Upstream Price Discrimination and Social Welfare in the Presence of Licensing

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Abstract

In this paper, we examine the welfare effect of price discrimination in the presence of fixed-fee licensing in a vertically related market with one upstream monopolist and $n$ downstream oligopolists. It is found that price discrimination by the upstream monopolist may significantly change the licensing behavior of an innovator. If the upstream monopolist is not allowed to price discriminate the downstream firms, the innovator will have an incentive to license its technology to fewer firms in order to suppress the rent to be extracted by the upstream monopolist. However, this incentive is absent when price discrimination is allowed. This incentive plays an important role in the welfare effects of upstream price discrimination. It is found that price discrimination by the upstream monopolist may raise social welfare as long as the efficiency gain from more licenses dominates the efficiency loss from input market distortion. This result is opposite to the general outcome of the literature when technology licensing is absent.

Key words: Price Discrimination, Licensing, Vertically related Markets

JEL Classification: L11, L24

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1. INTRODUCTION

There are policy disputes over price discrimination. Robinson (1933) first investigated the output effects of price discrimination with two distinct markets. If both markets have a linear demand and the monopolist has a constant marginal cost, price discrimination does not change the total output. Schmalensee (1981) extends Robinson’s results to an N-market case and points out that social welfare goes up only if price discrimination leads to a higher output level. Varian (1985) generalizes their result by incorporating into the model interdependent markets and increasing marginal cost. Both Schmalensee (1981) and Varian (1985) show that, given that market demands are linear, price discrimination always lowers social welfare since there is a negative welfare effect arising from output redistribution among markets. However, their result does not hold in a spatial framework. Hwang and Mai (1990) show that, with an endogenous plant location, price discrimination may enhance social welfare.

The literature on price discrimination focused primarily on output markets until Katz (1987) who was the first to discuss price discrimination in relation to input markets. He shows that price discrimination in input markets necessarily lowers social welfare as it distorts market efficiency by charging the efficient (inefficient) downstream firms a higher (lower) input price. DeGraba (1990) puts forward a similar argument, concluding that price discrimination would suppress downstream firms’ incentives to engage in R&D and thus lower social welfare. Yoshida (2000) uses a generalized model to show that an increase in the total output of the final good is a sufficient condition for welfare deterioration as it deepens the market inefficiency of the input markets. In what follows, we shall show that if there is an innovator in the
model who can license its technology to the downstream firms, price discrimination in the input markets can then raise final goods output and social welfare as well.

In practice, technology may be licensed by means of a fixed fee or a royalty rate or a combination of the two. In this paper, we shall focus on fixed-fee licensing only for two reasons. First, it is often costly or sometimes impossible for an innovator to monitor licensees’ output. Second, Kamien and Tauman (1984, 1986, 2002), and Kamien et al. (1992) have shown that the patentee would license to all the downstream firms if the technology is licensed by means of a royalty rate. Under such a circumstance, all the downstream firms in our model will become identical, and there will be no room for price discrimination.

Another feature of the present model is that we discuss licensing behaviors in vertically related markets. Our model is more general than the existing literature in the sense that it is more relevant to the real world practices since production for a final good usually involves several vertically related processes. However, most of the literature has ignored the importance of input markets. Arya and Mittendorf (2006) is the first paper to discuss licensing behaviors in vertically related markets. They assume that there is an upstream firm, as well as two downstream firms with different productivities in the model in which the upstream firm can engage in price discrimination among the downstream firms. It is found that the efficient downstream firm would license out its technology to the inefficient firm even if the innovation were drastic and the goods were homogeneous.

The main result of our paper is that price discrimination may raise the final outputs, and therefore the social welfare, mainly because price discrimination can induce the innovator to license its technology to more firms. The remainder of this paper is organized as follows. In Section 2, we briefly introduce the model setup and its game structure. The equilibrium with the upstream monopolist charges a uniform
pricing strategy is examined in Section 3 followed by the price discrimination equilibrium in Section 4. A comparison between the two pricing regimes and their impacts on social welfare are discussed in Section 5. We present a brief conclusion in Section 6.

2. A FIXED-FEE LICENSING MODEL

Assume that there is an outside innovator who would like to license his advanced production technology to downstream firms and charge a fixed fee. There are $n$ downstream firms producing a homogeneous product and competing in quantities in the final goods market. The innovator will only license to $k$ out of the $n$ firms. With this advanced technology, each downstream firm can lower its process cost from $c$ to $c - \varepsilon$, where $\varepsilon$ stands for an innovation level. In addition, we assume that producing one unit of the final good needs one unit of the input. The monopolistic upstream firm, $M$, supplies its input to all downstream firms by charging either a common input price $w$ or discriminatory input prices, $\hat{w}$, for licensed downstream firms and $\hat{w}^*$ for non-licensed downstream firms. For simplicity, we assume that the production cost of the input is zero. The inverse demand function for the final goods is $P = P(Q) = a - Q$, where $Q = \sum_{i \in S} x_i + \sum_{i \notin S} y_i$ is the quantity of the final goods, $x_i$ ($y_i$) is the output supplied by the licensees (non-licensees), and $S$ represents the set of the licensees.

