5 Oligopoly Pricing – Static

As in the monopoly problem, we start the analysis of oligopolistic markets with pricing issues. Always have it in mind that from now on we have situations with more than one firm in the market. In most cases, there are (strategic) interactions among firms. We should try to derive intuitions from a firm’s perspective: if I change my behavior (action), e.g., price or quantity, how would my opponents (rivals; competing firms) respond, knowing that they also have the calculation in their minds (and knowing that my rivals know that I know they have the calculation in their minds, and so on, *ad infinitum*)? In some cases, it is useful to derive intuitions from the best response (or reaction) functions.

5.1 Strategic substitutes and complements

Let $a_i$ denote firm $i$’s action. It could be a choice of how many to produce (quantity decision), how much to ask (pricing decision), or others such as investment decisions. Let $\Pi^i(a_i, a_j)$ be the profit function of firm $i$ when it has action $a_i$ and its rival, firm $j$, has action $a_j$. Let $\Pi^i(\cdot, \cdot)$ be twice continuously differentiable, and (strictly) concave everywhere in $i$’s action; i.e., $\Pi^i_{ii} < 0 \; \forall (a_i, a_j)$, where $\Pi^i_{ii} = \partial^2 \Pi^i / \partial a_i^2$. We can define a Nash equilibrium as a pair of action $(a_i^*, a_j^*)$ such that: $\forall i$,

$$\Pi^i(a_i^*, a_j^*) \geq \Pi^i(a_i, a_j^*), \forall a_i.$$ 

To derive a Nash equilibrium strategies, we can use the FOCs:

$$\Pi^i(\cdot, a_j^*) = 0, \forall i; \text{ where: } \Pi^i(\cdot, \cdot) \equiv \frac{\partial \Pi^i(\cdot, \cdot)}{\partial a_i}.$$ 

Or, we can define reaction functions as:

$$R_i(a_j) \equiv a_i(a_j) \equiv \arg \max_{a_i} \Pi^i(a_i(\cdot), a_j), \forall i,$$

and solve for a Nash equilibrium as:

$$R_i(a_j^*) = a_i^*, \forall i.$$ 

From the FOC, we have:

$$\Pi^i(R_i(a_j), a_j) \equiv 0.$$
Differentiate the above identity w.r.t. \( a_j \), we have:

\[
\Pi_{ii}(R_i(a_j), a_j)R'_i(a_j) + \Pi_{ij}(R_i(a_j), a_j) = 0.
\]

Hence, the slope of the reaction function \( R'_i(a_j) \):

\[
R'_i(a_j) = \frac{\Pi_{ij}(R_i(a_j), a_j)}{\Pi_{ii}(R_i(a_j), a_j)};
\]

or,

\[
\text{sign}(R'_i(a_j)) = \text{sign}(\Pi_{ij}(R_i(a_j), a_j)).
\]

We now define:

\[
\begin{cases}
\text{strategic substitutes:} & \Pi_{ij} < 0; \\
\text{strategic complements:} & \Pi_{ij} > 0.
\end{cases}
\]

The reaction functions are useful in that they give a sense of the kind of strategic response that firms have in minds. But be careful that in simultaneous action one-stage games, there is no “reaction” whatsoever. Once the actions are made, payoffs are realized, and the game is over.
5.2 Bertrand paradox

With only one firm in the market, the monopoly charges the monopoly price which is higher than the competitive one (marginal cost). Let’s bring in one additional firm (a duopolistic setting) which produces the same product (homogenous product). Consider a situation where there are two firms in the market with demand: \( q = D(p) \). Let the marginal cost of production be a constant: \( c \). The demand for firm \( i \) is as follows:

\[
D_i(p_i, p_j) = \begin{cases} 
D(p_i) & \text{if } p_i < p_j \\
D(p_i)/2 & \text{if } p_i = p_j \\
0 & \text{otherwise.}
\end{cases}
\]

The assumption that firms share the market when they charge the same price is not crucial. Hence, firm \( i \)'s profit, \( \Pi^i(p_i, p_j) \), is:

\[
\Pi^i(p_i, p_j) = (p_i - c)D_i(p_i, p_j).
\]

Note that:

- \( \pi(\cdot) \) is well-defined, but is not continuous because the discontinuity of \( D(\cdot) \);
- Reaction functions in a homogenous-product Bertrand game (with continuous price) are not defined.

