Economic Growth Models

● The Harrod- Domar Growth Model

If we assume that there is some direct economic relationship between the size of the total capital stock, K, and total GNP, Y－for example, if $3 of capital is always necessary to produce a $1 stream of GNP. Suppose that this relationship, known in economics as the **capital-output ratio,** is roughly 3 to 1. If we define the capital-output ratio as *k* and assume further that the national **saving ratio**, s, is a fixed proportion of national output. (e.g., 6%) and that total new investment is determined by the level of total savings.

1. Saving(S) is some proportion, *s*, of national income(Y) such that we have the simple equation

$$S=sY$$

2. Net investment (I)is defined as the change in the capital stock, *K*, and can be represented by ΔK such that

$$I=\Delta K$$

Because the total capital stock, *K*, bears a direct relationship to total national income or output, Y, as expressed by capital-output ratio, *k*, it follows that

$$\frac{K}{Y}=k$$

or

$$\frac{\Delta K}{\Delta Y} = k$$

or, finally,

$$\Delta K = k\Delta Y$$

net national saving, *S*, must equal net investment, *I*,

$$S=I$$

$$I=\Delta K = k\Delta Y$$

The “identity” of saving equaling investment

$$S=sY=k\Delta Y = \Delta K =I$$

or simply as

$$sY=k\Delta Y$$

$$\frac{\Delta Y}{Y} = \frac{s}{k}$$

$\frac{\Delta Y}{Y}$, represents the rate of change or rate of growth of GNP, the rate of growth of GNP ($\frac{\Delta Y}{Y}$) is determined jointly by the national saving ratio, *s*, and the national capital-output ratio, *k*. The more they can save and invest, the faster they can grow.

If we assume that the national capital-output ratio in some less developed country is, say, 3 and the aggregate saving ratio is 6% of GNP, this country can grow at rate of 2% per year

$$\frac{\Delta Y}{Y} = \frac{s}{k} = \frac{6\%}{3} = 2\%$$

Now if the national net savings rate can somehow be increased from 6% to, say, 15% GNP growth can be increased from 2% to 5%

$$\frac{\Delta Y}{Y} = \frac{s}{k} = \frac{15\%}{3} = 5\%$$

And the basic reason they didn't work was not because more saving and investment isn't a **necessary condition** for accelerated rates of economic growth — it is — but rather because it is not a **sufficient condition**.

● Solow Model

Special case of the Model

Assuming a C-D (Cobb-Douglas) production function,

$y = AK^{\alpha}L^{1-\alpha}$, Y = gross domestic product

K =stock of Capital
L = Labour
A = Technology
F denotes a function
α and $1-\alpha$ are positive parameters

• Marginal productivities are decreasing,

$$\ln y = \ln A + \alpha \ln K + (1-\alpha)\ln L$$

$$\Rightarrow \frac{\dot{y}}{y} = \frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}$$

$\frac{\dot{y}}{y} = \frac{\Delta Y}{Y}$, represents the rate of change or rate of growth of GNP

$\frac{\dot{A}}{A}$ is rate of growth of technology ; $\frac{\dot{K}}{K}$ is rate of growth of capital ; $\frac{\dot{L}}{L}$ is rate of growth of labor.

General Form

Assuming a production function,

$y = F(L, K, A)$ which exhibits constant returns to scale, let technology be represented by A, labor be represented by L, and capital be represented by K.

$$\frac{\frac{dy}{dt}}{y} = \frac{F_L \cdot L}{y}\frac{\frac{dL}{dt}}{L} + \frac{F_K \cdot K}{y}\frac{\frac{dK}{dt}}{K} + \frac{\frac{\partial F}{\partial A} \cdot A}{y}\frac{\frac{dA}{dt}}{A}$$

$$\Rightarrow \frac{\dot{y}}{y} = \varepsilon_L \frac{\dot{L}}{L} + \varepsilon_K \frac{\dot{K}}{K} + \phi \frac{\dot{A}}{A}$$

$\varepsilon_L = \dfrac{\partial y / y}{\partial L / L}$ is output elasticity of labor

$\varepsilon_K = \dfrac{\partial y / y}{\partial K / K}$ is output elasticity of capital

$\phi = \dfrac{\partial y / y}{\partial A / A}$ is output elasticity of technology