

## Final Exam in Econometrics (II)

Date: June, 2005

Time Allowed: 3 hours

Answer all questions

46 marks

- (4 marks) Consider the regression model  $y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i$ . Suppose that you are concerned with that both  $x_2$  and  $x_3$  are measured with error, and that  $z_2$  and  $z_3$  are considered to be possible instruments for  $x_2$  and  $x_3$ , respectively. How would you perform a Hausman test to evaluate the presence or absence of measurement error?
- (4 marks) Suppose  $y = \beta x + \varepsilon$  where all the classical regression assumptions hold except that the variance of the error term  $\varepsilon$  is a constant  $K$  times  $x^2$ . Then the BLUE is the average of the  $y$  divided by the average of the  $x$ . True, false or uncertain. Explain.
- (8 marks) Suppose that  $y = (\alpha + \beta x)\varepsilon$  where the multiplicative error term  $\varepsilon$  has a mean equal to 1,  $E(\varepsilon) = 1$ . (a) How would you estimate  $\alpha$  and  $\beta$ ? *Hints:* Express  $\varepsilon$  as one plus a new error. (b) How would you estimate  $\alpha$  and  $\beta$ , if in addition you knew that  $\varepsilon$  is distributed normally? Be explicit.
- Suppose  $y = \beta x + \varepsilon$  and you wish to calculate the heteroskedasticity-consistent estimate of the variance of the OLS estimator. The error term  $\varepsilon$  has a mean 0 with possible heteroskedasticity of unknown form. Note that  $x$  is just a scalar. Define a transformation matrix  $P$  with the inverse of the OLS residuals on the diagonal and zeros elsewhere. Transform  $y$  and  $x$  to obtain  $y^* = Py$ , and  $x^* = Px$ , and create  $w = P^{-1}\varepsilon$ . (a) (4 marks) Show that the IV estimator of  $y^*$  regressed on  $x^*$ , using  $w^*$  as a set of instruments for  $x^*$ , is just  $\beta^{ols}$ . (b) (4 marks) Use the formula for the variance of the IV estimator to find the estimated variance-covariance matrix of this estimator. (c) (*Bonus, 4 marks*) How is the estimated variance-covariance matrix related to the White's heteroskedasticity-consistent estimate of the variance of the OLS estimator.
- (4 marks) Suppose the regression model applied to  $y = \alpha_0 + \alpha_1 x + \alpha_2 w + \varepsilon$  except that estimated values of  $w$  has been employed in the samples. If the measured  $w$  is the true  $w$  plus a random error distributed uniformly between 0 and 4, what are the implications for your estimates of the  $\alpha_i$ ?
- (12 marks) Consider  $y = \alpha x + \varepsilon$  where  $x$  is measured with error (Note: not intercept in the model). Some argue that an variable  $z$  taking value 1 if  $x$  is greater than median of  $x$ , or 1 otherwise might be a qualified instrument for  $x$ .
  - Explain how to test  $E(z\varepsilon) = 0$ , the validity of the instrument  $z$ .
  - Derive the IV estimator for  $\beta$  using the instrument  $z$ .
  - Show the consistency of the derived estimator.
- (6 marks) Explain how to conduct a simulation study to compare OLS and IV estimators in the context of measurement errors.