The Nash equilibrium can be defined as the pair of prices \((p_i^*, p_j^*)\) such that:

\[
\Pi^i(p_i^*, p_j^*) \geq \Pi^i(p_i, p_j^*), \forall i.
\]

The unique equilibrium is:

\[
(p_i^*, p_j^*) = (c, c).
\]

For uniqueness, we need to rule out the following situations as potential equilibria:

(i). \( p_i^* > p_j^* > c \);
(ii). \( p_i^* = p_j^* > c \);
(iii). \( p_i^* > p_j^* = c \).

Therefore: 1. Firms price at marginal cost; 2. Firms make no (normal) profits; 3. It is welfare optimal. Moreover, the results remain the same if we add a third or more firms. An
immediate implication is that the equilibrium price is *independent* of the number of firms in the market as long as there are no less than two firms in the market.

What’s the equilibrium when one firm has the cost advantage, say $c_i < c_j$? The unique (Nash) equilibrium for this simultaneous pricing game (*Bertrand game*) becomes (when the equilibrium is defined at the limit):

$$(p_i^*, p_j^*) = (c_j, c_j),$$

where firm $j$ makes no profit while firm $i$ has profit $\Pi^i(p_i^*, p_j^*)$ as:

$$\Pi^i(p_i^*, p_j^*) = \begin{cases} (c_j - c_i)D(c_j) & \text{if } c_j \leq p^m(c_i) \\ \max (p - c_i)D(p) & \text{otherwise.} \end{cases}$$

The **Bertrand paradox** is that a mere introduction of one additional firm (a “slight” change in the number of firms) causes the equilibrium price to change abruptly (from monopoly price to marginal cost). It describes a situation of *intense and tough* competition among firms. How to resolve is “paradox”? Let’s attack the underlying assumptions: 1. Maybe firms are not playing the pricing game; 2. Maybe firms cannot supply the market demand (capacity-constrained pricing game); 3. Firms may play the game many many times (intertemporal consideration; repeated games); 4. Maybe the product are not perfect substitutes so that firms enjoy some “monopoly” power. For the time being, we put aside 3 & 4 and focus on only 1 & 2.

Of course, there are many other ways to resolve the Bertrand paradox. Some of them are complicated. The easiest one is to introduce discreteness in price.
5.3 Quantity competition (Cournot game)

Consider the one-stage game where firms choose quantities simultaneously. The profit function \( \Pi_i(q_i, q_j) \) is said to have the exact Cournot form, if
\[
\Pi_i(q_i, q_j) = P(q_i + q_j)q_i - C_i(q_i).
\]

Each firm maximizes its profit given the quantity choice of the other firm. The FOC is:
\[
\Pi_i'(q_i, q_j) = \frac{P(q_i + q_j) - C_i'(q_i)}{P'(q_i + q_j)} + \underbrace{q_i P'(q_i + q_j)}_{\text{an additional unit}} = 0.
\]

Now we know where the negative externality comes from: When choosing its output, firm \( i \) takes into account the adverse effect \( (P'(q_i + q_j) < 0) \) on its own output \( q_i \), rather than the effect on aggregate output \( Q = q_i + q_j \), as in the monopoly case). Therefore each firm tends to choose an output that exceeds the optimal output from the industry’s point of view, and the overall profit is lower than that of the monopoly’s.