We use a three-stage game to illustrate the licensing behaviors in the vertically related market. In the first stage, the innovator determines the optimal number of licenses and the related licensing fee, $F$. Given this fixed licensing fee, the downstream firms decide whether to accept it or not. In the second stage, the upstream firm determines the input price. Finally, taking the licensing contract and the input prices as given, the downstream firms compete in a Cournot fashion. As usual, the
sub-game perfect equilibrium is derived by backward induction.

3. THE EQUILIBRIUM UNDER UNIFORM PRICING

In this section, we shall assume the innovation to be non-drastic,\(^1\) i.e., no downstream firms are driven out of the market, and focus on the uniform pricing case, that is, firm \(M\) can not price discriminate between the downstream firms. When innovation is non-drastic, all the non-licensees stay in the market. The profit functions for a representative licensee, \(\pi_i\), and a representative non-licensee, \(\pi_{i^*}\), are as follows:

\[
\pi_i(x_i, y_i; w, k) = \left[P(Q) - w - (c - \varepsilon)\right]x_i, \quad \forall i \in S ,
\]

\[
\pi_{i^*}(x_i, y_i; w, k) = \left[P(Q) - w - c\right]y_i, \quad \forall i \notin S .
\]

The net profit of the representative licensee (non-licensee) after technology licensing is \(\phi_i(x, y; w, k, \varepsilon) = \pi_i(x, y; w, k, \varepsilon) - F\) (\(\phi_{i^*}(x, y; w, k, \varepsilon) = \pi_{i^*}(x, y; w, k, \varepsilon)\)). The first-order conditions for profit maximization for each representative downstream firm are as follows:

\[
\frac{\partial \pi_i}{\partial x_i} = P(Q) + P'(Q)x_i - w - c + \varepsilon = 0, \quad \forall i \in S ,
\]

\[
\frac{\partial \pi_{i^*}}{\partial y_i} = P(Q) + P'(Q)y_i - w - c = 0, \quad \forall i \notin S .
\]

Solving these first-order conditions simultaneously, we can derive the equilibrium outputs as follows:

\[
x_i = x(w, k, \varepsilon) = \frac{a - w - c + \varepsilon(n - k + 1)}{1 + n}, \quad \forall i \in S,
\]

\[
y_i = y(w, k, \varepsilon) = \frac{a - w - c - \varepsilon k}{1 + n}, \forall i \notin S .
\]

From (1), it is straightforward to show that \(\varepsilon < (a - w - c)/k\) for the innovation being non-drastic. Summing over these first-order conditions, we can derive the

\(^1\) The definition of “non-drastic” innovation follows Arrow (1962).
aggregate first-order conditions for the representative licensees and non-licensees as follows:

\[
\sum_{i \in S} \frac{\partial \pi_i}{\partial x_i} = k \left[ P(Q) - w - c \right] + P'(Q)X = 0, \quad \forall i \in S, \\
\sum_{i \in S} \frac{\partial \pi_i^*}{\partial y_i} = (n - k) \left[ P(Q) - w - c \right] + P'(Q)Y = 0, \quad \forall i \not\in S,
\]

where \( X = \sum_{i \in S} x_i \) and \( Y = \sum_{i \in S} y_i \). It can be verified that both the second-order conditions and the stability condition hold. By totally differentiating the two aggregate first-order conditions, we obtain the following comparative static effects:

\[
\begin{align*}
X_u &= -k / (1 + n) < 0, \quad Y_u = -(n - k) / (1 + n) < 0, \\
X_e &= k(n - k + 1) / (1 + n) > 0, \quad Y_e = -k(n - k) / (1 + n) < 0, \\
X_\delta &= y + \varepsilon(n - k + 1) / (1 + n) > 0, \quad \text{and} \quad Y_\delta = -y - \varepsilon(n - k) / (1 + n) < 0.
\end{align*}
\]

Moreover, the corresponding comparative static effects for each individual firm are as follows:

\[
\begin{align*}
x_u &= -1 / (1 + n) < 0, \quad y_u = -1 / (1 + n) < 0, \quad x_e = (n - k + 1) / (1 + n) > 0, \\
y_e &= -k / (1 + n) < 0, \quad x_\delta = -\varepsilon / (1 + n) < 0, \quad \text{and} \quad y_\delta = -\varepsilon / (1 + n) < 0.
\end{align*}
\]

These comparative static effects show that raising the input price lowers the output of both licensees and non-licensees; and the larger the innovation, the greater (smaller) the output of the licensees (non-licensees). Moreover, increasing the number of the licenses also lowers the outputs of each licensee and non-licensee. Nevertheless, the aggregate outputs of the licensees increase with the number of licenses. In other words, the aggregate derived demand for the input increases with the number of licenses.

