: } & \quad E(R_p) = .4(.20) + .4(.25) + .2(.60) = .3000, \text{ or } 30.00\% \\
\text{Normal: } & \quad E(R_p) = .4(.15) + .4(.11) + .2(.05) = .1140, \text{ or } 11.40\% \\
\text{Bust: } & \quad E(R_p) = .4(.01) + .4(-.15) + .2(-.50) = -.1560, \text{ or } -15.60\%
\end{align*}
\]

And the expected return of the portfolio is:
E(R_p) = .30(.30) + .45(.114) + .25(–.156) = .1023, or 10.23%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, than add all of these up. The result is the variance. So, the variance and standard deviation of the portfolio is:

\[ \sigma^2_p = .30(0.30 - 0.1023)^2 + .45(0.114 - 0.1023)^2 + .25(-0.156 - 0.1023)^2 \]
\[ \sigma^2_p = 0.02847 \]
\[ \sigma_p = (0.02847)^{1/2} = 0.1687, \text{ or } 16.87\% \]

b. The risk premium is the return of a risky asset, minus the risk-free rate. T-bills are often used as the risk-free rate, so:

\[ \text{RP}_t = E(R_p) - R_f = 0.1023 - 0.038 = 0.0643, \text{ or } 6.43\% \]

c. The approximate expected real return is the expected nominal return minus the inflation rate, so:

Approximate expected real return = 0.1023 - 0.035 = 0.0673, or 6.73%

To find the exact real return, we will use the Fisher equation. Doing so, we get:

\[ 1 + E(R_i) = (1 + h)(1 + e(r_i)) \]
\[ 1.1023 = (1.0350)(1 + e(r_i)) \]
\[ e(r_i) = (1.1023/1.035) - 1 = 0.0650, \text{ or } 6.50\% \]

The approximate real risk-free rate is:

Approximate expected real return = 0.038 - 0.035 = 0.003, or 0.30%

And using the Fisher effect for the exact real risk-free rate, we find:

\[ 1 + E(R_i) = (1 + h)(1 + e(r_i)) \]
\[ 1.038 = (1.0350)(1 + e(r_i)) \]
\[ e(r_i) = (1.038/1.035) - 1 = 0.0029, \text{ or } 0.29\% \]
The approximate real risk premium is the approximate expected real return minus the risk-free rate, so:

Approximate expected real risk premium = .0673 – .003 = .0643, or 6.43%

The exact real risk premium is the exact real return minus the risk-free rate, so:

Exact expected real risk premium = .0650 – .0029 = .0621, or 6.21%

23. We know the total portfolio value and the investment of two stocks in the portfolio, so we can find the weight of these two stocks. The weights of Stock A and Stock B are:

\[ X_A = \frac{180,000}{1,000,000} = .18 \]
\[ X_B = \frac{290,000}{1,000,000} = .29 \]

Since the portfolio is as risky as the market, the \( \beta \) of the portfolio must be equal to one. We also know the \( \beta \) of the risk-free asset is zero. We can use the equation for the \( \beta \) of a portfolio to find the weight of the third stock. Doing so, we find:

\[ \beta_p = 1.0 = X_A(0.85) + X_B(1.40) + X_C(1.45) + X_{rf}(0) \]

Solving for the weight of Stock C, we find:

\[ X_C = .30413793 \]

So, the dollar investment in Stock C must be:

Invest in Stock C = \( .30413793 \times 1,000,000 \) = $304,137.93

We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weight we know, or:

\[ 1 = X_A + X_B + X_C + X_{rf} \]
\[ 1 = .18 + .29 + .30413793 + X_{rf} \]
\[ X_{rf} = .22586207 \]
So, the dollar investment in the risk-free asset must be:

Invest in risk-free asset = \( .22586207 \times 1000000 \) = $225,862.07

24. We are given the expected return and \( \beta \) of a portfolio and the expected return and \( \beta \) of assets in the portfolio. We know the \( \beta \) of the risk-free asset is zero. We also know the sum of the weights of each asset must be equal to one. So, the weight of the risk-free asset is one minus the weight of Stock X and the weight of Stock Y. Using this relationship, we can express the expected return of the portfolio as:

\[
E(R_p) = .1122 = X_X(.1535) + X_Y(.0940) + (1 - X_X - X_Y)(.045)
\]

