7.3 Vertical differentiation

A few notations:

- \( U \): surplus derived from the consumption of the good. Consumers have 0-1 demand for the product; and,
  \[
  U = \begin{cases} 
  \theta s - p & \text{if buys;} \\
  0 & \text{otherwise.}
  \end{cases}
  \]

- \( s > 0 \): quality;

- \( \theta > 0 \): taste parameter; \( \theta \) distributed normally between \( \underline{\theta} \) and \( \overline{\theta} \).

Let there be two firms. Firm \( i \) produces a good of quality \( s_i \). Let \( s_1 < s_2 \) (think about it as the first stage outcome of a two-stage quality-price game). The unit cost of production is \( c \). We look for an equilibrium in which high-\( \theta \) consumers buy \( s_2 \), low-\( \theta \) consumers buy \( s_1 \), and the whole market is served (covered). A consumer with parameter \( \theta \) is indifferent between \( s_1 \) and \( s_2 \) if:

\[
\theta s_1 - p_1 = \theta s_2 - p_2;
\]

or,

\[
\theta = \frac{p_2 - p_1}{s_2 - s_1}.
\]

Hence,

\[
\begin{align*}
D_1(p_1, p_2) &= \frac{p_2 - p_1}{s_2 - s_1} - \underline{\theta} \\
D_2(p_1, p_2) &= \overline{\theta} - \frac{p_2 - p_1}{s_2 - s_1}.
\end{align*}
\]

In equilibrium, firm \( i \):

\[
\max_{p_i} (p_i - c)D_i(p_1, p_2).
\]

The maximization problems yield:

\[
\begin{align*}
p_1 &= R_1(p_2) = \left[p_2 + c - \frac{\theta(s_2 - s_1)}{2}\right] / 2 \\
p_2 &= R_2(p_1) = \left[p_1 + c + \frac{\theta(s_2 - s_1)}{2}\right] / 2.
\end{align*}
\]
Solving for the equilibrium, we have:

\[
\begin{align*}
\hat{p}_1^c &= c + \frac{(\bar{s}_2 - s_1) - 2\theta(s_2 - s_1)}{3} = c + \frac{\bar{s} - 2\theta}{3} (s_2 - s_1) \\
\hat{p}_2^c &= c + \frac{2\bar{s}(s_2 - s_1) - \theta(s_2 - s_1)}{3} = c + \frac{2\bar{s} - \theta}{3} (s_2 - s_1).
\end{align*}
\]

It is natural to have \( \hat{p}_2^c > \hat{p}_1^c \) so that consumers would buy from the low quality good. With \( \hat{p}_1^c \) and \( \hat{p}_2^c \),

\[
\begin{align*}
D_1^c &= (\bar{s} - 2\theta)/3 \\
D_2^c &= (2\bar{s} - \theta)/3;
\end{align*}
\]

and,

\[
\begin{align*}
\Pi_1^1(s_1, s_2) &= (\bar{s} - 2\theta)^2(s_2 - s_1)/9 \\
\Pi_2^2(s_1, s_2) &= (2\bar{s} - \theta)^2(s_2 - s_1)/9.
\end{align*}
\]

Note also, when \( s_2 = s_1 \), we have the Bertrand result. So, the principle of differentiation can again be justified when we look at the choice of quality. Note also that we need to assume: \( \bar{s} \geq 2\theta \); i.e., the amount of consumer heterogeneity (in tastes) is sufficiently large.

Now let us consider a quality-choice stage before the price competition. Let the choice of qualities be costless, and firms choose \( s \in [s, \bar{s}] \). Based on \( \Pi_1^1(s_1, s_2) \) and \( \Pi_2^2(s_1, s_2) \), we know that in equilibrium \( s_1 \neq s_2 \), WLOG, let \( s_1 < s_2 \). Since both firms make more profits when they are more differentiated; firm 1 gains by reducing its quality toward \( s \), and firm 2 gains by increasing its quality toward \( \bar{s} \). Hence, a pure-strategy equilibrium is: \( \{s_1^c, s_2^c\} = \{s, \bar{s}\} \). Since firms are symmetric, when \( s_1 > s_2 \), the equilibrium is: \( \{s_1^c, s_2^c\} = \{\bar{s}, s\} \). The important thing is: Again, we have maximal differentiation. Note that this result cannot be an equilibrium if \( s \) is “too” low. We need to assume: \( \bar{s} s - [c + ((\bar{s} - 2\theta)/3)(\bar{s} - s)] \geq 0 \); i.e., the consumer with the lowest taste parameter buys the low quality product so that the whole market is served.

The maximal-differentiation result is interesting because it formalizes the effect of strategic behavior in an extreme way. Even though quality is costless to produce, the low-quality firm gains from reducing its quality to the minimum because this softens price competition. Although this result is not robust (lots of assumptions and specifications), the principle of differentiation is quite general.
7.4 The principle of differentiation

The principle of differentiation is important in that it adds to our understanding of the nature of price rivalry. According to which, firms generally do not want to produce the same product (neither horizontally nor qualitatively). Firms usually wish to differentiate themselves from other firms. However, not all technically feasible goods will be produced. Often a few of them are selected even though thousands are possible \textit{a priori}. This incomplete spectrum of goods is most likely related to the existence of fixed costs (capital, personnel, R&D). Fixed costs limit the spectrum.