In the second stage, given the aggregate derived demand, firm \( M \) determines a uniform input price to maximize its profit. The profit function of firm \( M \) is specified as follows:

\[
\Omega(w; k, \varepsilon) = wQ(w; k, \varepsilon).
\]

Taking the first-order condition with respect to \( w \), we can derive the optimal input
price as follows:
\[ w = w(k, \varepsilon) = \frac{a - c + k \varepsilon}{2} \frac{k \varepsilon}{2n}, \quad (2) \]
Substituting (2) into (1), we can rewrite the final goods’ outputs as follows:
\[ x = x(k, \varepsilon) = \frac{n(a - c) + \varepsilon \left[ 2n(1 + n) - k(1 + 2n) \right]}{2n(1 + n)}, \quad (3) \]
\[ y = y(k, \varepsilon) = \frac{n(a - c) - \varepsilon k(1 + 2n)}{2n(1 + n)}. \]
The profit of the upstream firm is \( \Omega = \Omega(k, \varepsilon) = w(X + Y). \)

By totally differentiating the first-order condition, we can have the comparative static results of the equilibrium, \( w_i = dRw/dk > 0 \) and \( w_e = dw/d\varepsilon > 0 \). The second-order condition is also satisfied. From \( w_i > 0 \), we can know that the innovator can lower the input price by reducing the number of licenses. The intuition for this result is clear. As the number of licenses declines, the aggregate derived demand for the input decreases since there are now fewer technology-licensed (and more efficient) firms. As a result, the monopolistic upstream firm would charge a lower input price. On the other hand, the effects of the number of licenses and the innovation on the equilibrium outputs are as follows:
\[ \frac{dX}{dk} = X_w w_i + X_i > 0, \]
\[ \frac{dY}{dk} = Y_w w_i + Y_i < 0, \]
\[ \frac{dX}{d\varepsilon} = X_w w_e + X_e > 0, \]
\[ \frac{dY}{d\varepsilon} = Y_w w_e + Y_e < 0. \]
Raising the number of licenses increases (decreases) the aggregate outputs of the licensees (non-licensees). The greater the innovation, the larger (smaller) are the aggregate outputs of the licensees (non-licensees). It can also be verified that \( dx/\varepsilon < 0, \ dy/\varepsilon < 0, \ dx/\varepsilon > 0, \) and \( dy/\varepsilon < 0 \).
It is worth mentioning that for the innovation to be non-drastic, the following inequality must hold, \[ k < n(a - c) / \varepsilon (1 + 2n) \equiv \bar{k} \], that is the innovator cannot license its technology to too many firms. Given this constraint, the innovator determines the optimal number of licenses and the related fixed licensing fee in the first stage. The optimal licensing fee is equal to the difference in profit for each licensee between accepting and rejecting the licensing offer, that is, \[ F \leq \pi(k, \varepsilon) - \pi^*(k - 1, \varepsilon) \]. Given that the innovator has all the bargaining power and can extract all the rent from the licensees, the profit maximization problem of the innovator for non-drastic innovation is as follows:

\[
\max_k \Lambda(k; \varepsilon) = k[\pi(k; \varepsilon) - \pi^*(k - 1; \varepsilon)] \\
\text{st. } k \leq \bar{k}.
\]

The first derivative is as follows:

\[
\frac{d\Lambda}{dk} = \left[ (P(k) - c + \varepsilon)x(k) - (P(k - 1) - c)y(k - 1) \right] + \left[ -w(k)x(k) + w(k - 1)y(k - 1) \right] \\
+ k \left[ P'(Q)x(k) \frac{dy(k)}{dk} - P'(Q)y(k - 1) \frac{dx(k - 1)}{dk} \right] - \left[ w_1x(k) - w_2y(k - 1) \right].
\]

The first two brackets on the RHS represent the fixed-fee revenue effect which is positive. By licensing to an additional downstream firm, the innovator can acquire additional fixed-fee revenue. However, part of the rent from the technology licensing goes to the upstream firm through a higher input price, as represented by the second bracket of the fixed-fee revenue effect. We call this the upstream firm’s rent-extracting effect. The last two terms in the brace on the RHS represent the licensing number effect. They are negative. The first part is related to the revenue. If the innovator licenses to an extra downstream firm, the profits of each licensee will become lower, as will the fixed-fee revenue acquired by the innovator. The second part concerns the cost. As the number of the licenses declines, the upstream firm will
lower the input price, which will increase the profits of the downstream firms. We call this the cost-saving effect. This cost-saving effect combined with the upstream firm’s rent-extracting effect is referred to as the input price effect. Compared to Kamien and Tauman (1986), we find that licensing is less profitable in a vertically related market.