And the \( \beta \) of the portfolio is:

\[
\beta_p = .96 = X_X(1.55) + X_Y(0.70) + (1 - X_X - X_Y)(0)
\]

We have two equations and two unknowns. Solving these equations, we find that:

\[
X_X = -0.2838710
\]
\[
X_Y = 2.0000000
\]
\[
X_{r_f} = -0.7161290
\]

The amount to invest in Stock X is:

Investment in stock X = \(-0.28387\times 100000\) = \(-$28,387.10\)

A negative portfolio weight means that you short sell the stock. If you are not familiar with short selling, it means you borrow a stock today and sell it. You must then purchase the stock at a later date to repay the borrowed stock. If you short sell a stock, you make a profit if the stock decreases in value. The negative weight on the risk-free asset means that we borrow money to invest.

25. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

\[
E(R_A) = .33(.102) + .33(.115) + .33(.073) = .0967, \text{ or } 9.67\%
\]
\[
E(R_B) = .33(-.045) + .33(.148) + .33(.233) = .1120, \text{ or } 11.20\%\]
To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation of Stock A are:

\[
\sigma^2 = .33(.102 - .0967)^2 + .33(.115 - .0967)^2 + .33(.073 - .0967)^2 = .00031
\]

\[
\sigma = (.00031)^{1/2} = .0176, \text{ or } 1.76\%
\]

And the standard deviation of Stock B is:

\[
\sigma^2 = .33(-.045 - .1120)^2 + .33(.148 - .1120)^2 + .33(.233 - .1120)^2 = .01353
\]

\[
\sigma = (.01353)^{1/2} = .1163, \text{ or } 11.63\%
\]

To find the covariance, we multiply each possible state times the product of each assets’ deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

\[
\text{Cov}(A,B) = .33(.102 - .0967)(-.045 - .1120) + .33(.115 - .0967)(.148 - .1120) \\
+ .33(.073 - .0967)(.233 - .1120)
\]

\[
\text{Cov}(A,B) = -.001014
\]

And the correlation is:

\[
\rho_{A,B} = \frac{\text{Cov}(A,B)}{\sigma_A \sigma_B}
\]

\[
\rho_{A,B} = -.001014 / (.0176)(.1163)
\]

\[
\rho_{A,B} = -.4964
\]

26. The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

\[
E(R_A) = .25(-.020) + .60(.138) + .15(.218) = .1105, \text{ or } 11.05\%
\]

\[
E(R_B) = .25(.034) + .60(.062) + .15(.092) = .0595, \text{ or } 5.95\%
\]
To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation of Stock A are:

$$\sigma_A^2 = .25(0.020 - 0.1105)^2 + .60(0.138 - 0.1105)^2 + .15(0.218 - 0.1105)^2 = 0.00644$$

$$\sigma_A = (0.00644)^{1/2} = 0.0803, \text{ or } 8.03\%$$

And the standard deviation of Stock B is:

$$\sigma_B^2 = .25(0.034 - 0.0595)^2 + .60(0.062 - 0.0595)^2 + .15(0.092 - 0.0595)^2 = 0.00032$$

$$\sigma_B = (0.00032)^{1/2} = 0.0180, \text{ or } 1.80\%$$

To find the covariance, we multiply each possible state times the product of each assets’ deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

$$\text{Cov}(A,B) = .25(0.020 - 0.1105)(0.034 - 0.0595) + .60(0.138 - 0.1105)(0.062 - 0.0595) + .15(0.218 - 0.1105)(0.092 - 0.0595)$$

$$\text{Cov}(A,B) = 0.001397$$

And the correlation is:

$$\rho_{A,B} = \frac{\text{Cov}(A,B)}{\sigma_A \sigma_B}$$

$$\rho_{A,B} = 0.001397 / (0.0803)(0.0180)$$

$$\rho_{A,B} = 0.9658$$

27. a. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$$E(R_p) = X_f E(R_f) + X_g E(R_g)$$

$$E(R_p) = .30(.10) + .70(.15)$$

$$E(R_p) = 0.1350, \text{ or } 13.50\%$$

b. The variance of a portfolio of two assets can be expressed as:
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\[ \sigma^2_P = X^2_F \sigma^2_F + X^2_G \sigma^2_G + 2X_F X_G \sigma_F \sigma_G \rho_{F,G} \]
\[ \sigma^2_P = .30^2(.43^2) + .70^2(.62^2) + 2(.30)(.70)(.43)(.62)(.25) \]
\[ \sigma^2_P = .23299 \]

So, the standard deviation is:

\[ \sigma_P = (.23299)^{1/2} = .4827, \text{ or 48.27\%} \]

28.  

a.  

