Final Exam in Econometrics (II)
Date: June, 2005
Time Allowed: 3 hours
Answer all questions
46 marks

1. (4 marks) Consider the regression model
   \[ y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i. \]
   Suppose that you are concerned with that both \( x_2 \) and \( x_3 \) are measured with error, and that \( z_2 \) and \( z_3 \) are considered to be possible instruments for \( x_2 \) and \( x_3 \), respectively. How would you perform a Hausmen test to evaluate the presence or absence of measurement error?

2. (4 marks) Suppose \( y = \beta x + \varepsilon \) where all the classical regression assumptions hold except that the variance of the error term \( \varepsilon \) is a constant \( K \) times \( x^2 \). Then the BLUE is the average of the \( y \) divided by the average of the \( x \). True, false or uncertain. Explain.

3. (8 marks) Suppose that \( y = (\alpha + \beta x)\varepsilon \) where the multiplicative error term \( \varepsilon \) has a mean equal to 1, \( E(\varepsilon) = 1 \). (a) How would you estimate \( \alpha \) and \( \beta \)? Hints: Express \( \varepsilon \) as one plus a new error. (b) How would you estimate \( \alpha \) and \( \beta \), if in addition you knew that \( \varepsilon \) is distributed normally? Be explicit.

4. Suppose \( y = \beta x + \varepsilon \) and you wish to calculate the heteroskedasticity-consistent estimate of the variance of the OLS estimator. The error term \( \varepsilon \) has a mean 0 with possible heteroskedasticity of unknown form. Note that \( x \) is just a scalar. Define a transformation matrix \( P \) with the inverse of the OLS residuals on the diagonal and zeros elsewhere. Transform \( y \) and \( x \) to obtain \( y^* = Py \), \( x^* = Px \), and create \( w = P^{-1}x \). (a) (4 marks) Show that the IV estimator of \( y^* \) regressed on \( x^* \), using \( w^* \) as a set of instruments for \( x^* \), is just \( \beta_{ols} \). (b) (4 marks) Use the formula for the variance of the IV estimator to find the estimated variance-covariance matrix of this estimator. (c) (Bonus, 4 marks) How is the estimated variance-covariance matrix related to the White’s heteroskedasticity-consistent estimate of the variance of the OLS estimator.

5. (4 marks) Suppose the regression model applied to \( y = \alpha_0 + \alpha_1 x + \alpha_2 w + \varepsilon \) except that estimated values of \( w \) has been employed in the samples. If the measured \( w \) is the true \( w \) plus a random error distributed uniformly between 0 and 4, what are the implications for your estimates of the \( \alpha_i \)?

6. (12 marks) Consider \( y = \alpha x + \varepsilon \) where \( x \) is measured with error (Note: not intercept in the model). Some argue that an variable \( z \) taking value 1 if \( x \) is greater than median of \( x \), or 1 otherwise might be a qualified instrument for \( x \).
   (a) Explain how to test \( E(z\varepsilon) = 0 \), the validity of the instrument \( z \).
   (b) Derive the IV estimator for \( \beta \) using the instrument \( z \).
   (c) Show the consistency of the derived estimator.

7. (6 marks) Explain how to conduct a simulation study to compare OLS and IV estimators in the context of measurement errors.