We summarize the above results in the following proposition:

**Proposition 1. By licensing to fewer firms, the innovator can suppress the upstream firm’s rent-extracting effect and intensify the cost-saving effect.**

Setting \( d\Lambda/dk = 0 \), we can derive the optimal number of licenses for non-drastic innovation as follows:

\[
k_r = \begin{cases} 
n \\
\frac{2n(a-c) + \varepsilon[1+2n(2+n)]}{4(1+2n)} \\
\end{cases}
\]

\[
k^I = \begin{cases} 
n \\
\frac{2n(a-c) + \varepsilon[1+2n(2+n)]}{4(1+2n)} \\
\end{cases}
\]

if \( 0 < \varepsilon \leq \frac{2n(a-c)}{6n^2-1} \)

\[
\frac{2n(a-c)}{6n^2-1} < \varepsilon \leq \bar{\varepsilon};
\]

(4)

where subscript \( F \) denotes the variables that are associated with the equilibrium of the first-stage game and superscript \( I \) is used to denote the variables that are associated with the interior solution. Furthermore, \( k^I \leq \bar{k} \) if and only if \( \varepsilon \leq 2n(a-c)/(1+4n+2n^2) = \bar{\varepsilon} \). By totally differentiating the first-order condition, we can obtain the comparative static effects as follows: \( dk/d\varepsilon < 0 \). The larger the innovation is, the smaller the number of licenses will be.

Substituting (4) into \( w, x \) and \( y \) in (2) and (3), we can obtain the equilibrium input prices and the final outputs for the downstream firms as follows:

\[
w_r = \begin{cases} 
\frac{a-c + \varepsilon}{2} \\
\frac{2n(3+4n)(a-c) + \varepsilon(1+4n+2n^2)}{8n(1+2n)} \\
\end{cases}
\]

if \( 0 < \varepsilon \leq \frac{2n(a-c)}{6n^2-1} \)

\[
\frac{2n(a-c)}{6n^2-1} < \varepsilon < \bar{\varepsilon};
\]


The equilibrium downstream firms’ profits are

\[ \phi_r = \pi_r - F = (x_r)^2 - F, \]

\[ \phi_r^* = \pi^*_r = (y_r^*)^2, \]

where

\[ F = \left\{ \begin{array}{ll}
\frac{a-c+\varepsilon}{2(1+n)} & \text{if } 0 < \varepsilon < \frac{2n(a-c)}{6n^2-1} \\
\frac{2n(a-c)+\varepsilon(-1+4n+6n^2)}{8n(1+n)} & \text{if } 2n(a-c) < \varepsilon < \varepsilon, \\
\frac{a-c-\varepsilon(1+2n)}{2(1+n)} & \text{if } 0 < \varepsilon < \frac{2n(a-c)}{6n^2-1} \\
\frac{2n(a-c)-\varepsilon(1+4n+2n^2)}{8n(1+n)} & \text{if } 2n(a-c) < \varepsilon < \varepsilon.
\end{array} \right. \]

The equilibrium profit of the innovator on the other hand is \( \Lambda_r = k_r F \), and the equilibrium profit of the monopolistic upstream firm is \( \Omega_r = w_r Q_r \), where

\[ Q_r = k x_r + (n-k) y_r. \]

4. THE EQUILIBRIUM UNDER THIRD-DEGREE PRICE DISCRIMINATION

Under this regime, the upstream firm can charge different input prices to licensees (\( \hat{w} \)) and non-licensees (\( \hat{w}^* \)), where the symbol “hat” stands for the case where there is price discrimination. The profit functions of the downstream firms are as follows:

\[ \hat{\pi}_i(\hat{x}, \hat{y}; \hat{w}, \hat{k}) = \left[ P(\hat{Q}) - \hat{w} - (c-\varepsilon) \right] \hat{x}_i, \quad \forall i \in S, \]

\[ \hat{\pi}_i^*(\hat{x}, \hat{y}; \hat{w}, \hat{k}) = \left[ P(\hat{Q}) - \hat{w}^* - c \right] \hat{y}_i, \quad \forall i \notin S. \]

The net profit of the licensee (non-licensee) is

\[ \hat{\phi}(\hat{x}, \hat{y}; \hat{w}, \hat{k}, \hat{\varepsilon}) = \hat{\pi}_i(\hat{x}, \hat{y}; \hat{w}, \hat{k}, \hat{\varepsilon}) - \hat{F}. \]
Choosing \( \hat{x}_i \) to maximize \( \hat{x}_i^* \) and \( \hat{y}_i \) to maximize \( \hat{y}_i^* \), we can derive the first-order conditions for profit maximization for the downstream firms as follows:

\[
\frac{\partial \hat{x}_i}{\partial \hat{x}_i} = P(\hat{Q}) + P'(\hat{Q}) \hat{x}_i - \hat{w} - c + \varepsilon = 0, \quad \forall i \in S,
\]

\[
\frac{\partial \hat{y}_i^*}{\partial \hat{y}_i^*} = P(\hat{Q}) + P'(\hat{Q}) \hat{y}_i - \hat{w}^* - c = 0, \quad \forall i \notin S.
\]