The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

\[ E(R_P) = X_A E(R_A) + X_B E(R_B) \]
\[ E(R_P) = .35(.09) + .65(.15) \]
\[ E(R_P) = .1290, \text{ or 12.90\%} \]

The variance of a portfolio of two assets can be expressed as:

\[ \sigma^2_P = X^2_A \sigma^2_A + X^2_B \sigma^2_B + 2X_A X_B \sigma_A \sigma_B \rho_{A,B} \]
\[ \sigma^2_P = .35^2(.36^2) + .65^2(.62^2) + 2(.35)(.65)(.36)(.62)(.50) \]
\[ \sigma^2_P = .22906 \]

So, the standard deviation is:

\[ \sigma_P = (.22906)^{1/2} = .4786, \text{ or 47.86\%} \]

b.  

\[ \sigma^2_P = X^2_A \sigma^2_A + X^2_B \sigma^2_B + 2X_A X_B \sigma_A \sigma_B \rho_{A,B} \]
\[ \sigma^2_P = .35^2(.36^2) + .65^2(.62^2) + 2(.35)(.65)(.36)(.62)(-.50) \]
\[ \sigma^2_P = .12751 \]

So, the standard deviation is:

\[ \sigma = (.12751)^{1/2} = .3571, \text{ or 35.71\%} \]

c.  

As Stock A and Stock B become less correlated, or more negatively correlated, the standard deviation of the portfolio decreases.

29.  
a.  

(i)  

Using the equation to calculate beta, we find:
\[ \beta_A = (\rho_{A,M}) \left( \frac{\sigma_A}{\sigma_M} \right) \]

\[ 0.85 = (\rho_{A,M})(0.31) / 0.20 \]

\[ \rho_{A,M} = 0.55 \]

(ii) Using the equation to calculate beta, we find:

\[ \beta_B = (\rho_{B,M}) \left( \frac{\sigma_B}{\sigma_M} \right) \]

\[ 1.40 = (.50)(\sigma_B) / 0.20 \]

\[ \sigma_B = 0.56 \]

(iii) Using the equation to calculate beta, we find:

\[ \beta_C = (\rho_{C,M}) \left( \frac{\sigma_C}{\sigma_M} \right) \]

\[ \beta_C = (.35)(.65) / 0.20 \]

\[ \beta_C = 1.14 \]

(iv) The market has a correlation of 1 with itself.

(v) The beta of the market is 1.

(vi) The risk-free asset has zero standard deviation.

(vii) The risk-free asset has zero correlation with the market portfolio.

(viii) The beta of the risk-free asset is 0.

b. Using the CAPM to find the expected return of the stock, we find:

\[ E(R_A) = R_f + \beta_A[E(R_M) - R_f] \]

\[ E(R_A) = 0.05 + 0.85(0.12 - 0.05) \]

\[ E(R_A) = .1095, \text{ or } 10.95\% \]

According to the CAPM, the expected return on Firm A’s stock should be 10.95 percent. However, the expected return on Firm A’s stock given in the table is only 10 percent. Therefore, Firm A’s stock is overpriced, and you should sell it.

\[ \text{Firm B:} \]
\[ E(R_B) = R_f + \beta_B[E(R_m) - R_f] \]
\[ E(R_B) = 0.05 + 1.4(0.12 - 0.05) \]
\[ E(R_B) = 0.1480, \text{ or } 14.80\% \]

According to the CAPM, the expected return on Firm B’s stock should be 14.80 percent. However, the expected return on Firm B’s stock given in the table is 14 percent. Therefore, Firm B’s stock is overpriced, and you should sell it.

**Firm C:**
\[ E(R_C) = R_f + \beta_C[E(R_m) - R_f] \]
\[ E(R_C) = 0.05 + 1.14(0.12 - 0.05) \]
\[ E(R_C) = 0.1296, \text{ or } 12.96\% \]

According to the CAPM, the expected return on Firm C’s stock should be 12.96 percent. However, the expected return on Firm C’s stock given in the table is 16 percent. Therefore, Firm C’s stock is underpriced, and you should buy it.