The principle of differentiation also provides theoretical justification for the marketing recommendations concerning segmentation such as: developing market niches and/or clienteles. However, there are also forces that work against maximal differentiation. They are:

- Where is the beef?
  - Recall the market-share effect in the linear city model;
  - Extend the linear city model;
  - Demand concentration.

- Positive externality among firms:
  - On the supply side, firms locate near a source of raw materials;
  - On the demand side: consumer search costs.

- The absence of price competition:
  - Price regulation;
  - TV news broadcasting.
7.5 Advertising and information

7.5.1 Views on advertising and monopolistic competition

In economics, the view on advertising varies. The two benchmarks are: the partial view (information advertising) and the adverse view (image advertising). The partial view sees advertising as providing information to the customers and enabling them to make rational choices. Advertising announces the existence of a product, quotes its price, informs consumers about retail locations, and describes the product’s quality. Therefore, advertising reduces product differentiation associated with the lack of information about some products and fosters competition. Moreover, it facilitates the entry of new firms and encourages the production of high-quality goods. In general, the partial view considers advertising fosters price competition. Benham (1972) argues that the average price of eyeglasses is significantly higher in states where advertising is not allowed. See also Cady (1976) for a study on prescription drugs.

The adverse view claims that advertising is meant to persuade and to “fool” consumers. It creates differentiation when products are essentially the same rather than reducing informational differentiation. In general, the adverse view considers advertising reduces product competition and increases barriers to entry.

Before we start our discussion of the effect of advertising on consumer demand and product differentiation. Let’s detour a little bit to look at a concept that used frequently in IO: monopolistic competition. Monopolistic competition, in its simplest form, is used to formalize the following industry configuration: Consider a market where each firm produces at most one product, monopolistic competition requires:

(i) Each firm faces a downward-sloping demand;

(ii) Each firm makes zero profit;

(iii) A price change by one firm has only a negligible effect on the demand of any other firm.
Note that in general properties (i) and (ii) are satisfied in most oligopolistic competition models with product differentiation and free entry (e.g., the circular city model). It is property (iii) that distinguishes monopolistic competition from other configurations. It says that each firm or product has no direct “neighbors” in the product space. Hence, the point of monopolistic competition is not to study strategic aspects between products, but rather to abstract from them to simplify the analysis and study other issues, such as the number of products offered.

7.5.2 Information advertising – monopolistic competition

Consider the following situation: All firms in the market offer the same product (no horizontal or vertical differentiation). Production of the good requires a constant marginal cost $c$. There are $N$ homogenous consumers with unit demands of the good. A consumer receives: $U = \pi - p$ if she buys one unit at $p$; and, 0 if she does not buy. Consumer are not allow to search, they receive information on prices through advertising. The only way that a firm can reach a consumer is to send advertisement (ads) at random. A consumer can receive 0, 1, 2, ..., ads. If she receives none, she buys nothing. If she receives one or more prices, she buys from the one with the lowest price (as long as $p \leq \pi$). An ad conveys information on the existence and the price of a product in the market (the partial view).

Let $m$ denotes the total number of ads sent out by all firms. A consumer’s probability of not getting an ad at all is:

$$1 - \Phi = \left(1 - \frac{1}{N}\right)^m \approx e^{-m/N},$$

for $N$ large. Thus, to make sure a fraction $\Phi \in (0, 1)$ of consumers receives at least one ad, the number of ads is:

$$m = N \ln \left(\frac{1}{1 - \Phi}\right).$$

Let $c'$ be the (constant) cost of sending an ad. The cost of sending ads to make sure that a fraction $\Phi$ of consumers receiving at least one ad is:

$$A(\Phi) = c'm = c'N \ln \left(\frac{1}{1 - \Phi}\right).$$
Suppose firms face no entry costs or fixed costs. In this free-entry equilibrium, any price \( p \in [c + c', \bar{s}] \) is advertised by some firm. Let \( x(p) \) denote the probability that an ad at \( p \) is accepted by the consumer who receives it (the consumers buys from the corresponding firm); i.e., the probability that this ad is the only ad that she receives, or the lowest-price one from all the ads received. This \( x(p) \) can be thought of as a demand function. Clearly, it is downward-sloping. The zero-profit condition requires: for all \( p \in [c + c', \bar{s}] \),

\[
(p - c)x(p) - c' = 0.
\]

The main focus here is the level of advertising. Note that in equilibrium the probability that a consumer does not receive an ad, \( 1 - \Phi^c \), is equal to \( x(\bar{s}) \), the probability that an ad at the highest price would induce a sale (the only chance that such an ad will trigger a sale is if it is the only ad received by the consumer). Hence,

\[
1 - \Phi^c = x(\bar{s}) \quad \text{the zero profit condition} \quad \frac{c'}{\bar{s} - c}.
\]

