7 Product Differentiation

Two of the most common ways to specify product differentiation are: horizontal (spatial) differentiation and vertical (qualitative) differentiation. Let’s use them for our first look of price competition versus non-price competition. Recall that in horizontal product differentiation, a product is the same for two consumers who live at the same place; a product is different for two consumers if they are not at the same location. What makes product differentiated is the transportation cost paid by consumers. In the vertical differentiation case, it is the “quality” that makes product differentiated.

As alluded to before, when products are differentiated; the cross-elasticity of demand is not infinite at equal prices. Prices above marginal cost can and will be sustained in equilibrium under product differentiation. Diversity prevents cut-throat price competition, even in a non-repeated relationship. It is then nature to ask:

- The determination of prices under product differentiation;
- The choice of products.

7.1 The linear city (spatial differentiation)

For the linear city model, we assume the following:

- Consumers have 0–1 demand for a product and live uniformly on the [0,1] interval, and there are \( N \) of them. WLOG, let \( N = 1 \);
- There is fixed surplus derived from the consumption of the product; call it \( \bar{s} \). Let \( \bar{s} \) be sufficiently large so that consumers always buy;
- The unit transportation cost is \( t \);
- There are two firms, 1 and 2, sell the same (physical) product.
Let us look for the equilibrium of a two-stage game: at the first stage, each firm chooses a location; at the second stage, firms compete in prices. It is natural to setup the location–price game this way for firms can adjust prices faster than locations.

Let firm 1 locate at $a \geq 0$, and firm 2 at $1 - b$, where $b \geq 0$. WLOG, let $1 - b - a \geq 0$; i.e., firm 1 is located to the left of firm 2. Note that $a = b = 0$ corresponds to maximal differentiation, and $a + b = 1$ corresponds to no (minimal) differentiation. A consumer who is indifferent between the two firms is located at:

- When linear transportation cost is linear,

$$p_1 + t(x - a) = p_2 + t(1 - b - x);$$

or,

$$x = a + \frac{1 - b - a}{2} + \frac{p_2 - p_1}{2t}.$$

Hence,

$$D_1(a, b, p_1, p_2) = x = \begin{array}{c}
\text{back yard} \quad a \\
\text{half in-between} \quad \frac{1 - b - a}{2} \\
\text{price difference} \quad \frac{p_2 - p_1}{2t}
\end{array}$$

$$D_2(a, b, p_1, p_2) = 1 - x = b + \frac{1 - b - a}{2} + \frac{p_1 - p_2}{2t}.$$

Now consider a consumer located at $y \geq 1 - b$ (to the right of firm 2). This consumer belongs to firm 2’s “turf” or “back yard.” Her choice between the two firms is determined by the comparison between: $p_1 + t(y - a)$, and: $p_2 + t[y - (1 - b)]$, i.e., between $p_1 + t(1 - b - a)$ and $p_2$. Hence, all consumers located to the right of firm 2 always make the same choice as the consumers located at firm 2’s location. This means that at $p_1 + t(1 - b - a) = p_2$, the demand functions are discontinuous; all consumers on firm 2’s turf switch to firm 1 for a small reduction in $p_1$. The discontinuity of demand functions causes discontinuity and non-concaveness in the profit function.

Pure-strategy equilibrium may not exist (when both firms are close to the center; d’Aspremont, Gabszewicz, and Thisse, 1979). One way to sidestep this technical issue is to have the transportation cost been quadratic.
• In the case that the transportation cost is quadratic,

\[ p_1 + t(x - a)^2 = p_2 + t(1 - b - x)^2; \]

or,

\[ x = a + \frac{1 - b - a}{2} + \frac{p_2 - p_1}{2t(1 - b - a)}. \]

Hence,

\[ D_1(a, b, p_1, p_2) = x = a + \frac{1 - b - a}{2} + \frac{p_2 - p_1}{2t(1 - b - a)}. \]

\[ D_2(a, b, p_1, p_2) = 1 - x = b + \frac{1 - b - a}{2} + \frac{p_1 - p_2}{2t(1 - b - a)}. \]

Now the demand functions and profit functions are well-behaved (continuous and concave). In the quadratic setting, the marginal transportation cost increases with the distances to the store. This prevents the problem that we encountered in the linear setting. Hence, we use the quadratic transportation setup in the following.

The firms’ problems are:

\[ \max_{p_i} (p_i - c)D_i(a, b, p_i, p_j). \]

These yield:

\[ p_1^c(a, b) = c + t(1 - b - a)(1 + \frac{a - b}{3}); \]

\[ p_2^c(a, b) = c + t(1 - b - a)(1 + \frac{b - a}{3}). \]

An immediate observation is that: \( p_i^c \geq c, \forall i = 1, 2 \). With these Nash equilibrium prices, we can now discuss firms’ location choice problem. Before we get into that, let’s take a closer look of these prices. Suppose \( a = b = 0 \) (maximal differentiation), \( p_1^c = p_2^c = c + t \), and \( \Pi^1 = \Pi^2 = t/2 \). Suppose \( a + b = 1 \) (minimal differentiation; same location), then \( p_1^c = p_2^c = c \), and \( \Pi^1 = \Pi^2 = 0 \).

Going back to the first stage, what are firms’ locations choices? Each firm anticipates that the choice of location affects not only the demand function but also the intensity of
price competition in the second stage. Write down firms’ profit functions:

$$\Pi^i(a, b) = (p^e_i(a, b) - c) D_i(a, b, p^e_i(a, b), p^e_2(a, b)) .$$

One can then solve the following pair of problems:

$$\begin{align*}
\max_a \Pi^1(a, b) \\
\max_b \Pi^2(a, b),
\end{align*}$$

to derive a Nash equilibrium. But this is tedious, we can do better.