30. Because a well-diversified portfolio has no unsystematic risk, this portfolio should lie on the Capital Market Line (CML). The slope of the CML equals:

\[ \text{Slope CML} = \frac{E(R_m) - R_f}{\sigma_M} \]
\[ \text{Slope CML} = \frac{.12 - .05}{.22} \]
\[ \text{Slope CML} = .31818 \]

a. The expected return on the portfolio equals:

\[ E(R_p) = R_f + \text{Slope CML}(\sigma_p) \]
\[ E(R_p) = .05 + .31818(.09) \]
\[ E(R_p) = .0786, \text{ or } 7.86\% \]

b. The expected return on the portfolio equals:

\[ E(R_p) = R_f + \text{Slope CML}(\sigma_p) \]
\[ .20 = .05 + .31818(\sigma_p) \]
\[ \sigma_p = .4714, \text{ or } 47.14\% \]

31. First, we can calculate the standard deviation of the market portfolio using the Capital Market Line (CML). We know that the risk-free rate asset has a return of 4 percent and a
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standard deviation of zero and the portfolio has an expected return of 7 percent and a standard deviation of 10 percent. These two points must lie on the Capital Market Line. The slope of the Capital Market Line equals:

\[ \text{Slope}_{CML} = \frac{\text{Rise}}{\text{Run}} \]
\[ \text{Slope}_{CML} = \frac{\text{Increase in expected return}}{\text{Increase in standard deviation}} \]
\[ \text{Slope}_{CML} = \frac{(.07 - .04)}{(.10 - 0)} \]
\[ \text{Slope}_{CML} = .30 \]

According to the Capital Market Line:

\[ E(R_i) = R_f + \text{Slope}_{CML}(\sigma_i) \]

Since we know the expected return on the market portfolio, the risk-free rate, and the slope of the Capital Market Line, we can solve for the standard deviation of the market portfolio which is:

\[ E(R_M) = R_f + \text{Slope}_{CML}(\sigma_M) \]
\[ .12 = .04 + (.30)(\sigma_M) \]
\[ \sigma_M = (.12 - .04) / .30 \]
\[ \sigma_M = .2667, \text{ or } 26.67\% \]

Next, we can use the standard deviation of the market portfolio to solve for the beta of a security using the beta equation. Doing so, we find the beta of the security is:

\[ \beta_i = \frac{(\rho_{i,M})(\sigma_i)}{\sigma_M} \]
\[ \beta_i = (.45)(.55) / .2667 \]
\[ \beta_i = 0.93 \]

Now we can use the beta of the security in the CAPM to find its expected return, which is:

\[ E(R_i) = R_f + \beta_i[E(R_M) - R_f] \]
\[ E(R_i) = .04 + .93(.12 - .04) \]
\[ E(R_i) = .1143, \text{ or } 11.43\% \]

32. First, we need to find the standard deviation of the market and the portfolio, which are:

\[ \sigma_M = (.0382)^{1/2} \]
\[ \sigma_M = 0.1954, \text{ or } 19.54\% \]

\[ \sigma_Z = (0.3285)^{1/2} \]
\[ \sigma_Z = 0.5731, \text{ or } 57.31\% \]

Now we can use the equation for beta to find the beta of the portfolio, which is:

\[ \beta_Z = \frac{\rho_{Z,M} \sigma_Z}{\sigma_M} \]
\[ \beta_Z = (0.28)(0.5731) / 0.1954 \]
\[ \beta_Z = 0.82 \]

Now, we can use the CAPM to find the expected return of the portfolio, which is:

\[ E(R_Z) = R_f + \beta_Z [E(R_M) - R_f] \]
\[ E(R_Z) = 0.042 + 0.82(0.109 - 0.042) \]
\[ E(R_Z) = 0.0970, \text{ or } 9.70\% \]

**Challenge**

33. The amount of systematic risk is measured by the \( \beta \) of an asset. Since we know the market risk premium and the risk-free rate, if we know the expected return of the asset we can use the CAPM to solve for the \( \beta \) of the asset. The expected return of Stock I is:

\[ E(R_I) = 0.15(0.11) + 0.55(0.18) + 0.30(0.08) = 0.1395, \text{ or } 13.95\% \]