The social planner’s problem is to:

\[
\max_{\Phi} \Phi(\bar{s} - c) - c' \ln \left( \frac{1}{1 - \Phi} \right).
\]

The FOC yields:

\[
\bar{s} - c - \frac{c'}{1 - \Phi^*} = 0,
\]

or,

\[
\Phi^* = \Phi^c.
\]

The market (free-entry) equilibrium level of advertising is socially optimal. Note that a firm’s incentive to send out an ad with price \( p \) is: \((p - c)x(p)\). Given the zero-profit condition, this incentive is the same as the gain from sending an ad at price \( \bar{s} \). Such an ad yields \( \bar{s} - c \) with a probability equal to the probability that the consumer receives no ad. But this is exactly the social planner’s concern. From the social planner’s point of view, an ad increases the social welfare by \( \bar{s} - c \) only when the consumer receives no other ad.
7.5.3 Information advertising – oligopolistic competition

Note that the previous free-entry equilibrium is monopolistically competitive – there exists no strategic interaction among firms. To take into account oligopolistic interaction, let’s go back to the linear city setup with firms locating at the two ends, and any consumer (lives in \([0, 1]\)) can buy a product if and only if she receives an ad from the corresponding firm. Let \(\Phi_i\) \((i = 1, 2)\) denote the fraction of consumers who receive an ad from firm \(i\). All the consumers have equal chances of receiving a given ad (non-localized advertisement; non-targeted advertisement). Let the cost of reaching fraction \(\Phi_i\) of consumers be \(A(\Phi_i)\), with: \(A' > 0\), and \(A'' > 0\).

Recall that the demand for firm 1 under full information is: \((p_2 - p_1 + t)/2t\). We can write down the demand for firm 1’s at the current setting:

\[
D_1 = \frac{\Phi_1(1 - \Phi_2) \cdot 1}{\text{prob. of receiving ads only from firm 1}} + \frac{\Phi_1\Phi_2 \cdot \frac{p_2 - p_1 + t}{2t}}{\text{prob. of receiving ads from both firms}}
\]

\[
= \Phi_1 \left[ (1 - \Phi_2) + \Phi_2 \left( \frac{p_2 - p_1 + t}{2t} \right) \right].
\]

Note that the elasticity of demand (symmetric): \(\varepsilon_1 = \Phi p/(2 - \Phi)t\) is increasing in \(\Phi\).

Consider the game that the two firms simultaneously choose prices and advertising levels. Firm’s behavior is described by:

\[
\max_{p_1, \Phi_1} . (p_1 - c)D_1 - A(\Phi_1).
\]

We cannot go further without specifying \(A(\cdot)\). Let \(A(\Phi_i) = a\Phi_i^2/2\), quadratic advertising costs. The FOCs yields:

\[
p_1 = \frac{p_2 + t + c}{2} + \frac{1 - \Phi_2}{\Phi_2},
\]

and,

\[
a\Phi_1 = (p_1 - c) \left[ (1 - \Phi_2) + \Phi_2 \left( \frac{p_2 - p_1 + t}{2t} \right) \right].
\]

Let’s look at a symmetric equilibrium \((p_1^c = p_2^c = p^c, \text{ and } \Phi_1^c = \Phi_2^c = \Phi^c)\) where not all customers receive both ads \((\Phi^c < 1)\) and firms do compete for the “common demand” \((a\text{ is not too large, so that if a firm charges a high price and focuses on its own “turf” would...})\)
not yield enough demand). This way, we can compare this equilibrium with the one in the linear city model. By symmetry, we have:

\[ p^c = c + t \left( 2 - \frac{\Phi^c}{\Phi^c} \right) = c + \sqrt{2at}, \]

and,

\[ \Phi^c = \frac{2}{1 + \sqrt{2a/t}}. \]

And,

\[ \Pi^1 = \Pi^2 = \frac{2a}{\left( 1 + \sqrt{2a/t} \right)^2}. \]

Note that we need to ensure \( \Phi^c < 1 \), therefore we need to impose the assumption: \( a \geq t/2 \). As a matter of fact, we need also to derive an upper bound for \( a \). But it is not important.

Conclusions are:

- \( p^c > (c + t) \), the full information price. \( p^c \uparrow \) as \( t \uparrow \), but not as rapid as it is in the full information case. Here, as \( t \uparrow \), more horizontal differentiation yields higher profit margins and therefore encourages more advertising (see below) and creates a bigger common demand area (more competition). Intuitively, as \( a \uparrow \), \( p^c \uparrow \).

- \( \Phi^c \uparrow \) as:
  - \( a \downarrow \);
  - \( t \uparrow \).

- \( \Pi^i \uparrow \) as:
  - \( t \uparrow \);
  - \( a \uparrow \). Given \( p \) and \( \Phi \), \( \Pi \downarrow \) as \( a \uparrow \) (direct effect). However, as \( a \uparrow \), \( \Phi \downarrow \). Firms can then increases the price and therefore \( \Pi \uparrow \) (strategic effect).