Note that in equilibrium,

$$\frac{d\Pi^1}{da} = \frac{\partial\Pi^1}{\partial a} + \left(\frac{p^e_1}{\partial p_1 a} + \frac{\partial\Pi^1}{\partial p_2 a}\right) = (p^e_1 - c) \left(\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2 a}\right) ;$$

and,

$$\frac{\partial D_1}{\partial a} = \frac{1}{2} + \frac{p^e_1 - p^e_2}{2t(1-b-a)^2} = \frac{3 - 5a - b}{6(1-b-a)} ;$$

$$\frac{\partial D_1}{\partial p_2 a} = \left(\frac{1}{2t(1-b-a)}\right) t \left(-\frac{4}{3} + \frac{2a}{3}\right) = \frac{-2 + a}{3(1-b-a)} .$$

Since,

$$\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2 a} < 0$$

and $p^e_1 > c$, we conclude: $d\Pi^1/da < 0$. Therefore, when firm 1 is to the left of firm 2 (i.e., $1 - b > a$), it always wants to move leftward. Similarly, we can show: $d\Pi^2/db < 0$. Hence, when firm 2 is to the right of firm 1, it always wants to move rightward. The equilibrium location exhibits maximal differentiation. This is an extreme example of a more general result named: the principle of differentiation.
An important observation here is that when firm 1 is to the left of 1/2; i.e., \( a < 1/2 \), \( \partial \Pi^1 / \partial a > 0 \). This implies firm 1 will want to move toward the center to increase its market share given the price structure. This is called the **market-share effect**: for given prices, the two firms want to locate at or near the center. However, firm 1 acknowledges that the associated decrease in product differentiation forces firm 2 to lower its price. Our computations show that the **strategic effect** dominates the market-share effect (strategic effect > market-share effect).

It is also interesting to ask for the social planner’s location choices. Note that because consumption is fixed, social planner minimizes consumer’s average transportation cost and chooses 1/2 and 3/4. Therefore, the market outcome yields socially too much product differentiation.

### 7.2 The circular city (spatial differentiation)

The linear city model assumes that there are two firms in the market. This suggests some kind of **entry barriers** to block potential entrants entering the market. Solap (1979) uses a circular city model to study entry and location choice when there are no barriers to entry other than fixed cost or entry costs. The circular city model is “easier” in that the product space is completely homogenous (no location is *a priori* better than another). Let’s assume the following:

- Consumers are located uniformly on a circle with a perimeter equal to 1;
- Firms are also located around the circle;
- Travel occurs along the circle;
- There exists a large number of identical firms with the same marginal cost; each firm is allowed to locate at one location; entry fixed cost is \( f \);
- Consumers buy one unit of the good, with unit travel cost \( t \).
Consider the following two-stage game: in the first stage, potential entrants simultaneously choose whether or not to enter; at the second stage, firms compete in prices given the first-stage outcome. Suppose there are \( n \) firms entered in the first stage, given the principle of differentiation, we put these entering firms equidistant from one another on the circle. Thus, maximum differentiation is \textit{exogenously imposed}. The focus here is to study the extent of entry.

Let there be \( n \) firms entered the market in the first stage. We look for the symmetric Nash equilibrium price \( p \) in the second-stage. Note that any firm has only two real competitors – the two closest nearby ones. Suppose firm \( i \) chooses price \( p_i \). A consumer located at the distance \( x \in (0, 1/n) \) from firm \( i \) is indifferent between purchasing from firm \( i \) and purchasing from \( i \)'s closest neighbor if:

\[
p_i + tx = p + t(1/n - x).
\]

Then firm \( i \) faces a demand of:

\[
D_i(p_i, p) = 2x = \frac{p + t/n - p_i}{t}.
\]

So firm \( i \)'s problem is:

\[
\max_{p_i} \left( p_i - c \right) \left( \frac{p + t/n - p_i}{t} \right) - f.
\]

Using the FOC and letting \( p_i = p \), we have:

\[
p = c + t/n.
\]

Going back to the first stage, it cannot be an equilibrium if those \( n \) entering firms are making positive profits. The zero-profit condition determines the number of firms:

\[
(p - c)/n - f = t/n^2 - f = 0;
\]

or,

\[
n^c = \sqrt{t/f}.
\]

Hence, the equilibrium price \( p^c \):

\[
p^c = c + \sqrt{t/f}.
\]
Therefore,

- An increase in $f$ causes a decrease in $n^c$, and an increase in $p^c$;

- As $f \to 0$, $n^c \to \infty$, and $p^c \to c$. With very low entry cost, each consumer purchases a product very close to her preferred product, and the market is approximately competitive.

- An increase in $t$ causes an increase in $p^c$, and thereby an increase in $n^c$. From firms’ point of view, the increase in $t$ is an increase in the possibility of differentiation.

An important note is that the equilibrium price is above marginal cost, nevertheless, firms are making zero profit due to the fixed cost. In other words, a larger than zero price-cost margin does not imply positive profit. This is a very general result for this type of models.

The social planner’s problem is to minimize the total fixed cost plus the average transportation cost:

$$\min \; n f + \left( 0 + \frac{1}{2n} \right) t.$$ 

We have:

$$n^* = \frac{1}{2} \sqrt{\frac{t}{f}} = \frac{1}{2} n^c.$$ 

Again, the market outcome yields socially too much product differentiation (too many firms have entered).

Finally, a note on the maximal differentiation assumption. Economides (1984) uses a three-stage, entry-location-price, game to show that the exogenous maximal differentiation can be justified.