Using the CAPM to find the \( \beta \) of Stock I, we find:

\[ 0.1395 = 0.04 + 0.075\beta_I \]
\[ \beta_I = 1.33 \]

The total risk of the asset is measured by its standard deviation, so we need to calculate the standard deviation of Stock I. Beginning with the calculation of the stock’s variance, we find:

\[ \sigma_I^2 = 0.15(0.11 - 0.1395)^2 + 0.55(0.18 - 0.1395)^2 + 0.30(0.08 - 0.1395)^2 \]
\[ \sigma_I^2 = 0.00209 \]
\[ \sigma_I = (0.00209)^{1/2} = 0.0458, \text{ or } 4.58\% \]
Using the same procedure for Stock II, we find the expected return to be:

$$E(R_{II}) = .15(-.25) + .55(.11) + .30(.31) = .1160$$

Using the CAPM to find the \( \beta \) of Stock II, we find:

$$\beta_{II} = \frac{.1160 - .04}{.075} = 1.01$$

And the standard deviation of Stock II is:

$$\sigma_{II}^2 = .15(-.25 - .1160)^2 + .55(.11 - .1160)^2 + .30(.31 - .1160)^2$$
$$\sigma_{II}^2 = .03140$$
$$\sigma_{II} = (.03140)^{1/2} = .1772, \text{ or } 17.72\%$$

Although Stock II has more total risk than I, it has much less systematic risk, since its beta is much smaller than I’s. Thus, I has more systematic risk, and II has more unsystematic and more total risk. Since unsystematic risk can be diversified away, I is actually the “riskier” stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.

34. Here we have the expected return and beta for two assets. We can express the returns of the two assets using CAPM. If the CAPM is true, then the security market line holds as well, which means all assets have the same risk premium. Setting the reward-to-risk ratios of the assets equal to each other and solving for the risk-free rate, we find:

$$\frac{(.1228 - R_f)/1.35}{.80} = \frac{(.0854 - R_f)/.80}{.80}$$
$$\frac{.80(.1228 - R_f)}{1.35} = .80(.0854 - R_f)$$
$$\frac{.09824 - .80R_f}{1.35} = .01705$$
$$R_f = .031, \text{ or } 3.10\%$$

Now using CAPM to find the expected return on the market with both stocks, we find:

$$R_M = .0990, \text{ or } 9.90\%$$
35.  

a.  The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the expected return and standard deviation of each stock are:

**Asset 1:**

\[ E(R_1) = 0.15(0.20) + 0.35(0.15) + 0.35(0.10) + 0.15(0.05) = 0.1250, \text{ or } 12.50\% \]

\[ \sigma^2_1 = 0.15(0.20 - 0.1250)^2 + 0.35(0.15 - 0.1250)^2 + 0.35(0.10 - 0.1250)^2 + 0.15(0.05 - 0.1250)^2 \]

\[ \sigma_1 = (0.00213)^{1/2} = 0.0461 \text{ or } 4.61\% \]

**Asset 2:**

\[ E(R_2) = 0.15(0.20) + 0.35(0.10) + 0.35(0.15) + 0.15(0.05) = 0.1250, \text{ or } 12.50\% \]

\[ \sigma^2_2 = 0.15(0.20 - 0.1250)^2 + 0.35(0.10 - 0.1250)^2 + 0.35(0.15 - 0.1250)^2 + 0.15(0.05 - 0.1250)^2 \]

\[ \sigma_2 = (0.00213)^{1/2} = 0.0461 \text{ or } 4.61\% \]

**Asset 3:**

\[ E(R_3) = 0.15(0.05) + 0.35(0.10) + 0.35(0.15) + 0.15(0.20) = 0.1250, \text{ or } 12.50\% \]

\[ \sigma^2_3 = 0.15(0.05 - 0.1250)^2 + 0.35(0.10 - 0.1250)^2 + 0.35(0.15 - 0.1250)^2 + 0.15(0.20 - 0.1250)^2 \]

\[ \sigma_3 = (0.00213)^{1/2} = 0.0461 \text{ or } 4.61\% \]

