Mid-Term Exam in Econometrics (II)

Date: March 29, 2006

Time Allowed: 2.5 hours

32 points

1. (4 marks) Consider the t statistic printed out by regression packages for the coefficient estimate. Explain in detail how you would conduct a simulation study to verify that this statistic actually has t distribution when the null hypothesis is true.

2. (4 marks) The consistency of b depends on the probability limits of the two terms $\frac{1}{T}X'X$ and $\frac{1}{T}X'\varepsilon$. Give examples of models where plim $\frac{1}{T}X'X$ is zero, finite and infinite. Give an intuitive explanation why b is (in)consistent in these cases.

3. (24 marks) The true model is $y_i = \mu+\varepsilon_i$, where ε_i is iid with the density $f(\varepsilon_i) = 1-|\varepsilon_i|$ for $-1 \leq \varepsilon_i \leq 1$. Now you have observed the data $y_1,\ y_2, \cdots, y_T$.

   (a) Determine the log-likelihood based on the postulated density function, and show how to obtain the ML estimate for μ from the log-likelihood.

   (b) Determine how to obtain a solution for the ML estimate μ using $y_1,\ y_2, \cdots, y_T$. In the case that you can not obtain the closed form, you need to think of the numerical solution. Be precise about your answers.

   (c) Suppose $T = 10$ which is small so that the standard asymptotic theory for ML estimate for μ is not reliable in this context. Explain how to devise a bootstrap procedure to construct the confidence interval for the ML estimate μ at 5% significance level.

   (d) Explain how you would conduct a simulation study to verify the confidence interval based on the bootstrap procedures in (c) is more reliable than that based on the asymptotic theory.

   (e) Suppose you do not know the true data generating process about the density function of the disturbance, but estimate μ using the normality assumption. Derive the Fisher information matrix in this case.

   (f) Continuing from the above, how would you expect the QML estimate in terms of efficiency, compared with the ML estimate? Argue with the results obtained from the above.

   (g) Now suppose the true model is $y_i = \mu + \beta x_i + \varepsilon_i$, where $x_i \sim nid(0, 4)$. However, using the least squares method, you estimated the model $y_i = \alpha + \epsilon_i$, ie. there is an omitted variable. Argue whether or not the OLS estimate for α is consistent. Explain intuitively.

   (h) Explain in detail how you conduct the simulation study to verify your argument aforementioned.