Note that the FOC can be written as:
\[
L_i \equiv \frac{P(\cdot) - C_i'(\cdot)}{P(\cdot)} = \frac{\alpha_i}{\varepsilon} > 0,
\]
where: \( \varepsilon = -P'Q/P \) is the demand elasticity, and \( \alpha_i = q_i/Q \) is firm \( i \)'s market share. Hence, firms price higher than the marginal cost. Another easily seen feature is that: the industrial weighted average Lerner index is the elasticity adjusted Herfindahl index. Sum over \( i \) with weight \( \alpha_i \), we have:
\[
\sum_i \alpha_i L_i = \left( \sum_i \alpha_i^2 \right)/\varepsilon.
\]

Define reaction functions \( R_i \) as:
\[
\Pi_i(R_i(q_j), q_j) = 0.
\]
One can then solve the equilibrium \( (q_i^*, q_j^*) \) by solving the intersections of the reaction functions. For example, let \( D(p) = 1 - p \) and \( C_i(q_i) = c_i q_i \). the reaction functions are:
\[
q_i = R_i(q_j) = \frac{1 - c_i - q_j}{2}.
\]
Solving for the only intersection, one derives: \( q_i = (1 - 2c_i + c_j)/3 \). Observe that a firm’s output:

- Decreases with its marginal cost;
- Increases with its competitor’s marginal cost.

These two properties are very general. Decreasing in its own marginal cost is the same as in the monopoly situation. The intuition is that: given \( q_j \), firm \( i \) is a \textit{de facto} monopoly on the residual demand. Hence, the same proof applies. As to the result of increasing in its competitor’s marginal cost, it holds as long as the following conditions are satisfied: 1. Downward sloping reaction function (quantities are strategic substitutes); 2. Single crossing of the reaction functions (there exists an unique equilibrium) such as: \(|R_j'(q_i)| < |R_i'(q_j)|\) (the stability condition) in the \((q_1, q_2)\) space. This is best seen through a figure.

It is straightforward to generalize the above analysis to the case of \( n \) firms. Note that the FOC becomes:

\[
P(Q) - C_i'(q_i) + q_i P'(Q) = 0.
\]

Let \( P(Q) = 1 - Q \) and \( C_i(q_i) = cq_i \). The above equation becomes:

\[
1 - Q - c - q_i = 0.
\]

Since it is “natural” to look for the symmetric equilibrium in a symmetric game, we can plug in \( Q = nq \), and derive:

\[
q = \frac{1 - c}{1 + n},
\]

or,

\[
p = \frac{1 + nc}{n + 1},
\]

and,

\[
\Pi = \left( \frac{1 - c}{n + 1} \right)^2.
\]

The Cournot game is kind of “nice” in that it has a \textit{continuous transition} from the monopoly outcome to the competitive outcome when \( n \) goes from one to infinity. What’s the “problem”
(if any) with the Cournot game? Consider how does the market clear when firms choose quantities. In general, people assume that there exists a Cournot auctioneer who equates supply and demand after all the firms submit their outputs to her. This assumption is a bit “strange”. However, I am going to show you that we may be able to give a Cournot game a Bertrandian interpretation by taking into account the capacity constraints. But before that, let’s do two exercises to get more feeling about the quantity game.

Consider a market with 3 identical firms with demand: \( D(p) = 1 - p \), and \( c = 0 \). Then, \( q_i = 1/4 \) and \( \Pi_i = 1/16 \). Consider the case that two of them merge. Then, \( q_i = 1/3 \) and \( \Pi_i = 1/9 \). Note that: \( 2 \times 1/16 > 1/9 \). Therefore, merger is not profitable in simple quantity games. On the first thought, this is a bit counter-intuitive, but use the strategic substitutes reasoning and think again. In such games, merger weakens the strategic position of firms involved in the deal. On the contrary, merger is profitable in simple differentiated product price games.

