1. (3 points) Heights are normally distributed with means 1.7 meters (males) and 1.5 meters (females) and common variance .6 meter. Is it more likely that a sample has been drawn from the male population, if (a) the sample consists of a single person with height 1.6 meters, or (b) the sample consists of 6 persons with average heights 1.65 meters. Explain your reasoning.

2. (3 points) Comment on the following: MLE exploits the distribution information in estimation, whereas OLS does not. MLE is always superior to OLS.

3. True or false (explain why in any case)

   (a) (2 points) Consider a general non-linear regression, \( y_t = g(x_t; \beta) + u_t \), where \( u_t \) is n.i.d.(0, (\( \beta x_t \)^2)). \( \beta \) need not be estimated simultaneously with variance of \( u_t \).

   (b) (2 points) Consider the model \( y_t = x_t \beta + \sqrt{x_t} \gamma + u_t \), where \( u_t \) is i.i.d.(0, \( \sigma^2 \)), and \( x_t \) is n.i.d.(\( \mu, \sigma_x^2 \)). If \( y_t \) is regressed on \( \sqrt{x_t} \) alone to get \( \hat{\gamma} \), then the regression can not consistently estimate \( \gamma \) and \( \sigma^2 \).

4. Suppose that \( y_t = \alpha + \beta x_t + u_t \), where \( u_t \)'s are i.i.d. with p.d.f. \( f(u_t) = \lambda u_t^{-(\lambda+1)} \) where \( \lambda > 2 \) and \( 1 \leq u_t \leq \infty \).

   (a) (3 points) Are the OLS estimators of \( \alpha \) and \( \beta \) BLUE?

   (b) (3 points) Would prior knowledge of \( \lambda \) help in estimating \( \alpha \) and \( \beta \)? Why or why not?

   (c) (3 points) For \( \lambda \) unknown, explain what you would do to estimate \( \alpha \), \( \beta \) and \( \lambda \)?

5. Suppose a classical regression applied to \( y = \alpha + \beta x + \delta w + \epsilon \). Explain how to test \( \beta = \delta^2 \) using

   (a) (3 points) an ‘asymptotic’ t test.

   (b) (3 points) a Wald test.

6. Suppose \( x \) is a random variable with p.d.f. \( f(x) = k e^{kx} \) for \( x > 0 \), Given a sample of size \( N \),

   (a) (3 points) Find the MLE of \( k \);

   (b) (3 points) Use the Fisher information matrix to find the variance of the estimator;

   (c) (4 points) Show that the Wald and LM tests for the null hypothesis \( k = k_0 \) are identical.