Solving these first-order conditions simultaneously, we can have the equilibrium outputs as follows:

\[
\hat{x}_i = \hat{x}(\hat{w}, \hat{w}^*, \hat{k}, \varepsilon) = \frac{a - (n - \hat{k} + 1)\hat{w} + (n - \hat{k})\hat{w}^* - c + \varepsilon(n - \hat{k} + 1)}{1 + n}, \quad \forall i \in S,
\]

\[
\hat{y}_i = \hat{y}(\hat{w}, \hat{w}^*, \hat{k}, \varepsilon) = \frac{a + \hat{k}\hat{w} - (\hat{k} + 1)\hat{w}^* - c + \varepsilon\hat{k}}{1 + n}, \quad \forall i \notin S.
\]

It is straightforward to show that the second-order condition and the stability condition are both satisfied. Summing over these first-order conditions among licensees and non-licensees separately and totally differentiating the two aggregate first-order conditions, we can obtain the following comparative static effects:

\[
\hat{X}_a = -k(n - k + 1)/(1 + n) < 0, \quad \hat{Y}_a = k(n - k)/(1 + n) > 0,
\]

\[
\hat{X}_{a^*} = k(n - k)/(1 + n) > 0, \quad \hat{Y}_{a^*} = -(1 + k)(n - k)/(1 + n) < 0,
\]

\[
\hat{X}_e = k(n - k + 1)/(1 + n) > 0, \quad \hat{Y}_e = -(n - k)/(1 + n) < 0,
\]

\[
\hat{X}_\ell = [(n - k + 1)x + ky]/(1 + n) > 0, \text{ and } \hat{Y}_\ell = -[(n - k)x + (k + 1)y]/(1 + n) < 0.
\]

Moreover, the corresponding comparative static effects for each individual firm are as follows:

\[
\hat{x}_a = -(n - k + 1)/(1 + n) < 0, \quad \hat{y}_a = k/(1 + n) > 0, \quad \hat{x}_{a^*} = (n - k)/(1 + n) > 0,
\]

\[
\hat{y}_{a^*} = -(1 + k)/(1 + n) < 0, \quad \hat{x}_e = (n - k + 1)/(1 + n) > 0, \quad \hat{y}_e = -k/(1 + n) < 0,
\]

\[
\hat{x}_\ell = -(x - y)/(1 + n) < 0, \text{ and } \hat{y}_\ell = -(x - y)/(1 + n) < 0.
\]

In the second stage, the upstream firm determines the optimal input price to
maximize profits, given the derived demand. The profit function of the upstream firm is as follows:

\[ \hat{\Omega}(\hat{w}, \hat{w}^*; \hat{k}, \varepsilon) = \hat{w} \hat{X} (\hat{w}, \hat{w}^*; \hat{k}, \varepsilon) + \hat{w}^* \hat{Y} (\hat{w}, \hat{w}^*; \hat{k}, \varepsilon). \]

The first-order conditions for profit maximization are as follows:

\[ \hat{\Omega}_w = \hat{X} + \hat{w} \hat{X}_w + \hat{w}^* \hat{Y}_w = 0, \]
\[ \hat{\Omega}_{w^*} = \hat{Y} + \hat{w} \hat{X}_{w^*} + \hat{w}^* \hat{Y}_{w^*} = 0. \]

By simultaneously solving the two first-order conditions, we can derive the input prices as follows:

\[ \hat{w} = \hat{w}(\hat{k}, \varepsilon) = \frac{a - c + \varepsilon}{2} \quad \text{and} \quad \hat{w}^* = \hat{w}^*(\hat{k}, \varepsilon) = \frac{a - c}{2}. \quad (6) \]

By proceeding as before, we can derive the following comparative static effects:

\[ \hat{w}_\varepsilon > 0, \quad \hat{w}^*_\varepsilon = 0, \quad \hat{w}_\hat{k} = 0 \quad \text{and} \quad \hat{w}^*_\hat{k} = 0. \]

As the innovation becomes greater, the input price of the licensees (non-licensees) becomes higher (is unaffected), which leads to the following lemma:

**Lemma 1. If the upstream firm can practice price discrimination, the innovator cannot affect the input price through the number of licenses.**

Given the linear demand for the final good, the derived demands are independent of each other and are unaffected by the numbers of licenses when the upstream firm engages in price discrimination. This is the main difference between this regime and the uniform pricing regime. Furthermore, the effects of the number of licenses and the innovation on outputs are as follows:

\[ \frac{d\hat{X}}{d\varepsilon} = \hat{X}_\varepsilon \hat{w}_\varepsilon + \hat{X}_\varepsilon > 0, \]
\[ \frac{d\hat{X}}{d\hat{k}} = \hat{X}_\hat{k} > 0, \]
\[
\frac{d\hat{\gamma}_i}{ds} = \hat{Y}_i \hat{\psi}_e + \hat{\gamma}_e < 0, \\
\frac{d\hat{\gamma}_i}{dk} = \hat{Y}_i < 0.
\]