b.  To find the covariance, we multiply each possible state times the product of each assets’ deviation from the mean in that state. The sum of these products is the covariance. The correlation is the covariance divided by the product of the two standard deviations. So, the covariance and correlation between each possible set of assets are:
Asset 1 and Asset 2:
\[
\text{Cov}(1,2) = .15(.20 - .1250)(.20 - .1250) + .35(.15 - .1250)(.10 - .1250) \\
+ .35(.10 - .1250)(.15 - .1250) + .15(.05 - .1250)(.05 - .1250)
\]
\[
\text{Cov}(1,2) = .00125
\]
\[
\rho_{1,2} = \text{Cov}(1,2) / \sigma_1 \sigma_2
\]
\[
\rho_{1,2} = .00125 / (.0461)(.0461)
\]
\[
\rho_{1,2} = .5882
\]

Asset 1 and Asset 3:
\[
\text{Cov}(1,3) = .15(.20 - .1250)(.05 - .1250) + .35(.15 - .1250)(.10 - .1250) \\
+ .35(.10 - .1250)(.15 - .1250) + .15(.05 - .1250)(.20 - .1250)
\]
\[
\text{Cov}(1,3) = -.002125
\]
\[
\rho_{1,3} = \text{Cov}(1,3) / \sigma_1 \sigma_3
\]
\[
\rho_{1,3} = -.002125 / (.0461)(.0461)
\]
\[
\rho_{1,3} = -1
\]

Asset 2 and Asset 3:
\[
\text{Cov}(2,3) = .15(.20 - .1250)(.05 - .1250) + .35(.10 - .1250)(.10 - .1250) \\
+ .35(.15 - .1250)(.15 - .1250) + .15(.05 - .1250)(.20 - .1250)
\]
\[
\text{Cov}(2,3) = -.00125
\]
\[
\rho_{2,3} = \text{Cov}(2,3) / \sigma_2 \sigma_3
\]
\[
\rho_{2,3} = -.00125 / (.0461)(.0461)
\]
\[
\rho_{2,3} = -.5882
\]

c. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 2:
\[
E(R_P) = X_1E(R_1) + X_2E(R_2)
\]
\[
E(R_P) = .50(.1250) + .50(.1250)
\]
\[
E(R_P) = .1250, or 12.50\%
\]

The variance of a portfolio of two assets can be expressed as:
\[
\sigma^2_P = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + 2X_1X_2\sigma_1\sigma_2\rho_{1,2}
\]
\[
\sigma^2_P = .50^2(.0461^2) + .50^2(.0461^2) + 2(.50)(.50)(.0461)(.0461)(.5882)
\]
\[ \sigma_p^2 = 0.001688 \]

And the standard deviation of the portfolio is:

\[ \sigma_p = (0.001688)^{1/2} \]
\[ \sigma_p = 0.0411 \text{ or } 4.11\% \]

d. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 3:

\[ E(R_p) = X_1 E(R_1) + X_3 E(R_3) \]
\[ E(R_p) = 0.50 \times 0.1250 + 0.50 \times 0.1250 \]
\[ E(R_p) = 0.1250, \text{ or } 12.50\% \]

The variance of a portfolio of two assets can be expressed as:

\[ \sigma_p^2 = X_1^2 \sigma_1^2 + X_3^2 \sigma_3^2 + 2X_1X_3\sigma_1\sigma_3\rho_{1,3} \]
\[ \sigma_p^2 = 0.50^2 \times (0.0461^2) + 0.50^2 \times (0.0461^2) + 2 \times 0.50 \times 0.50 \times (0.0461) \times (0.0461) \times (-1) \]
\[ \sigma_p^2 = 0.000000 \]

Since the variance is zero, the standard deviation is also zero.

e. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 2 and Asset 3:

\[ E(R_p) = X_2 E(R_2) + X_3 E(R_3) \]
\[ E(R_p) = 0.50 \times 0.1250 + 0.50 \times 0.1250 \]
\[ E(R_p) = 0.1250, \text{ or } 12.50\% \]

The variance of a portfolio of two assets can be expressed as:

\[ \sigma_p^2 = X_2^2 \sigma_2^2 + X_3^2 \sigma_3^2 + 2X_2X_3\sigma_2\sigma_3\rho_{1,3} \]
\[ \sigma_p^2 = 0.50^2 \times (0.0461^2) + 0.50^2 \times (0.0461^2) + 2 \times 0.50 \times 0.50 \times (0.0461) \times (0.0461) \times (-0.5882) \]
\[ \sigma_p^2 = 0.000438 \]

And the standard deviation of the portfolio is:

\[ \sigma_p = (0.000438)^{1/2} \]
\[ \sigma_p = 0.0209 \text{ or } 2.09\% \]

\( f. \) As long as the correlation between the returns on two securities is below 1, there is a benefit to diversification. A portfolio with negatively correlated securities can achieve greater risk reduction than a portfolio with positively correlated securities, holding the expected return on each stock constant. Applying proper weights on perfectly negatively correlated securities can reduce portfolio variance to 0.