Consider a duopoly producing a homogenous product. Firm \( i \) uses 1 unit of labor and 1 unit of raw material to produce 1 unit of output, and firm \( j \) uses 2 units of labor and 1 unit of raw material to produce 1 unit of output. Let the unit cost of labor be \( w \) and the unit cost of raw material be \( r \). Demand is, again: \( Q = 1 - p \). Note that \( c_i = w + r \) and \( c_j = 2w + r \); hence,

\[
q_i = \frac{(1 - 2c_i + c_j)}{3} = \frac{1 - r}{3} \\
q_j = \frac{(1 - 2c_j + c_i)}{3} = \frac{1 - r - 3w}{3}.
\]

Now consider a small change in the labor cost \( w \) to firm \( i \)'s profit. Since

\[
\Pi^i = \max_{q_i} \left[ 1 - (q_i + q_j) - (w + r) \right] q_i,
\]

and,

\[
\frac{d\Pi^i}{dw} = \frac{\partial\Pi^i}{\partial q_i} \frac{\partial q_i^*}{\partial w} + \frac{\partial\Pi^i}{\partial q_j} \frac{\partial q_j^*}{\partial w} + \frac{\partial\Pi^i}{\partial w} \\
= 0 + \left( -\frac{\partial q_i^*}{\partial w} - 1 \right) q_i = 0.
\]

by envelope theorem
The increase in $w$ raises production cost and reduces both firms' profit. Nevertheless, it weakens firm $j$’s strategic position since firm $j$ is more labor intensive. This is good for firm $i$’s profit. In this particular example, these two effects cancel each other exactly.
5.4 Rationing rules

At any price $p$, a firm is not willing to supply more than its competitive supply $S_i(p)$ defined as:

$$p = C_i'(S_i(p)).$$

When firms are not willing to supply the whole market, e.g., $p_i < p_j$ and $S_i(p_i) < D(p_i)$, or, when firms are capacity-constrained, we need to discuss assumptions on how the demand is rationed. The former can be considered as a self-imposed capacity constraint. By defining: $\overline{q}_i = S_i(p_i)$, we take into account both scenarios. The most common rationing rules are: the efficient (parallel) rationing rule and the randomized (proportional) rationing rules. In the following, let $p_i < p_j$.

5.4.1 The efficient (parallel) rationing rule

Suppose that $\overline{q}_i < D(p_i)$. The efficient rationing rule has firm $j$’s residual demand as:

$$\tilde{D}_j(p_j) = \begin{cases} D(p_j) - \overline{q}_i & \text{if } D(p_j) > \overline{q}_i \\ 0 & \text{otherwise.} \end{cases}$$

From the figure, it is obvious why this rationing rule is also called parallel rationing rule. But why it is efficient? Suppose consumer are heterogenous with unit demands. This rationing rule requires to sell $\overline{q}_i$, which is charged with a lower price $p_i$, to the consumers who value the product most. It is efficient because it is maximizing welfare. Some people also call the parallel rationing rule as the welfare- (surplus-) maximizing rationing rule.

What if consumers are identical (same downward sloping demand function)? All consumers are rationed fairly; i.e., each of $m$ consumers gets $\overline{q}_i/m$ at price $p_i$.

5.4.2 The randomized (proportional) rationing rule

Again, let $\overline{q}_i < D(p_i)$. The randomized rationing rule has firm $j$’s residual demand as:

$$\tilde{D}_j(p_j) = \left( \frac{D(p_j) - \overline{q}_i}{D(p_i)} \right) D(p_j).$$
Suppose consumers are heterogeneous with unit demands. This rationing rule is as if consumers arrive the market randomly and face the same probability of being rationed. Note that: \((D(p_i) - \bar{q}_i)/D(p_i)\) can be thought as the probability of not being able to buy from firm \(i\), the low price firm. It is called proportional because for consumers that have the same downward sloping demand, it is as if firm \(i\) serves a proportion: \(\bar{q}_i/D(p_i)\), and leaves: 
\[1 - \frac{\bar{q}_i}{D(p_i)}\]
to firm \(j\).