The greater the innovation is, the larger the aggregate outputs of the licensees and the smaller the aggregate outputs of the non-licensees will be. An increase in the number of licenses will also increase the aggregate outputs of the licensees and decrease the aggregate outputs of the non-licensees. Moreover, it is possible to derive that, 
\[
d\hat{x}/d\varepsilon > 0, \quad d\hat{x}/d\hat{k} < 0, \quad d\hat{\gamma}/d\varepsilon < 0 \quad \text{and} \quad d\hat{\gamma}/d\hat{k} < 0. \]

By substituting (6) into (5), we can rewrite the final goods' outputs as follows:
\[
\hat{x} = \hat{x}(\hat{k}, \varepsilon) = \frac{a - c + \varepsilon(n \hat{k} - 1)}{2(1+n)} \quad \text{and} \quad \hat{\gamma} = \hat{\gamma}(\hat{k}, \varepsilon) = \frac{a - c - \varepsilon \hat{k}}{2(1+n)}.
\]

As \( \hat{\gamma} \geq 0 \) based on the assumption of non-drastic innovation, it is required that the optimal licensing number (to be derived later) satisfy the following inequality:
\[
\hat{k} \leq (a - c)/\varepsilon.
\]

In the first stage, the innovator determines the optimal license number to maximize its profits. The profit function of the innovator is as follows:
\[
\hat{\Lambda}(\hat{k}; \varepsilon) = \hat{k}[\hat{x}(\hat{k}; \varepsilon) - \hat{x}^*(\hat{k} - 1; \varepsilon)]
\]

\[st. \hat{k} \leq (a - c)/\varepsilon.\]

The first-order condition for profit maximization may be derived as follows:
\[
\frac{d\hat{\Lambda}}{d\hat{k}} = \left[ (P(\hat{k}) - c + \varepsilon) \hat{x}(\hat{k}) - (P(\hat{k} - 1) - c) \hat{\gamma}(\hat{k} - 1) \right] + \left[ -\hat{w}(\hat{k}) \hat{x}(\hat{k}) + \hat{w}^*(\hat{k} - 1) \hat{\gamma}(\hat{k} - 1) \right]
\]

\[+\hat{k} \left[ P'(\hat{\gamma}) \hat{x}(\hat{k}) \frac{d\hat{\gamma}_i}{dk} - P'(\hat{\gamma}) \hat{\gamma}(\hat{k} - 1) \frac{d\hat{\gamma}_i}{dk} \right].
\]

By comparing this equation with the one derived under the uniform pricing case, we find that they are similar in terms of their effects except in regard to the cost-saving effect. Since \( \hat{w}_i = 0 \) and \( \hat{w}^*_i = 0 \), there is no cost-saving effect under discriminatory pricing. This together with the result in Lemma 1, which states that
reducing the number of licenses has no effect on the input price, leads to the following conclusion: the innovator will tend to license his technology to more firms under price discrimination than under uniform pricing. We can therefore establish the following proposition:

**Proposition 2.** Given the same innovation level, the innovator tends to license to more firms under discriminatory pricing than under uniform pricing.

Solving the above first-order condition for profit maximization, we can derive the optimal number of licenses as follows:

\[
\hat{k}_r = \begin{cases} 
\frac{n}{2} & \text{if } 0 < \varepsilon \leq \frac{2(a - c)}{-2 + 3n}, \\
\frac{a - c + n + 2}{4(1 + n)} & \text{if } \frac{2(a - c)}{-2 + 3n} < \varepsilon \leq \frac{2(a - c)}{2 + n}.
\end{cases}
\] (8)

Substituting (8) into \( \hat{x} \) and \( \hat{y} \) in (7), we can derive the equilibrium outputs of the downstream licensed and non-licensed firms as follows:

\[
\hat{x}_r = \begin{cases} 
\frac{a - c + \varepsilon}{2(1 + n)} & \text{if } 0 < \varepsilon \leq \frac{2(a - c)}{-2 + 3n}, \\
\frac{2(a - c) + \varepsilon(2 + 3n)}{8(1 + n)} & \text{if } \frac{2(a - c)}{-2 + 3n} < \varepsilon \leq \frac{2(a - c)}{2 + n}.
\end{cases}
\]

\[
\hat{y}_r = \begin{cases} 
\frac{a - c - n\varepsilon}{2(1 + n)} & \text{if } 0 < \varepsilon \leq \frac{2(a - c)}{-2 + 3n}, \\
\frac{2(a - c) - \varepsilon(1 + n)}{8(1 + n)} & \text{if } \frac{2(a - c)}{-2 + 3n} < \varepsilon \leq \frac{2(a - c)}{2 + n}.
\end{cases}
\]