**36. a.** The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each stock is:

\[
E(R_A) = 0.15(-0.10) + 0.60(0.09) + 0.25(0.32) = 0.1190, \text{ or } 11.90\%
\]

\[
E(R_B) = 0.15(-0.08) + 0.60(0.08) + 0.25(0.26) = 0.1010, \text{ or } 10.10\%
\]

\( b. \) We can use the expected returns we calculated to find the slope of the Security Market Line. We know that the beta of Stock A is 0.25 greater than the beta of Stock B. Therefore, as beta increases by 0.25, the expected return on a security increases by 0.018 (= 0.1190 – 0.1010). The slope of the security market line (SML) equals:

\[
\text{Slope}_{SML} = \frac{\text{Rise}}{\text{Run}}
\]

\[
\text{Slope}_{SML} = \frac{\text{Increase in expected return}}{\text{Increase in beta}}
\]

\[
\text{Slope}_{SML} = \frac{0.1190 - 0.1010}{0.25}
\]

\[
\text{Slope}_{SML} = 0.0720, \text{ or } 7.20\%
\]

Since the market’s beta is 1 and the risk-free rate has a beta of zero, the slope of the Security Market Line equals the expected market risk premium. So, the expected market risk premium must be 7.2 percent.

We could also solve this problem using CAPM. The equations for the expected returns of the two stocks are:

\[
.119 = R_f + (\beta_B + 0.25)(MRP)
\]

\[
.101 = R_f + \beta_B(MRP)
\]

We can rewrite the CAPM equation for Stock A as:

\[
.119 = R_f + \beta_B(MRP) + 0.25(MRP)
\]
Subtracting the CAPM equation for Stock B from this equation yields:

\[ 0.018 = 0.25 \times \text{MRP} \]
\[ \text{MRP} = 0.0720, \text{ or } 7.20\% \]

which is the same answer as our previous result.

37. a. A typical, risk-averse investor seeks high returns and low risks. For a risk-averse investor holding a well-diversified portfolio, beta is the appropriate measure of the risk of an individual security. To assess the two stocks, we need to find the expected return and beta of each of the two securities.

*Stock A:*

Since Stock A pays no dividends, the return on Stock A is simply: \( \frac{P_1 - P_0}{P_0} \). So, the return for each state of the economy is:

\[ R_{\text{Recession}} = \frac{(64 - 75)}{75} = -0.147, \text{ or } -14.70\% \]
\[ R_{\text{Normal}} = \frac{(87 - 75)}{75} = 0.160, \text{ or } 16.00\% \]
\[ R_{\text{Expanding}} = \frac{(97 - 75)}{75} = 0.293, \text{ or } 29.30\% \]

The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the stock is:

\[ E(R_A) = 0.20(-0.147) + 0.60(0.160) + 0.20(0.293) = 0.1253, \text{ or } 12.53\% \]

And the variance of the stock is:

\[ \sigma^2_A = 0.20(-0.147 - 0.1253)^2 + 0.60(0.160 - 0.1253)^2 + 0.20(0.293 - 0.1253)^2 \]
\[ \sigma^2_A = 0.0212 \]

Which means the standard deviation is:

\[ \sigma_A = (0.0212)^{1/2} \]
\[ \sigma_A = 0.1455, \text{ or } 14.55\% \]

Now we can calculate the stock’s beta, which is:
\[ \beta_A = \left( \rho_{A,M} \right) \left( \sigma_A / \sigma_M \right) \]
\[ \beta_A = (0.70)(0.1455) / 0.18 \]
\[ \beta_A = 0.566 \]