Note that efficient rationing rule is used more often in the literature because if one allows consumer arbitrage, the proportional rationing rule becomes the efficient one. Note also that firms “prefer” the proportional rationing rule because the high price firm would have a higher demand. This is just obvious from the figure; and mathematically,
\[
\left(\frac{D(p_i) - \bar{q}_i}{D(p_i)}\right) D(p_j) = D(p_j) - \frac{D(p_j)}{D(p_i)} \bar{q}_i \geq D(p_j) - \bar{q}_i.
\]

### 5.5 Capacity-constrained pricing games

Up until now we are discussing the pricing game under the assumption of constant returns to scale. What if firms are capacity-constrained, or more generally, the cost structure is decreasing returns to scale? The analysis in the Bertrand game seems to suggest an equilibrium price \(p^*\):
\[
p^* = C_i'(q_i) = C_j'(q_j),
\]
or,
\[
S_i(p^*) + S_j(p^*) = D(p^*).
\]
Well, as it turns out the competitive outcome is generally not an equilibrium outcome when we take into account capacity constraints or decreasing returns to scale. The intuition is that at the competitive equilibrium, firm \(i\) will not supply more if firm \(j\) raises the price because firm \(i\) is already on its supply curve. Thus, if firm \(j\) raises its price a little bit, it loses demand at the margin while profits from all inframarginal units.

In general, the equilibrium of capacity-constrained pricing game involves **mixed strategies**. One property of such an equilibrium is that both firms’ prices exceed the competitive
price. This property formalizes the notion that decreasing returns to scale (capacity constraints in particular) softens price competition. Let’s look at an example.

**Example 1** A pricing game with “small” capacities.

Let the demand be: \( D(q) = 1 - p \), or equivalently, \( p = 1 - Q = 1 - (q_i + q_j) \). The capacity constraints are: \( q_i \leq \overline{q}_i \) and \( q_j \leq \overline{q}_j \). The capacities are acquired previously at unit cost: \( c_0 \in [3/4, 1] \). Once the capacities are installed, the marginal cost of production is \( c \). WLOG, let \( c = 0 \). Finally, let the rationing rule be the efficient one.

Note first, we only need to consider \( \overline{q}_i \) and \( \overline{q}_j \in [0, 1/3] \). This is because even in the monopoly case, max \( p(1 - p) = 1/4 \), and \( 1/4 - c_0 \overline{q}_i < 0 \) when: \( \overline{q}_i > 1/3 \). Next, we show that: when \( \overline{q}_i \) and \( \overline{q}_j \in [0, 1/3] \),

\[
p^* = 1 - (\overline{q}_i + \overline{q}_j),
\]

is an equilibrium. In other words, at \( p^* \), both firms “dump” their capacities on the market and the consumers are not rationed.

Observe that there is no point to charge a lower price because both firms are capacity-constrained. There is also no point to charge a price higher than \( p^* \). This is because when \( p \geq p^* \), a firm’s profit is:

\[
p \left( 1 - p - \overline{q}_j \right) = (1 - q - \overline{q}_j)q,
\]

where \( q \) is the firm’s quantity sold at price \( p \). Since the derivative at \( q = \overline{q}_i \) is: \( 1 - 2\overline{q}_i - \overline{q}_j \), which is greater or equal to zero when \( \overline{q}_i \) and \( \overline{q}_j \in [0, 1/3] \), raising the price above \( p^* \) (or, equivalently, lowering the output below \( \overline{q}_i \)) is not optimal.

So, after solving the price competition, the firms’ reduced form profit functions (gross of investment costs) are:

\[
\Pi^g_i(\overline{q}_i, \overline{q}_j) = [1 - (\overline{q}_i + \overline{q}_j)]\overline{q}_i;
\]

and (net of investment cost),

\[
\Pi^i_i(\overline{q}_i, \overline{q}_j) = [1 - (\overline{q}_i + \overline{q}_j) - c_0]\overline{q}_i.
\]
Familiar? This is in the exact Cournot reduced form! Everything is as if the two firms put outputs equal to their capacities on the market and an auctioneer equaled supply and demand (a Cournot interpretation). But now we have firms choosing the market-clearing price themselves (A Bertrand interpretation). In this “small” capacity pricing games, the reduced-form profit function have the exact Cournot form. For “large” capacities, there is no pure-strategy equilibrium (of course the capacities are not binding). Can we be a bit more clear about “small” or “large”? Let’s do this.