By substituting the outputs in the profit functions, we can derive the optimal fixed fee charged by the innovator as follows:

\[
\hat{F} = \begin{cases} 
\frac{\varepsilon n[2(a - c) - \varepsilon(n - 2)]}{4(1 + n)^2} & \text{if } 0 < \varepsilon \leq \frac{2(a - c)}{-2 + 3n}, \\
\frac{\varepsilon n[2(a - c) + \varepsilon(2 + n)]}{8(1 + n)^2} & \text{if } \frac{2(a - c)}{-2 + 3n} < \varepsilon \leq \frac{2(a - c)}{2 + n}.
\end{cases}
\]
The equilibrium profit of the innovator is therefore $\hat{\Lambda}_f = \hat{k}_f \hat{F}$, and the equilibrium profit of the monopolistic upstream firm is $\hat{\Omega}_f = \hat{w}_f \hat{X}_f + \hat{w}_f^* \hat{Y}_f$.

5. SOCIAL WELFARE UNDER UNIFORM AND DISCRIMINATORY PRICING

We are interested in the comparison of social welfare under uniform pricing and price discrimination in the input market. It is generally believed in the literature on input price discrimination that social welfare is higher under uniform pricing than under price discrimination. The intuition goes as follows. When price discrimination is allowed, the more efficient firms are charged with a higher input price which creates a distortion in output allocation and therefore lowers the social welfare level. See for example, Yoshida (2000). In this paper, we have shown that with technology licensing from an outside innovator, the social welfare under price discrimination is at least as good as that under uniform pricing. Before we compare the welfare under the two regimes, let us pay attention to the output, consumer surplus and profit effects first. By comparing the total outputs under the two regimes, we can establish the following proposition:

**Proposition 3.** The total output, and hence the consumer surplus, under price discrimination is no less than that under uniform pricing.

The total output under uniform pricing, $Q_f$, can be expressed as follows:

$$Q_f = \frac{n}{1 + n} \left\{ a - \left( w_f + c - \frac{k_f}{n} \varepsilon \right) \right\}.$$  

By proceeding as before, we can derive the total output under discriminatory pricing, $\hat{Q}_f$, as follows:
\[
\hat{Q}_r = \frac{n}{1+n}\left\{ a - \left( \frac{\hat{k}_r \hat{w}_r + (n - \hat{k}_r) \hat{w}_r^*}{n} + c \frac{\hat{k}_r}{n} \right) \right\}.
\]

Let
\[
\bar{w}_r \equiv \left[ \frac{\hat{k}_r \hat{w}_r + (n - \hat{k}_r) \hat{w}_r^*}{n} \right].
\]
By substituting (6) into \( \bar{w}_r \), this leads to:
\[
\bar{w}_r = \frac{a - c + \hat{k}_r \varepsilon}{2n}.
\]

By comparing \( \bar{w}_r \) with \( w_r \), it is easy to show that if \( k_r = \hat{k}_r \), then \( w_r = \bar{w}_r \). As \( k \) decreases, the average production cost increases, and hence the total output decreases. Since the number of licenses under discriminatory pricing is no less than that under uniform pricing, we have \( \hat{Q}_r \geq Q_r \), which implies that the consumer surplus under discriminatory pricing is also greater than that under uniform pricing.

This result for output differs from Katz (1987), DeGraba (1990), and Yoshida (2000). They assume no technology licensing and conclude that total output remains unchanged under price discrimination and uniform pricing, as the average production cost is unchanged. However, in our model, due to the technology licensing, the innovator will license to more downstream firms, which leads to a lower average production cost under price discrimination. As a result, the total output is greater when price discrimination is allowed.

Discriminatory pricing definitely benefits the input monopolist. However, it is not clear how it affects the profits of the downstream firms. Let
\[
\Delta \Phi(k_r, \hat{k}_r; \varepsilon) = \Phi(k_r; \varepsilon) - \Phi(\hat{k}_r; \varepsilon),
\]
where \( \Phi(k_r; \varepsilon) \) is the profits of the downstream firms under uniform pricing and can be specified as follows:
\[
\Phi(k_r; \varepsilon) = k_r [\phi_r(k_r; \varepsilon)] + (n - k_r) [\phi^*_r(k_r; \varepsilon)] = k_r [\pi^*_r(k_r - 1; \varepsilon)] + (n - k_r) [\pi^*_r(k_r; \varepsilon)].
\]
Similarly, we can also define the profits of the downstream firms under discriminatory
pricing as follows:

\[ \hat{\Phi}(\hat{k}_r; \varepsilon) = \hat{k}_r \left[ \hat{\phi}_r(\hat{k}_r; \varepsilon) \right] + \left( n - \hat{k}_r \right) \left[ \hat{\pi}^{*}_r(\hat{k}_r; \varepsilon) \right] = \hat{k}_r \left[ \hat{\Phi}^{*}_r(\hat{k}_r - 1; \varepsilon) \right] + \left( n - \hat{k}_r \right) \left[ \hat{\Phi}^{*}_r(\hat{k}_r; \varepsilon) \right] \]