For Stock B, we can directly calculate the beta from the information provided. So, the beta for Stock B is:

\[ \beta_B = \left( \rho_{B,M} \right) \left( \sigma_B / \sigma_M \right) \]
\[ \beta_B = (0.24)(0.34) / 0.18 \]
\[ \beta_B = 0.453 \]

The expected return on Stock B is higher than the expected return on Stock A. The risk of Stock B, as measured by its beta, is lower than the risk of Stock A. Thus, a typical risk-averse investor holding a well-diversified portfolio will prefer Stock B. Note, this situation implies that at least one of the stocks is mispriced since the higher risk (beta) stock has a lower return than the lower risk (beta) stock.

b. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

\[ E(R_P) = X_A E(R_A) + X_B E(R_B) \]
\[ E(R_P) = 0.70(0.1253) + 0.30(0.14) \]
\[ E(R_P) = 0.1297, \text{ or } 12.97\% \]

To find the standard deviation of the portfolio, we first need to calculate the variance. The variance of the portfolio is:

\[ \sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \sigma_A \sigma_B \rho_{A,B} \]
\[ \sigma_P^2 = (0.70)^2(0.1455)^2 + (0.30)^2(0.34)^2 + 2(0.70)(0.30)(0.1455)(0.34)(0.36) \]
\[ \sigma_P^2 = 0.02825 \]

And the standard deviation of the portfolio is:

\[ \sigma_P = \sqrt{0.02825} \]
\[ \sigma_P = 0.1681 \text{ or } 16.81\% \]
c. The beta of a portfolio is the weighted average of the betas of its individual securities. So the beta of the portfolio is:

\[ \beta_p = 0.70(0.566) + 0.30(0.453) \]
\[ \beta_p = 0.532 \]

38. a. The variance of a portfolio of two assets equals:

\[ \sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \text{Cov}(A,B) \]

Since the weights of the assets must sum to one, we can write the variance of the portfolio as:

\[ \sigma_P^2 = X_A^2 \sigma_A^2 + (1-X_A)^2 \sigma_B^2 + 2X_A (1-X_A) \text{Cov}(A,B) \]

To find the minimum for any function, we find the derivative and set the derivative equal to zero. Finding the derivative of the variance function with respect to the weight of Asset A, setting the derivative equal to zero, and solving for the weight of Asset A, we find:

\[ X_A = \frac{\sigma_B^2 - \text{Cov}(A,B)}{\sigma_A^2 + \sigma_B^2 - 2 \text{Cov}(A,B)} \]

Using this expression, we find the weight of Asset A must be:

\[ X_A = \frac{0.62^2 - 0.001}{0.33^2 + 0.62^2 - 2(0.001)} \]
\[ X_A = 0.7804 \]

This implies the weight of Stock B is:

\[ X_B = 1 - X_A \]
\[ X_B = 1 - 0.7804 \]
\[ X_B = 0.2196 \]

b. Using the weights calculated in part a, the expected return of the portfolio is:

\[ E(R_P) = X_A E(R_A) + X_B E(R_B) \]
\[ E(R_P) = 0.7804(0.09) + 0.2196(0.15) \]
\[ E(R_P) = 0.1032, \text{ or } 10.32\% \]
c. Using the derivative from part a, with the new covariance, the weight of each stock in the minimum variance portfolio is:

\[ X_A = \frac{\sigma_A^2 + \text{Cov}(A,B)}{\sigma_A^2 + \sigma_B^2 - 2\text{Cov}(A,B)} \]

\[ X_A = \frac{.62^2 + \color{red}{-0.05}}{.33^2 + .62^2 - 2\color{red}{(-.05)}} \]

\[ X_A = .7322 \]

This implies the weight of Stock B is:

\[ X_B = 1 - X_A \]
\[ X_B = 1 - .7322 \]
\[ X_B = .2678 \]

d. The variance of the portfolio with the weights on part c is:

\[ \sigma_P^2 = X_A^2 \sigma_A^2 + X_B^2 \sigma_B^2 + 2X_A X_B \text{Cov}(A,B) \]
\[ \sigma_P^2 = (.7322)^2(.33)^2 + (.2678)^2(.62)^2 + 2(.7322)(.2678)(-.05) \]
\[ \sigma_P^2 = .0663 \]

And the standard deviation of the portfolio is:

\[ \sigma_P = (0.0663)^{1/2} \]
\[ \sigma_P = .2576, \text{ or } 25.76\% \]