5.5.1 The pricing game

Let there be two firms, $i$ and $j$. Each firm has a rigid capacity constraint, $q_k$, $k = i, j$. WLOG, let the marginal cost of production after the capacities are installed be: $c = 0$. Assume a (strictly) concave demand function (to assure that reaction functions are well-defined decreasing functions) and efficient rationing. Note that (at the second stage) firms choose prices simultaneously. The following lemmas are useful.

**Lemma 2** In a pure-strategy equilibrium, $p_i = p_j = P(q_i + q_j)$. Firms sell up to capacity.

**Proof.** Note first, in equilibrium, it must be the case that $p_i = p_j$. If not, say, $p_i < p_j$, if firm $i$ is capacity-constrained, then it can increase $p_i$ without losing demand; if firm $i$ is not capacity-constrained, firm $i$ supplies the whole market. But then firm $j$ makes no profit and it is better off undercutting (charging a price $p_i - \varepsilon$ for some $\varepsilon > 0$).

Suppose $p_i = p_j > P(q_i + q_j)$. Then, at least one of the firms, say firm $i$, is not capacity-constrained, $q_i < q_i$. But then for some $\varepsilon > 0$, $(p_i - \varepsilon)q_i > p_i q_i$; i.e., firm $i$ would undercut.

Suppose $p_i = p_j < P(q_i + q_j)$. Then both firms are capacity-constrained; i.e., $q_i = q_i$ and $q_j = q_j$. But then raising price makes firms better off. ■

**Lemma 3** In a pure-strategy equilibrium, $p_i \geq P(R_i(q_j) + q_j)$. Firms will not charge lower than the price that corresponds to the optimal reaction to the other firm’s capacity.
**Proof.** Note that by the same argument as in the previous proof, \( p_i = p_j \) in equilibrium. We claim that in a pure-strategy equilibrium, \( p_i \geq P(R_i(\bar{q}_j) + \bar{q}_j) \). Suppose not, \( p_i = p_j < P(R_i(\bar{q}_j) + \bar{q}_j) \). If firm \( i \) is capacity-constrained, for some \( \epsilon > 0 \), \( (p_i + \epsilon)\bar{q}_i > p_i\bar{q}_i \). If firm \( i \) is not capacity-constrained, firm \( j \) is (at least one of them must be constrained, because otherwise they would undercut). \( p_i < P(R_i(\bar{q}_j) + \bar{q}_j) \) implies the corresponding level of output \( q_i > R_i(\bar{q}_j) \), and firm \( i \)'s profit:

\[
\text{efficient rationing} \quad p_i \left( D(p_i) - \bar{q}_j \right) = P(q_i + \bar{q}_j)q_i > P(R_i(\bar{q}_j) + \bar{q}_j)R_i(\bar{q}_j),
\]

a contradiction (to the definition of \( R_i(\cdot) \)).

These two lemmas imply that a pure-strategy equilibrium exists only if: \( \bar{q}_i \leq R_i(\bar{q}_j) \). Suppose not, the first lemma suggests \( p_i = P(\bar{q}_i + \bar{q}_j) \). Then \( p_i < P(R_i(\bar{q}_j) + \bar{q}_j) \), a contradiction to the second lemma.

The reason to lower the price is to sell more. But at \( p_i = p_j = P(\bar{q}_i + \bar{q}_j) \), they simply cannot. Raising the price and sell less? \( q_i < \bar{q}_i \leq R_i(\bar{q}_j) \) cannot be in equilibrium.