From the two profits, we can show that if \( 0 < \varepsilon \leq 2n(a - c)/(6n^2 - 1) \), \( k_r = \hat{k}_r = n \), and also \( \Delta \Phi(n, n; \varepsilon) = n \left[ \pi^{*}_r(n - 1; \varepsilon) - \pi^{*}_r(n - 1; \varepsilon) \right] < 0 \) since the monopolistic upstream firm charges the less efficient firm a lower input price. Furthermore, it is possible to show that if \( 2n(a - c)/(6n^2 - 1) < \varepsilon \leq \bar{\varepsilon} \), then \( \Delta \Phi(k_r, \hat{k}_r; \varepsilon) < 0 \). Based on the above results, we can establish the following proposition:

**Proposition 4.** Price discrimination increases the profits of the downstream industry.

Finally, we can establish the following proposition:

**Proposition 5.** With technology licensing, third-degree price discrimination in an upstream market can improve social welfare.

Social welfare is defined as total benefit net of total cost. Total benefit is in proportion to total output. From Proposition 3, the total benefit under discriminatory pricing is higher than that under uniform pricing. Conversely, price discrimination encourages the innovator to license to more firms which reduces the average production cost and thus the total cost as well. It can be derived that \( \hat{SW}_f = SW_f \) if \( 0 < \varepsilon \leq 2n(a - c)/(6n^2 - 1) \), and \( \hat{SW}_r > SW_r \) if \( 2n(a - c)/(6n^2 - 1) < \varepsilon \leq \bar{\varepsilon} \), where \( SW \) stands for social welfare consisting of the consumer surplus, the overall downstream profits, the innovator’s profit, and the upstream monopolist’s profit.

\(^2\) See the appendix for the proof.
Katz (1987), DeGraba (1990), and Yoshida (2000) show that the total output is identical under uniform and discriminatory pricing regimes; however, the distribution of production is distorted as efficient firms are charged a high input price under price discrimination. This makes the social welfare lower under price discrimination. Moreover, Yoshida (2000) shows that an increase in total output is a sufficient condition for welfare deterioration when price discrimination is allowed. However, if we introduce technology licensing into the model, all of the results are reversed.

6. CONCLUSIONS

In this paper, we examine the welfare effects of price discrimination in an input market in the presence of technology licensing. If the upstream firm adopts a uniform pricing strategy, the innovator can force the upstream monopolist to lower its input price by issuing fewer licenses. However, this is not the case when the upstream firm adopts a discriminatory pricing strategy, as the derived demand is independent of the marginal costs of the other downstream firms, which disconnect the influence of the innovator on the input price. Katz (1987) and DeGraba (1990) have shown that third-degree price discrimination by an upstream firm is welfare deteriorating as the upstream firm tends to charge efficient (inefficient) firms a higher (lower) input price which distorts the production efficiency. By contrast, we have shown that with technology licensing from an outside innovator, the innovator tends to issue more licenses to downstream firms which improves overall production efficiency and leads to higher social welfare under discriminatory pricing than under uniform pricing.

We have assumed that all the firms, upstream or downstream, are local firms. If we assume that the upstream firm is a foreign firm whose profit is thereby going to the foreign country and is not included in the domestic welfare calculation, our main
findings are reversed. That is, price discrimination in an input market hurts domestic social welfare, as the foreign upstream firm can extract more rent from the market under discriminatory pricing.

APPENDIX

In this appendix, we shall prove that if \( 2n(a - c)/(6n^2 - 1) < \varepsilon \leq \bar{\varepsilon} \), \( \Delta \Phi(k_r, \hat{k}_r; \varepsilon) > 0 \). Suppose that the number of the downstream firms is large and above the critical level, that is, \( n > 3 + \sqrt{10} \). We have \( k_r = k^i \) and \( \hat{k}_r = n \) if \( 2n(a - c)/(6n^2 - 1) < \varepsilon \leq 2(a - c)/(3n - 2) \); and \( k_r = k^i \) and \( \hat{k}_r = \hat{k}^i \) if \( 2(a - c)/(3n - 2) < \varepsilon \leq \bar{\varepsilon} \). On the contrary, for a small number of the downstream firms, that is, \( n \leq 3 + \sqrt{10} \), we have \( k_r = k^i \) and \( \hat{k}_r = n \) if \( 2n(a - c)/(6n^2 - 1) < \varepsilon \leq \bar{\varepsilon} \). From these results, we can derive \( d\Delta \Phi(k_r, \hat{k}_r; \varepsilon)/d\varepsilon > 0 \) and \( \Delta \Phi(k_r, \hat{k}_r; \varepsilon) < 0 \), which completes the proof.

REFERENCES