### 5.5.2 The choice of capacity

We now add a prior stage that firms choose capacities simultaneously. Let \( c_0 > 0 \) be the unit cost of capacity installation. In particular, we show that Cournot outcome when the capacity cost is not sunk is a pure-strategy equilibrium. The reaction function \( \tilde{R}(q_j) \) when the capacity cost is not sunk is defined as (note that due to symmetry, \( \tilde{R}(\cdot) = \tilde{R}_i(\cdot) = \tilde{R}_j(\cdot) \)):

\[
\arg \max_{q_i} [P(q_i + q_j) - c_0 - c]q_i.
\]

Denote \( q^{**} \) as the intersection of two reaction functions. Suppose firm \( j \) plays \( q^{**} \). Firm \( i \)'s optimal response is to play \( \tilde{R}(q^{**}) = q^{**} \), and the equilibrium price is: \( P(2q^{**}) \).

Conclusion: the Cournot equilibrium with cost \( c_0 + c \) is the equilibrium in the first stage capacity game; the second stage stage equilibrium price is \( P(2q^{**}) \).

Therefore, given the two-stage capacity-constrained-pricing-game interpretation of the
Cournot outcome, what we mean by quantity competition is really a choice of scale that determines the firm’s cost functions and thus determines the conditions of price competition. Capacity (and others) can be used as a competition soften instrument. Simple Bertrand and Cournot models should not be seen as two competing models giving contradictory predictions of the outcome of competition in a given market. Rather they are meant to describe markets with different cost structures. The Bertrand model may be a good approximation for industries with fairly flat marginal costs; the Cournot model may be better for those with sharply rising marginal cost.

5.6 Simultaneous price-quantity games

Same as in the two-stage capacity-constrained pricing games with “large” capacities, a simultaneous price-quantity game has no pure-strategy equilibrium. Let’s consider two firms choose $p$ and $q$ without first observing the choices made by their competitors. Let the marginal cost of output be the constant $c$ (constant returns to scale). Note a simultaneous price-quantity game can be interpreted as a two-stage capacity-constrained pricing game where capacities are imperfectly observable.

To see that there is no pure-strategy equilibrium, note first that if such an equilibrium exists, it has to the case that $p_i = p_j = p$. Note also, $p = c$. But $p_i = p_j = c$ cannot be an equilibrium either. This is because at the competitive price, at least one firm would supply strictly less than $D(c)$. This implies the other firm can raise its price slightly and makes a positive profit.

Gertner (1985) shows that a unique mixed-strategy equilibrium exists. He shows that in equilibrium, firms make zero expected profit. It is similar to the Bertrand equilibrium where firms make no profits. It also resembles the Cournot equilibrium in that the expected price exceeds the competitive price $c$. The idea is that without a commitment to capacity, firms cannot commit itself “not to flood the market”. This results in intensive competition in the Bertrandian fashion.
5.7 Concentration indices and profitability

Let there be $n$ firms in the market, and $\alpha_i$ denote the market share for firm $i$; i.e., $\alpha_i = q_i / Q$, where $Q = \sum_{i=1}^{n} q_i$. In general, one can define a concentration index as: $R \equiv R(\alpha_1, ..., \alpha_n)$. In practice, there are three indices that are commonly used in empirical studies: 1. The $m$-firm concentration ratio; 2. The Herfindahl index (or Herfindahl-Hirshman Index); 3. The entropy index.

- Rank firms according to $\alpha_i$ such that: $\alpha_1 \geq ... \geq \alpha_n$. The $m$-firm concentration ratio adds up the $m$ highest shares in the industry: $R_m \equiv \sum_{i=1}^{m} \alpha_i$.
- The Herfindahl index is the sum of squared market shares: $R_H \equiv \sum_{i=1}^{n} \alpha_i^2$.
- The entropy index is the sum of market shares times their logarithm: $R_e = \sum_{i=1}^{n} \alpha_i \ln \alpha_i$.

Encaoua and Jacquemin (1980) suggest that a “good” concentration index, $R(\alpha_1, ..., \alpha_n)$, should at least have the following properties:

1. Symmetric between firms;
2. A mean preserving spread (Lorenz condition) increases the index; and,
3. When firms are symmetric with equal market shares ($\alpha_i = 1/n$, $\forall i$), an increase in the number of firms decreases the index.

Given these conditions (axioms), one can show that if the index is of the following form:

$$R(\alpha_1, ..., \alpha_n) = \sum_{i=1}^{n} \alpha_i h(\alpha_i),$$

where: $h(\cdot)$ is non-decreasing function such that $\alpha_i h(\alpha_i)$ is convex, then it is a good concentration index (in the sense of Encaoua and Jacquemin). Therefore, the Herfindahl index and the entropy index both are good. Although the $m$-firm index does not belong to this family, it also satisfies all the conditions.
Can we say something about the relationship between the concentration indices and profitability? If firms are symmetric, then: \( R_m = m/n, \ R_H = 1/n, \) and \( R_e = -\ln n. \) And, if firms play the quantity game (Cournot game, quantity competition), then there is a positive relationship between concentration index (through \( n \)) and profitability. However, if firms compete in the Bertrand fashion (compete in prices, price competition, Bertrand game), we know that market price and profitability have nothing to do with the concentration indices.

Asymmetry among firms does seem to suggest a positive relationship between concentration indices and profitability. For Bertrand competition, the firm with the lowest cost, takes over the whole market, makes some profits: \( (c_{\text{second highest}} - c_{\text{lowest}})D(c_{\text{second highest}}), \) and the concentration index is the highest. With symmetric firms, firms make no profits, and the concentration index is lower. For Cournot competition, the same positive relationship exists. It is best seen through an example.

Let there be two firms with constant marginal costs: \( c_i \) and \( c_j. \) The market demand is: \( D(p) = 1 - p. \) We know that: \( q_i = (1 - 2c_i + c_j)/3 \) and \( q_j = (1 - 2c_j + c_i)/3, \) so that \( Q = q_i + q_j = (2 - c_i - c_j)/3. \) Now consider a decrease in \( c_i \) with an increase in \( c_j \) (firms become more asymmetric) such that: \( c \equiv (c_i + c_j)/2 \) remains the same. Note that as long as there is no change in the average cost, total output \( Q \) remains the same. But \( q_i \) increases and \( q_j \) decreases and this causes a mean-preserving spread in market shares. Hence, concentration indices increase. The rest is to show that total profit also increases. Note that since \( Q \) remains the same, so is \( p. \) Hence, the total revenue remains the same. As long as we can show that total cost decreases, we are done. Total cost \( TC(c_i, c_j) \) can be expressed as:

\[
TC(c_i, c_j) = c_i q_i + c_j q_j \\
= \frac{c_i(1 - 2c_i + c_j)}{3} + \frac{c_j(1 - 2c_j + c_i)}{3} \\
= \frac{c_i + c_j - 2(c_i + c_j)^2 + 6c_i c_j}{3}.
\]
Since \((c_i + c_j)/2\) remains the same, we can re-write \(TC(c_i, c_j)\) as \(TC(c_i, c)\):

\[
TC(c_i, c) = \frac{2c - 8c^2 + 6c_i(2c - c_i)}{3} = \frac{2c - 8c^2}{3} + 2c_i(2c - c_i).
\]

Observe that \(TC(c_i, c)\) is a concave function of \(c_i\), and \(TC(\cdot, c)\) is maximized when \(c_i = c\). Therefore, when \(c_i < c\) and \(c_i\) decreases, \(TC(\cdot, \cdot)\) decreases.

The idea is clear: Cost asymmetries ⇒ output asymmetries, this increases the concentration index. At the same time, low-cost firms enjoy a rent, thus the industry profit increases.